$\boxed{11.3}$

Type A: $\lambda_A = 100$ packets/s $L_A = 20$ bits $\Rightarrow E[x_A] = \frac{L_A}{C} = 2 \cdot 10^{-3}$ s

Type B: $\lambda_B = 20$ packets/s $L_B \sim Exp$, $\bar{L} = 100$ bits $\Rightarrow E[x_B] = 10 \cdot 10^{-3}$ s

$C = 10\,000$ bits/s $\rho_A = \lambda_A E[x_A] = 0.2$, $\rho_B = 0.2$

$E[x_A^2] = E[x_A]^2 = 4 \cdot 10^{-6} s^2$, $E[x_B^2] = 2 \cdot E[x_B]^2 = 200 \cdot 10^{-6}$ s

a) Non-preemptive priority, W_A, W_B, T_A, T_B

a.1. Type A has prio 1

$$\bar{R} = \frac{1}{2}\left[\lambda_A E[x_A^2] + \lambda_B E[x_B^2]\right] = \frac{1}{2}\left[400 \cdot 10^{-6} + 4000 \cdot 10^{-6}\right] = 2.2 \cdot 10^{-3} \text{ [s]}$$

$$W_A = \frac{\bar{R}}{1-\rho_A} = \frac{2.2 \cdot 10^{-3}}{0.8} = 2.75 \cdot 10^{-3} \text{ s} \qquad T_A = E[x_A] + W_A = 4.75 \cdot 10^{-3} \text{ s}$$

$$W_B = \frac{\bar{R}}{(1-\rho_A)(1-\rho_A-\rho_B)} = \frac{2.2 \cdot 10^{-3}}{(1-0.2)(1-0.4)} = 4.58 \cdot 10^{-3} \text{ s} \qquad T_B = 14.58 \cdot 10^{-3} \text{ s}$$

$$\bar{W} = \frac{\lambda_A}{\lambda_A+\lambda_B} \cdot W_A + \frac{\lambda_B}{\lambda_A+\lambda_B} \cdot W_B = 3.055 \cdot 10^{-3} \text{ s}$$

a.2. Same for opposite priorities, see solution in the compendium.

b) ~~Type A has~~ Preemptive - resume, Type A is prio 1

$$\bar{R}_A = \frac{\lambda_A \cdot E[x_A^2]}{2} = 0.2 \cdot 10^{-3} \text{ [s]} \qquad \bar{R}_B = \text{as before} = 2.2 \cdot 10^{-3} \text{ [s]}$$

$$W_A = \frac{\bar{R}_A}{1-\rho_A} = 0.25 \cdot 10^{-3} \text{ [s]} \qquad T_A = E[x_A] + W_A = 2.25 \cdot 10^{-3} \text{ [s]}$$

$$W_B = \frac{\bar{R}_B}{(1-\rho_A)(1-\rho_A-\rho_B)} = 4.58 \cdot 10^{-3} \text{ s}$$

as for non-preemptive.

$$E[x_B'] = \frac{E[x_B]}{1-\rho_A} = \frac{10 \cdot 10^{-3}}{0.8} = 12.5 \cdot 10^{-3}$$

$$T_B = E[x_B'] + W_B = 17.08 \cdot 10^{-3} \text{ s.}$$

$\boxed{11.5}$ M/G/1 with vacation

Type 1: $\lambda_1 = 60\ \text{jobs}/s$, $E[x_1] = 12 \cdot 10^{-3} s$, $x_1 \sim Exp$

Type 2: $\lambda_2 = 40\ \text{jobs}/s$, $E[x_2] = 6 \cdot 10^{-3} s$, $x_2 \sim Exp$

$\rho = \lambda_1 E[x_1] + \lambda_2 E[x_2] = 0.96$

Maintenance: $V \sim Exp$, $E[V] = 2 \cdot 10^{-3} s \Rightarrow E[V^2] = 2 \cdot E[V]^2 = 8 \cdot 10^{-6}$

a)

Remaining maintenance time, including the $\emptyset$ at the busy periods: Derived in class:

$$R_V = \frac{(1-\rho)}{2} \frac{E[V^2]}{E[V]} = \frac{0.04}{2} \cdot \frac{8 \cdot 10^{-6}}{2 \cdot 10^{-3}} = 0.08 \cdot 10^{-3}\ s$$

Remaining maintenance time, when job arrives to an empty system?

- Vacation period is exponential, so the remaining time after the first arrival is still exponential.

$$\Rightarrow R_{V|empty} = E[V] = 2 \cdot 10^{-3} s$$

b) T, T_1, T_2

$T = \bar{W} + E[x]$ $T_1 = \bar{W} + E[x_1]$, $T_2 = \bar{W} + E[x_2]$

$$W = \frac{\lambda E[x^2]}{2(1-\rho)} + \frac{E[V^2]}{2E[V]} = \dots = 0.254\ s$$

Details:

$\lambda = \lambda_1 + \lambda_2$

$$E[x^2] = \frac{\lambda_1}{\lambda_1+\lambda_2} E[x_1^2] + \frac{\lambda_2}{\lambda_1+\lambda_2} E[x_2^2]$$

$$\rho = \lambda_1 E[x_1] + \lambda_2 E[x_2] = \lambda \cdot E[x]$$