ALPHA

$$\alpha_n(j) = P(\overbrace{x_1 --- x_n}^{X_1^n}, z_n = s_j)$$

$$= P(x_1 --- x_{n-1}, x_n, z_n = s_j) =$$

$$= \sum_{i=1}^{M} P(x_1 --- x_{n-1}, x_n, z_{n-1} = s_i, z_n = s_j) =$$

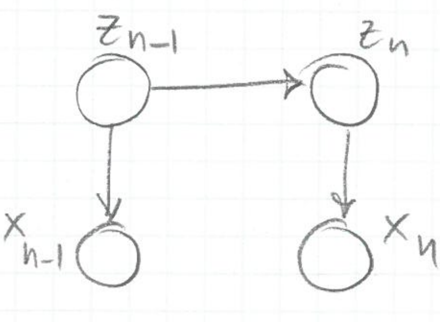
$$P(A, B) = P(A|B)\,P(B) \qquad A = x_n, z_n = s_j$$

$$B = X_1^{n-1}, z_{n-1} = s_i$$

$$= \sum_{i=1}^{M} P(x_n, z_n = s_j \mid X_1^{n-1}, z_{n-1} = s_i)\, \underbrace{P(X_1^{n-1}, z_{n-1} = s_i)}_{\alpha_{n-1}(i)}$$

$$A = x_n \quad B = z_n = s_j$$

$$= \sum_{i=1}^{M} \underbrace{P(x_n \mid z_n = s_j, \cancel{X_1^{n-1}}, \cancel{z_{n-1} = s_i})}_{\phi_j(x_n)}\, \underbrace{P(z_n = s_j \mid \cancel{X_1^{n-1}}\, z_{n-1} = s_i)}_{a_{ij}}\, \alpha_{n-1}(i)$$



$$= \sum_{i=1}^{M} \left(\alpha_{n-1}(i)\, a_{ij}\right)\, \phi_j(x_n)$$

BETA

$$\beta_n(i) \equiv P(x_{n+1}, \cdots x_N \mid z_n = s_i) =$$

$$= P\left(x_{n+1}, X_{n+2}^{N} \mid z_n = s_i\right) =$$

$$= \sum_{j=1}^{M} P\left(x_{n+1}, X_{n+2}^{N}, z_{n+1} = s_j \mid z_n = s_i\right)$$

$$P(A, B) = P(A \mid B)\, P(B) \qquad A = x_{n+1}$$

$$B = X_{n+2}^{N}, z_{n+1} = s_j$$

$$= \sum_{j=1}^{M} \underbrace{P\left(x_{n+1} \mid \cancel{X_{n+2}^{N}}, z_{n+1} = s_j, \cancel{z_n = s_i}\right)}_{\phi_j(x_{n+1})} P\left(X_{n+2}^{N}, z_{n+1} = s_j \mid z_n = s_i\right)$$

$$\begin{tikzcd}
z_n \arrow[r] \arrow[d] & z_{n+1} \arrow[d] \\
x_n & x_{n+1}
\end{tikzcd}$$

$$A = X_{n+2}^{N}$$

$$B = z_{n+1} = s_j$$

$$= \sum_{j=1}^{M} \phi_j(x_{n+1}) \underbrace{P\left(X_{n+2}^{N} \mid z_{n+1} = s_j, z_n = s_i\right)}_{\beta_{n+1}(j)} \underbrace{P\left(z_{n+1} = s_j \mid z_n = s_i\right)}_{a_{ij}}$$

$$= \sum_{j=1}^{M} \left[a_{ij}\, \phi_j(x_{n+1})\, \beta_{n+1}(j)\right]$$

GAMMA

$$P(A|B) = \frac{P(A B)}{P(B)}$$

$$\gamma_n(i,j) = P(z_{n-1}=s_i, z_n=s_j \mid X, \theta) =$$

$$= \frac{P(z_{n-1}=s_i, z_n=s_j, X \mid \theta)}{P(X\mid\theta)} \qquad P(X\mid\theta) = \sum_{k=1}^{M} \alpha_N(k)$$

Numerator!

$$P(z_{n-1}=s_i, z_n=s_j, X_1^{n-1}, x_n, X_{n+1}^N) =$$

$$P(A,B) = P(A|B)\,P(B) \qquad A = x_n, z_n=s_j, X_{n+1}^N$$

$$B = z_{n+1}=s_i, X_1^{n-1}$$

$$= P(x_n, z_n=s_j, X_{n+1}^N \mid z_{n-1}=s_i, \cancel{X_1^{n-1}})\; \underset{\alpha_{n-1}(i)}{P(X_1^{n-1}, z_{n-1}=s_i)} =$$

$$A = x_n, X_{n+1}^N \qquad B = (z_n=s_j)$$

$$= \alpha_{n-1}(i)\; P(x_n, X_{n+1}^N \mid z_n=s_j, \cancel{z_{n-1}=s_i})\; \underset{a_{ij}}{P(z_n=s_j \mid z_{n-1}=s_i)}$$

$$= \alpha_{n-1}(i)\, a_{ij}\; \underbrace{P(x_n \mid \cancel{X_{n+1}^N}, z_n=s_j)}_{\phi_j(x_n)}\; \underbrace{P(X_{n+1}^N \mid z_n=s_j)}_{\beta_n(j)}$$

$$\gamma_n(i,j) = \frac{\alpha_{n-1}(i)\, a_{ij}\, \phi_j(x_n)\, \beta_n(j)}{\sum_{k=1}^{M} \alpha_N(k)}$$