Institutionen för matematik, KTH

Homework assignment to Lecture 8.

The Hilbert transform of a function $u(x) = \sum_{n=-\infty}^{\infty} \hat{u}(n) e_n(x)$, $u \in L^2(S^1)$ is defined as a function $v(x) = \sum_{n=-\infty}^{\infty} -i \operatorname{sign}(n)\, \hat{u}(n) e_n(x)$. Here, $\operatorname{sign}(n)$ has value 1 for $n \geq 1$, value -1 for $n \leq -1$ and the value 0 at $n = 0$.

Compute explicit formula for the Hilbert transform of the function u determined by equations $u(x) = x$ for $|x| < 1/2$ and $u(x+1) = u(x)$ for all x.

This example shows that the Hilbert transform of a bounded function is not necessarily bounded.