EP2200 Home Assignment

1. Poisson process

Poisson process: $p_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$ $E[k,t] = \lambda t$

a) 50 pkt's transmitted together

$\Rightarrow$ first packet needs to wait for 49 other packets

Interarrival time: $X_i \sim Exp(\lambda)$, $X = \sum_{i=1}^{49} X_i$

$E[X] = \sum_i E[X_i] \Rightarrow W = X = 49 \cdot \frac{1}{\lambda} = 49\,msec$

b) After the first packet 10ms waiting

$P(k \text{ packets in a block}) = P(k-1 \text{ arrivals in } 10ms) = \frac{10^{k-1}}{(k-1)!} e^{-10}$

(Note, only $k-1$ is needed, since the first one is already there!)

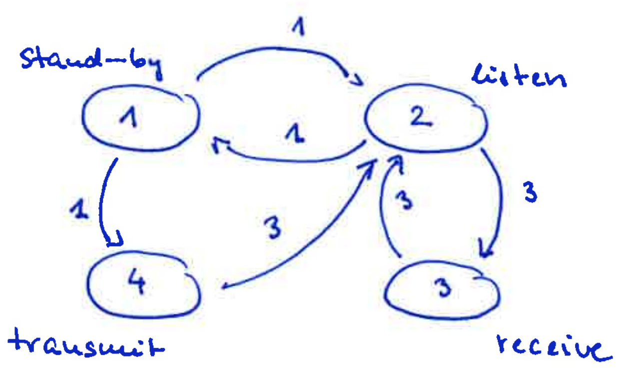
N = average number of packets in a block $= E[k,t]+1 =$
$= \lambda t + 1 = 10 + 1 = 11$

(Again, one packet was already there at the beginning of the 10ms)

2. Markov-chain

a) $q_{ii} = -1 \sum_i q_{ij}$

$$Q = \begin{bmatrix} -2 & 1 & 0 & 1 \\ 1 & -4 & 3 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & 3 & 0 & -3 \end{bmatrix}$$



b) ◦ If in stand-by: waits for Exp(1) then "wakes up" or, gets packet to transmit with rate 1

$\Rightarrow$ stays in stand-by for Exp(1+1), $\tau_1 = \frac{1}{2}$

◦ In listening: goes back to stand-by, if nothing happens in Exp(1) time, or starts to receive after Exp(3) time

$\Rightarrow$ stayes in listening for Exp(1+3), $\tau_2 = \frac{1}{4}$

◦ In transmit: transmission takes Exp(3) time, then moves to listen

◦ In receive: receiving takes Exp(3) time, then moves to listen.

c) $[p_1, p_2, p_3, p_4]\left[\, Q \,\right] = [0,0,0,0] \Rightarrow \left[\frac{3}{16}, \frac{3}{8}, \frac{3}{8}, \frac{1}{16}\right]$

$P(\text{stand-by}) = p_0 = \frac{3}{16}$

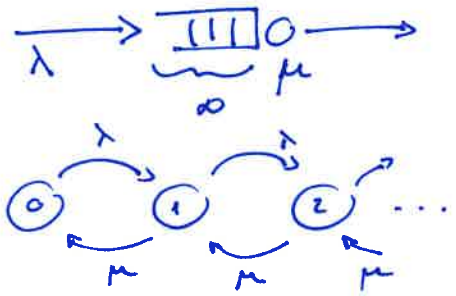
3. Little's result

a) $\left.\begin{array}{l} T = 3 \text{ min} \\ N = 5 \end{array}\right\}$ $\lambda = \frac{N}{T} = \frac{5}{3}$ req./min = 2400 req./day (M/M/∞ system!)

b) $\left.\begin{array}{l} T = 40 \text{ min} \\ N = 250 \end{array}\right\}$ $\lambda = \frac{N}{T} = 375$ ~~pers~~ visitor per hour

4. M/M/1

a)



$\lambda = \frac{1}{2}$ packet/msec

$x = 1\,ms \Rightarrow \mu = \frac{1}{x} = 1$

$\Big\} \rho = \frac{1}{2}$

b) Using equations derived in class:

$P_k = (1-\rho)\rho^k = \frac{1}{2} \cdot \frac{1}{2}^k = \left(\frac{1}{2}\right)^{k+1}$

$P_0 = P(\text{system empty}) = 1-\rho = \frac{1}{2}$

$N_q = N - N_s = \frac{\rho}{1-\rho} - \lambda \cdot \bar{x} = 1 - \frac{1}{2} = \frac{1}{2}$

c) At least n packets in the system

$P(k \geq n) = \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^{k+1} = \frac{\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} = \left(\frac{1}{2}\right)^n$

d) Waiting time more than 5ms – equation ~~derived in class~~ in formula sheet

$P(W > 5ms) = \rho \cdot e^{-(\mu-\lambda)t} = \frac{1}{2} e^{-\frac{5}{2}}$

5. M / M / m / m

a) $\left.\begin{array}{l}\lambda = 1 \text{ call/min} \\ x = 5 \text{ min}\end{array}\right\} \quad a = \lambda x = 5 \text{ Erlang}$

$P(\text{block}) \leq 0.01 \overset{\text{Erlang table}}{\Longrightarrow} m \geq 11 \qquad (p_m = 0.0082)$

$$P(\text{empty system}) = p_0 = \frac{1}{\sum_{i=0}^{m} \frac{a^i}{i!}} = 0.0068$$

Comment: to avoid the long sum use the known p_m!

$$p_m = \frac{a^m}{m!} p_0 \Rightarrow p_0 = \frac{p_m}{a^m / m!}$$

b) We divide the cell into two.

$\left.\begin{array}{l}\lambda_i = \frac{\lambda}{2} = 0.5 \text{ call/min} \\ x = 5 \text{ min}\end{array}\right\} \quad a_i = \lambda_i x = 2.5 \text{ Erlang}$

$P(\text{block}) \leq 0.01 \Rightarrow m_i \geq 7 \qquad m = \sum m_i = 14$

$p_{i,0} = 0.082$

Conclusion

- single cell is cheaper to build (BS costs a lot)
- two-cells may need only 7 channels, if they use the same channels. Otherwise more spectrum is needed
- the small cells can be switched off more frequently, but the consumed energy depends on the power needed.

  e.g.: same power: $\underline{11 \cdot P \cdot (1 - 0.006)} < 14 \cdot P (1 - 0.08)$

  $P \propto r^\alpha, \quad r' = \frac{r}{2} \Rightarrow P' = \frac{P}{4}$

  $11 \cdot P (1 - 0.006) > \underline{14 \cdot \frac{P}{4} (1 - 0.08)}$