KTH, Matematik, Maurice Duits

SF2795 Fourier Analysis
Homework Assignment for Lecture 12

1. (2.7.9) Give a new proof of the Fourier inversion formula for $C^\infty_\downarrow(\mathbb{R})$ based on the series

$$\sum_{k\in\mathbb{Z}} f(x+kT) = \frac{1}{T}\sum_{n=-\infty}^{\infty} \hat{f}(n/T)\mathrm{e}^{2\pi \mathrm{i} xn/T}.$$

2. (a) (2.11.5) Check the identity in $\mathbb{R}^2$

$$\frac{1}{2\pi t}\sum_{Z} \exp(-|\omega|^2/2t) = \frac{1}{Area(T)}\sum_{Z'} \exp(-2\pi^2 t|\omega'|^2)$$

   (b) (2.11.6) Prove the formula of Gauss for the number $\#(n)$ of points on the standard lattice $\mathbb{Z}^2$ at distance $\sqrt{n}$ from the origin:

$$t^{-1/2}\sum_{n=0}^{\infty} \#(n)\exp(-\pi n/t) = t^{1/2}\sum_{n=0}^{\infty} \#(n)\exp(-\pi n t).$$

3. (2.11.7) Give an example showing that the inequality condition in Minkowski's Theorem can not be improved. *Hint:* $n = 2$ is enough.