

KTH, Matematik, Maurice Duits

**SF2795 Fourier Analysis**  
**Homework Assignment for Lecture 9**

1. (Exercise: 2.1.2) Prove that the product  $\circ$  defined by

$$f \circ g(x) = \int_{-\infty}^{\infty} f(y)g(x-y)dx$$

is associative, commutative and verify the bound

$$\|f \circ g\|_1 \leq \|f\|_1 \|g\|_1$$

2. (Exercise: 2.1.3) Check that translation is continuous both in  $\mathbb{L}^1(\mathbb{R})$  and  $\mathbb{L}^2(\mathbb{R})$ , i.e. verify that in either space  $f_y(x) = f(x+y)$  is close to  $f$  for small  $|y|$ . (There is a hint in the book!)
3. (Exercise: 2.4.1) Give a proof of the fact that for any  $f \in \mathbb{L}^1(\mathbb{R})$

$$\lim_{|\gamma| \rightarrow \infty} \hat{f}(\gamma) = 0,$$

by using that

$$\hat{f}(\gamma) = - \int f(x)e^{-2\pi i(x-y)}dx = - \int f(x+y)e^{-2\pi x\gamma}dx$$

for  $y = 1/2\gamma$ . (There is hint in the book! The previous exercise is helpful).