

Homework assignments to Lecture 6.

1. (Section 1.7, ex. 7) Prove that the family of functions $f_n(x) = \sqrt{2} \sin(\pi nx)$, $n \geq 1$ is a unit-perpendicular family in $L^2(0, 1)$.

2. (Section 1.7, ex. 8) Prove that the same family as before is a unit-perpendicular basis in $L^2(0, 1)$, i.e. it spans the whole space (There is a hint in the book!)

3. Assume that $f \in C[0, 1]$ and that $f(0) = f(1) = 0$. We form Fourier coefficients of f with respect to the system f_n from previous exercises, i.e. we define $c_n = (f, f_n)$, where the inner product is in $L^2(0, 1)$. We define next the partial sums of the Fourier series

$$S_N(x) = \sum_{n=1}^N c_n f_n(x)$$

and the Cesaro means $\sigma_N(x) = \frac{1}{N} (S_1(x) + \cdots + S_N(x))$.

Prove that $\sigma_N(x)$ converge to $f(x)$ as $N \rightarrow \infty$ uniformly on the interval $[0, 1]$.