Department of Mathematics
SF2729 Groups and Rings
Period 3, 2016

Homework 10

Submission. The solutions should be typed and converted to .pdf. Deadline for submission is Monday February 29, 14.00. Either hand in the solutions in class, in the black mailbox for homework outside the math student office at Lindstedtsvägen 25, or by email to `skjelnes@kth.se`.

Score. For each set of homework problems, the maximal score is 3 points, and calulated as $\min\{3, \Sigma/2\}$, where Σ is the score obtained on the homework. The total score from all twelve homeworks will be divided by four when counted towards the first part of the final exam.

Problem 1. Let $R = \mathbb{Z}[\sqrt{-5}] = \mathbb{Z}[x]/(x^2+5)$.

(a) Prove that the elements $2, 3, 1+x, 1-x$ are all irreducible, and no two are associate. Conclude that 6 has not a unique factorization. **(1 p)**

(b) Determine if the lattice of integer linear combinations of the given vectors is an ideal **(2 p)**

$$a) \quad (5, 1+x) \quad b) \quad (7, 1+x) \quad c) \quad (4-2x, 2+2x, 6+4x).$$

(c) Determine wheter or not 11 is an irreducible element and whether or not (11) is a prime ideal. **(1 p)**

(d) Factor the ideal (14) into prime ideals. **(1 p)**

Problem 2.

(a) Factor the ideal $(6) \subseteq \mathbb{Z}[x]/(x^2+6)$ into prime ideals. **(1 p)**

(b) Find a generator for the principal ideal $(2+x, 2-x) \subseteq \mathbb{Z}[x]/(x^2+2)$. **(1 p)**

(c) Let $d \geq 3$. Prove that 2 is not a prime element in $\mathbb{Z}[\sqrt{-d}] = \mathbb{Z}[x]/(x^2+d)$, but that 2 is irreducible. **(2 p)**