Questions to prepare for the written exam, SF2522

List to be extended

1. Define what is meant by a probability space.

2. Define what is meant by a random variable.

3. Define what is meant by a stochastic process. Give an example of a stochastic process.

4. Formulate the basic properties of a Wiener process.

5. Define the Ito integral by the limit of the forward Euler method and show that the limit is independent of the partition used in the time discretization.

6. State and prove Grönwall's lemma.

7. Show that the Ito and Stratonovich integrals (with the same integrand) may be different.

8. Show by Ito's formula that if u solves Kolmogorov's backward equation with data $u(T, x) = g(x)$, then
$$u(t, x) = E[g(X(T))|X(t) = x],$$
where X is the solution of a certain SDE (which?).

9. Formulate and prove a theorem on error estimates for weak convergence of the forward Euler method for SDE's.

10. Show that weak convergence of a sequence of random variables does not imply strong convergence.

11. State the Geometric Brownian motion equation and derive its solution.

12. State the Ornstein-Uhlenbeck equation. Derive its stationary distribution.

13. Motivate the use of Monte-Carlo methods to compute European options based on a basket of several stocks and discuss some possibilities of methods of variance reduction.

14. State and derive Ito's formula.

15. State and derive the Feynman-Kac formula.

16. State and derive the Fokker-Planck equation.

17. State and derive the central limit theorem.