1

A

$M/M/1/\infty$

$\mu_i = \min\{i\mu, m\mu\}$, $m=5$

$\Rightarrow \mu, 2\mu, 3\mu, 4\mu, 5\mu, 5\mu \dots$

B

$M/M/5/\infty$

$\lambda = 1 \quad \mu = \frac{1}{4} \quad \Rightarrow \quad \rho = \frac{\lambda}{\mu} = 4$

a)

b) From balance equations: $p_0 = \frac{1}{77}$

$P(2 \text{ customer waits}) = p_2$

$P(2 \text{ customer waits}) = p_7$

This is a standard queue. Let us work with this first. Some results may be true for A.

$N_{q_B} = \frac{\lambda}{m\mu - \lambda} \cdot P(\text{wait}) = \dots = 2.21$

$P(\text{wait}) = \frac{5 \cdot E_5(4)}{5 - 4(1 - E_5(4))} = 0.554$

c) $N_q = ?$

$N_{q_A} = \lambda T - N_S = 6.21 - 0.987 = 5.22$

$\bar{T}_B = \frac{N}{\lambda} = \frac{N_q + N_s}{\lambda} = \frac{N_q + \rho}{\lambda} = 6.21 \text{ [s]}$

d) $T_A = ?$

This is true for A as well!

e) Service time changes with state!

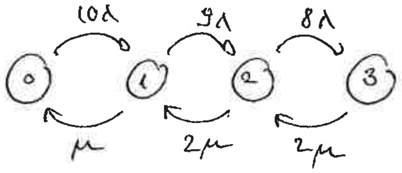
$T_S = \frac{\bar{N}_S}{\lambda} = \frac{1 - p_0}{\lambda} = \frac{76}{77} = 0.987$

$T_S = \{Exp(\mu)\} = 4\,s$

A, B } 5 + 5 customers $\lambda = 2$ calls/hour

$\bar{x} = 0.1$ hour, $\mu = 10$ calls/hour

a) M / M / 2 / 3 // 10



b) Time blocking: p_3

Call blocking: $\dfrac{p_3 \cdot 7}{p_0 10 + p_1 \cdot 9 + p_2 \cdot 8 + p_3 \cdot 7}$

$$\left.\begin{aligned} p_0 10\lambda &= p_1 \mu \\ p_1 9\lambda &= p_2 2\mu \\ p_2 8\lambda &= p_3 2\mu \end{aligned}\right\} \Rightarrow p_0 = \frac{25}{156},\ p_1 = \frac{50}{156},\ p_2 = \frac{45}{156},\ p_3 = \frac{36}{156}$$

Time blocking $= p_3 = \dfrac{36}{156} = \dfrac{3}{13} \approx 0.23$

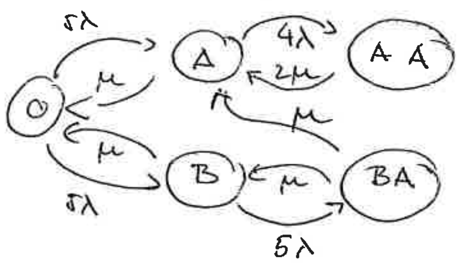
Call blocking $= \dots = 0.192$

c) Expected waiting time

- only one queuing position:

$E[\text{waiting}] = E[\text{time until first service}] = \dfrac{1}{2\mu} = \dfrac{1}{20}$ hour $= 3$ min

d) State: {Type of customers in the servers}



③ 2015

$M / E_2{+}M / 1 / 1$ $\quad C = 32 \cdot 10^6$ Git/s

Stream 1: $\lambda_1 = 2000 \frac{pkt}{sec.}$

$L_1 \sim Exp(\beta_1)$ $\quad E[L] = \frac{1}{\beta_1} = 500$ B

$X_1 \sim Exp(\mu_1)$ $\quad E[X] = \frac{1}{\mu_1} = \frac{500 \cdot 8}{32 \cdot 10^6} = 125 \cdot 10^{-6}$ s $\quad \mu_1 = 8 \cdot 10^{-3} \frac{packet}{sec.}$

Stream 2: $\lambda_2 = 3000 \frac{pkt}{sec.}$

$L_2 \sim$ Erlang-2(β_2) $\quad E[L] = \frac{2}{\beta_2} = 500$B

$E[X] = 125 \cdot 10^{-6}$ s $= \frac{2}{\mu_2}$ $\quad \mu_2 = 16 \cdot 10^{-3} \frac{packet}{sec.}$

a) Arrival process $\sim$ Poisson $(\lambda_1 + \lambda_2)$

Packet size distribution

$E[L] = p_1 E[L_1] + p_2 E[L_2] = \frac{\lambda_1}{\lambda_1+\lambda_2} E[L_1] + \frac{\lambda_2}{\lambda_1+\lambda_2} E[L_2] = 500$ B

Lin. comb.

$E[L^2] = p_1 E[L_1^2] + p_2 E[L_2^2] = \left[\frac{4}{5} + \frac{3}{5} \cdot \frac{3}{2}\right] \cdot 250\,000 = 425\,000$ B^2

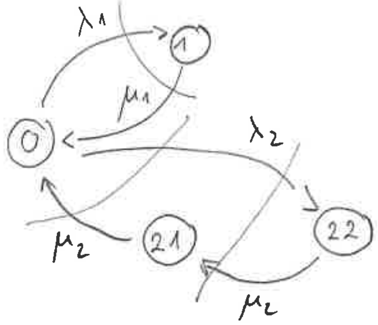
Exp: $E[L_1^2] = \frac{2}{(\beta_1)^2} = 2 \cdot E[L_1]^2 = 50\,00\,00$

Erlang-2 $\quad E[L_2^2] = Var[L_2] + E[L_2]^2 = \cancel{2} \cdot Var[L_2^*] + E[L_2]^2 =$

$= \cancel{2} \cdot \frac{1}{\beta_2^2} + \left(\frac{2}{\beta_2}\right)^2 = \frac{6}{\beta_2^2} = \frac{3}{2} \cdot \left(\frac{2}{\beta_2}\right)^2 = \frac{3}{2} \cdot 250\,000$

b) $M/G/1/1$ or $M/E_2{+}M/1/1$

c)



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1.

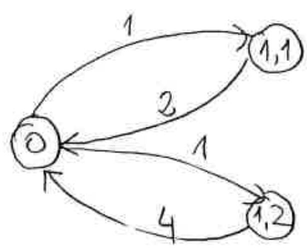
$\lambda = 2$

$$S^*(s) = \frac{1}{s+2} + \frac{2}{s+4} = 0.5 \cdot \frac{2}{s+2} + 0.5 \cdot \frac{4}{s+4} \Rightarrow$$

Service time is hyperexponential :

- $Exp(\mu_1 = 2)$ with probability 0.5
- $Exp(\mu_2 = 4)$ with probability 0.5

a) $M/H_2/1/1$



b)

$$P_{1,1} = \frac{1}{2} P_0 \quad , P_{1,2} = \frac{1}{4} P_0$$

$$P_0 = \frac{1}{1 + \frac{1}{2} + \frac{1}{4}} = \frac{4}{7} \quad , \quad P_{1,1} = \frac{2}{7} \quad , \quad P_{1,2} = \frac{1}{7}$$

$$P_{block} = P_{1,1} + P_{1,2} = 1 - P_0 = \frac{3}{7}$$

$$\text{Utilization} = \rho \cdot (1 - P_{block}) = \lambda \cdot \bar{x} \, (1 - P_{block}) =$$

$$= \underbrace{2 \cdot \left(\frac{1}{2} \cdot 0.5 + \frac{1}{4} \cdot 0.5\right)}_{\lambda \cdot \bar{x} \, = \, \frac{3}{4}} \left(1 - \frac{3}{7}\right) = 0.4286$$

c) $M/H_2/1$

$$\bar{W} = \frac{\lambda E[S^2]}{2(1-\rho)}$$

$$E[S^2] = S^*(s)'' \Big|_{s=0} = \frac{2}{(s+2)^3} + \frac{4}{(s+4)^3} \Big|_{s=0} = \frac{5}{16}$$

$$= p_1 E[S_1^2] + p_2 E[S_2^2] = \frac{1}{2} \cdot \frac{2}{\mu_1^2} + \frac{1}{2} \cdot \frac{2}{\mu_2^2} = \frac{1}{4} + \frac{1}{16} = \frac{5}{16}$$

(4.2)

$$\overline{W} = \frac{2 \cdot \frac{5}{16}}{2\left(1-\frac{3}{4}\right)} = \frac{5}{4}$$

d) Same as in c) (same service times)

e)

$M \sim Exp(1)$: maintenance time

$S_{hp} = S + M \quad \Rightarrow \quad \rho_{hp} = \lambda_{hp}\bar{x}_{hp} = 0.1\lambda \cdot (\bar{x}+1) = 0.275$

$S_{ep} = S \quad \Rightarrow \quad \rho_{ep} = \lambda_{ep}\bar{x} = 0.9\lambda \cdot \bar{x} = 0.675$

$E[S_{hp}^2] = E[(S+M)^2] = E[S^2] + 2E[S]E[M] + E[M^2] = \frac{49}{16}$

$E[S_{ep}^2] = E[S^2] = \frac{5}{16}$

$\overline{R} = \frac{1}{2}\left(\lambda_{hp}\overline{S_{hp}^2} + \lambda_{ep}\overline{S_{ep}^2}\right) = \frac{1}{2}\left(0.2 \cdot \frac{49}{16} + 1.8\,\frac{5}{16}\right) = 0.5875$

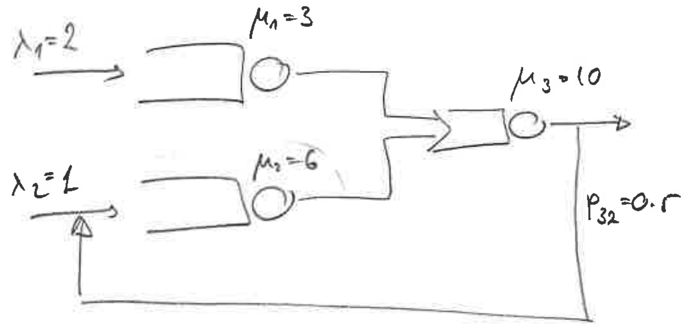
$\overline{W}_{hp} = \frac{\overline{R}}{1-\rho_{hp}} = \frac{0.5875}{1-0.275} = 0.81$

$\overline{W}_{ep} = \frac{\overline{R}}{(1-\rho_{hp})(1-\rho_{hp}-\rho_{ep})} = \frac{0.5875}{(1-0.275)(1-0.275-0.675)} = 16.21$

$\overline{W} = 0.1 \cdot \overline{W}_{hp} + 0.9\,\overline{W}_{ep} = 14.67$

(5)

a) M/M/1 → Poisson departure,



b) $\lambda_1^* = \lambda_1 = 2$

$\lambda_2^* = \lambda_2 + 0.5 \cdot \lambda_3^*$

$\lambda_3^* = \lambda_1^* + \lambda_2^*$

$\lambda_2^* = \lambda_2 + 0.5(\lambda_1^* + \lambda_2^*)$

$\lambda_2^* = \lambda_2 + 0.5 \cdot \lambda_1 + 0.5 \cdot \lambda_2^*$

$\lambda_2^* = \dfrac{0.5 \cdot \lambda_1 + \lambda_2}{1-0.5} = 2(0.5\lambda_1 + \lambda_2) =$

$= 2(1+1) = 4$

$\lambda_3^* = 2 + 4 = 6$

$\Rightarrow \rho_1 = \dfrac{\lambda_1^*}{\mu_1} = \dfrac{2}{3}, \quad \rho_2 = \dfrac{\lambda_2^*}{\mu_2} = \dfrac{4}{6} = \dfrac{2}{3}, \quad \rho_3 = \dfrac{\lambda_3^*}{\mu_3} = \dfrac{6}{10} = \dfrac{3}{5}$

Product form

c) $P(\text{Network is empty}) = p_{10} \cdot p_{20} \cdot p_{30} = (1-\rho_1)(1-\rho_2)(1-\rho_3) = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{5} = \frac{2}{45}$

d) $N = N_1 + N_2 + N_3 = \sum \dfrac{\rho_i}{1-\rho_i} = 2 + 2 + \dfrac{3}{2} = \dfrac{11}{2} = 5.5$

e) $T = \dfrac{N}{\lambda_1 + \lambda_2} = \dfrac{11}{2 \cdot 3} = \dfrac{11}{6}$, $\quad T_1 = \dfrac{N_1}{\lambda_1^*} = \dfrac{2}{2} = 1 \quad T_2 = \dfrac{N_2}{\lambda_2^*} = \dfrac{1}{2} \quad T_3 = \dfrac{1}{4}$; $\frac{3}{4}$

$T = p_1 \overbrace{(T_1 + T_f)}^{T_{S1}} + p_2 \overbrace{(T_2 + T_f)}^{T_{S2}} = \frac{2}{3}(1 + T_f) + \frac{1}{3}(\frac{1}{2} + T_f) = \frac{2}{3} + \frac{1}{6} + T_f = \frac{5}{6} + T_f = \frac{11}{6}$

$\Rightarrow \underline{\underline{T_f = 1}} \Rightarrow T_{S1} = 2 \quad T_{S2} = \dfrac{3}{2}$

f) $V = \dfrac{\sum \lambda_i^*}{\sum \lambda_i} = \dfrac{2+4+6}{3} = 4$

Or: With Bernoulli-trials → next page

5. e - other solution

$T_{S1} = T_1 + T_3 + N_f T_{23}$

$T_{S2} = T_2 + T_3 + N_f T_{23}$

N_f: average number of node 2 - nod 3 cycles

T_{23}: time of nod 2-nod 3 cycle:

$T = T_2 + T_3 = \frac{3}{4}$

N_f: each time, leave the network with $p = 1 - p_{32}$, or go back otherwise.

$$N_f = (1-p_{32}) \cdot 0 + (1-p_{32}) p_{32} \cdot 1 + (1-p_{32}) p_{32}^2 \cdot 2 \cdots$$

$$= (1-p_{32}) \cdot \sum_{i=0}^{\infty} i \, p_{32}^i = \dots = \frac{p_{32}}{1-p_{32}} = 1$$

$T_{S1} = 1 + \frac{1}{4} + 1 \cdot \frac{3}{4} = 2$

$T_{S2} = \frac{1}{2} + \frac{1}{4} + \frac{3}{4} = \frac{3}{2}$