KTH, Matematik, Maurice Duits

SF2795 Fourier Analysis
Homework Assignment for Lecture 14

1. (3.3.3) Check that if $f \in \mathbb{L}^1$, if $\hat{f}$ is an entire functions of exponential type less than or equal to $2\pi T$, and if $B > T$, then
$$|\hat{f}(\gamma)| \leq \text{constant} \times \mathrm{e}^{2\pi BR\sin\theta}$$
on the close upper half place. *Hint:* Apply Phragmén -Lindelöf to $\mathrm{e}^{2\pi \mathrm{i} B\gamma}\hat{f}(\gamma)$.

2. (3.4.1) Check that a rational function $\hat{f} \in \mathbb{L}^2(\mathbb{R})$ is Hardy iff all its poles lie in the lower half plane.
Hint: Use Cauchy's Theorem to prove that $f = \hat{f}^{\vee} = 0$ for $x < 0$ if the poles lie in the lower half plane.

3. (3.4.2) Check that the functions
$$e_n = \frac{1}{\sqrt{\pi}(\mathrm{i}\gamma - 1)}\left(\frac{\mathrm{i}\gamma + 1}{\mathrm{i}\gamma - 1}\right)^n, \qquad n = 0, \pm 1, \pm 2, \ldots$$
form a unit perpendicular family.

4. (3.4.3) Check that $e_n : n \geq 0$ spans the Hardy space H^{2+}, while $e_n : n < 0$ spans H^{2-} for $n < 0$. *Hint:* Use exercise (3.4.1) to check that e_n belongs to H^{2+} for $n \geq 0$ and to H^{2-} for $n < 0$. Then prove that the whole family space $\mathbb{L}^2$. The fact that $(\mathrm{i}\gamma - 1)^{-1}(\mathrm{i}\gamma + 1)$ maps $\mathbb{R}$ onto the unit circle is helpful.