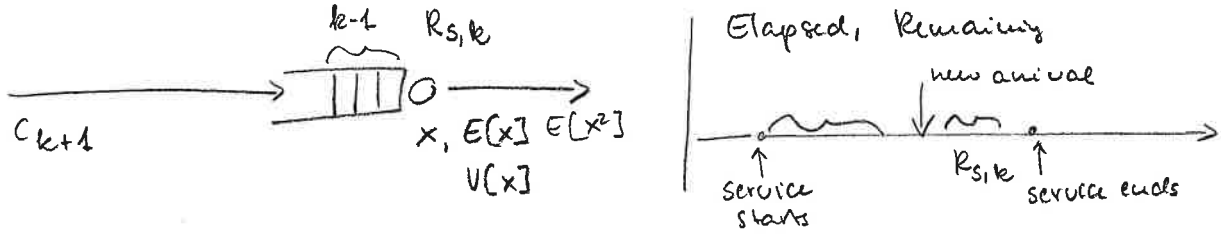
M/G/1

2014



Remaining service time for Exp(μ) and D?

Exp(μ): $R_S = \frac{\lambda}{2}\frac{2}{\mu^2} = \frac{\lambda}{\mu^2} = \varrho \cdot \frac{1}{\mu} = (1-\varrho)\cdot 0 + \varrho \cdot \frac{1}{\mu}$ ← Confirms the memoryless property!

D: x $\quad R_S = \frac{\lambda}{2}\cdot x^2 = \varrho \cdot \frac{x}{2}$

Example M/G/1

Arrival $\lambda = 0.1$ / minute

Service: $X = X_1 + X_2$, $\quad X_1 \sim Exp(\mu_1)$, $X_2 \sim Exp(\mu_2)$ $\quad \mu_1 = 1$ $\quad \mu_2 = \frac{1}{2}$

$E_1[x] = \frac{1}{\mu_1} = 1$ $\quad E_2[x] = \frac{1}{\mu_2} = 2$ $\quad \Rightarrow E[x] = 3$

$E_1[x^2] = \frac{2}{\mu_1^2} = 2$ $\quad E_2[x] = \frac{2}{\mu_2^2} = 8$

$V_1[x] = \frac{1}{\mu_1^2} = 1$ $\quad V_2[x] = \frac{1}{\mu_2^2} = 4$ $\quad V[x] = 5$

$C_x^2 = \frac{5}{9}$

$$W = \frac{\varrho\, E[x]}{2(1-\varrho)}(1+C_x^2) = \frac{0.1\cdot 3\cdot 3}{2\,(1-0.1\cdot 3)}\left(1+\frac{5}{9}\right) = \frac{0.1\cdot 9}{2\cdot 0.7}\cdot\frac{14}{9} = 1\,[\text{min}]$$

%

Transforms!

P-K Transform forms for M/M/1

Number of customers, N

$$B(s) = \frac{\mu}{s+\mu}$$

$$G(z) = \frac{\mu}{\lambda - \lambda z + \mu} \cdot \frac{(1-\rho)(1-z)}{\frac{\mu}{\lambda - \lambda z + \mu} - z} = \frac{(\mu-\lambda)(1-z)}{\mu - z\lambda + z^2\lambda - z\mu} = \frac{(\mu-\lambda)(1-z)}{(\mu - z\lambda)(1-z)} =$$

$$= \frac{\mu-\lambda}{\mu - \lambda z} = \frac{1-\rho}{1-\rho z} \quad ; \text{ Geometric distribution} \Rightarrow p_k = (1-\rho)\rho^k$$

System time

$$T^*(s) = \frac{\mu}{s+\mu} \cdot \frac{s(1-\rho)}{s - \lambda + \lambda \frac{\mu}{s+\mu}} = \frac{s(\mu-\lambda)}{(s+\mu)(s-\lambda)+\lambda\mu} = \frac{s(\mu-\lambda)}{s^2 + \mu s - \lambda s + \cancel{\lambda\mu} + \cancel{\lambda\mu}} =$$

$$= \frac{\mu-\lambda}{s+\mu-\lambda} \quad : \text{Exp}(\mu-\lambda) \Rightarrow T(t) = 1 - e^{-(\mu-\lambda)t}$$

Example

$$B^*(s) = \frac{\mu_1}{s+\mu_1} \cdot \frac{\mu_2}{s+\mu_2}$$

$$W^*(s) = \frac{s(1-\rho)}{s - \lambda + \lambda B^*(s)} = \frac{0.7s^2 + 1.05s + 0.35}{s^2 + 1.4s + 0.35} = \frac{A}{B}$$

$$E[W] = \frac{d}{ds} W^*(s)\Big|_{s=0} = \frac{A'B - AB'}{B^2}\Big|_{s=0} = \dots = 1$$

Calculate at home!