$\boxed{12.2}$

a) $T^*(s)$, $T(t)$?

$T = T_1 + T_2$

$T^*(s) = T_1^*(s) T_2^*(s) = \left(\frac{\mu-\lambda}{s+\mu-\lambda}\right)^2$ $\Leftarrow$ Remember from M/M/1

$T(t)$ ~~Laplace table~~ ? Sum of two Exp! Erlang-2

$f_T(t) = (\mu-\lambda)^2 \cdot t \cdot e^{-(\mu-\lambda)t}$

b) $E[T]$, $E[T^2]$, $V[T]$ $\leftarrow$ see with $\mathcal{L}$-transform in the compendium.

$E[T] \overset{E[T_1]+E[T_2]}{=} 2 \cdot E[T_1] = \frac{2}{\mu-\lambda}$

$V[T] = 2 \cdot V[T_1] = \frac{2}{(\mu-\lambda)^2}$

$E[T^2] = V[T] + E[T]^2 = \frac{6}{(\mu-\lambda)^2}$

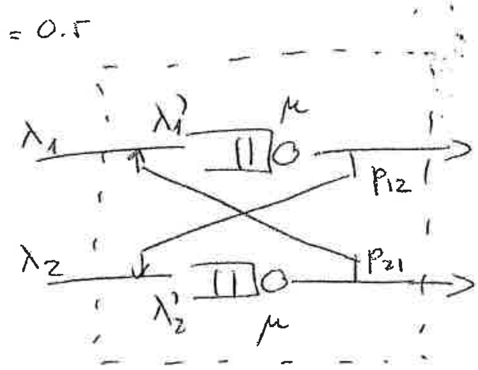
c) $P(\tau > t) = 1 - F_T(t) =$ ~~P(...)~~

$= P($the two $Exp(\mu-\lambda)$ services are not finished within $t)$

$= P($ $\varnothing$ or 1 "arrivals" in a Poisson process, representing the services$) = e^{-(\mu-\lambda)t}[1 + (\mu-\lambda)t]$.

Exam 2009 Dec.

(2) $\lambda_1 = \lambda_2 = 1$, $\mu = 3$, $p_{12} = p_{21} = 0.5$



a) M/M/1 servers

$$\lambda_1' = \lambda_1 + p_{21}\lambda_2'$$
$$\lambda_2' = \lambda_2 + p_{12}\lambda_1'$$

$$\Rightarrow \lambda_1' = \lambda_2' = 2$$

b) Based on the independence of the queues:

$$P(\text{system empty}) = P_0^1 \cdot P_0^2 = (1-\rho_1)(1-\rho_2) = \left(1 - \frac{\lambda_1'}{\mu}\right)\left(1 - \frac{\lambda_2'}{\mu}\right) = \frac{1}{9}$$

c) Little theorem for the entire queuing nw:

$$\bar{T}_{sys} = \frac{\bar{N}}{\lambda_1+\lambda_2} = \frac{\bar{N}_1 + \bar{N}_2}{\lambda_1+\lambda_2} = \frac{4}{2} = 2$$

$$\bar{N}_i = \frac{\rho_i}{1-\rho_i} = 2$$

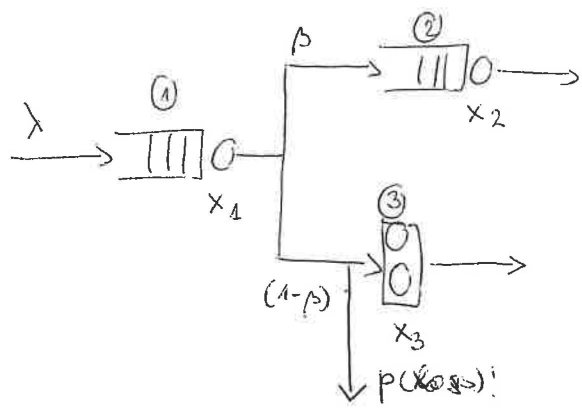
d) i. $$I = \frac{\lambda_1' + \lambda_2'}{\lambda_1 + \lambda_2} = 2$$

ii. Alt. solution:

Number of visits for one job: Geometric$\left(\frac{1}{2}\right)$

$$\Rightarrow I = \frac{1}{1/2} = 2$$

12.3



$\lambda = 4$ customers/min.

$x_1 = 10s \Rightarrow \mu_1 = \frac{1}{10}/s = 6/\text{min}$

$x_2 = 30s \quad \mu_2 = \frac{1}{30}/s = 2/\text{min}.$

$x_3 = 20s \quad \mu_3 = \frac{1}{20}/s = 3/\text{min}.$

$\beta = 0.2.$

a) Arrival rates: $\lambda_1 = \lambda = 4$ customers/min. $\Rightarrow \rho_1 = \frac{4}{6} = \frac{2}{3}$

$\lambda_2 = \beta \cdot \lambda = 0.8 = \frac{4}{5}$ $\qquad \rho_2 = \frac{2}{5}$

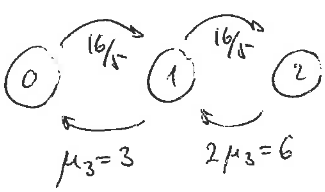
$\lambda_3 = 3.2. = \frac{16}{5}$

b) Mean number of customers at ① and ② $\Rightarrow M/M/1 \quad N = \frac{\rho}{1-\rho}$

$N_1 = \frac{\rho_1}{1-\rho_1} = \frac{2/3}{1/3} = 2 \qquad N_2 = \frac{\rho_2}{1-\rho_2} = \frac{2/5}{3/5} = \frac{2}{3}$

c) Number of rejected customers per minute at ③ $\quad \lambda_{loss}: \lambda = \lambda_{eff} + \lambda_{loss}$

M/M/2/2



$p_0 \cdot \frac{16}{5} = 3 \cdot p_1 \Rightarrow p_1 = \frac{16}{15} p_0$

$p_1 \cdot \frac{16}{5} = 6 \cdot p_2 \qquad p_2 = \frac{16}{30} \cdot p_1 = \frac{16 \cdot 16}{15 \cdot 30} p_0$

$p_0 \left(1 + \frac{16}{15} + \frac{16 \cdot 16}{15 \cdot 30}\right) = 1$

$p_0 \left(\frac{15 \cdot 30 + 16 \cdot 30 + 16 \cdot 16}{15 \cdot 30}\right) = 1 = p_0 \cancel{\frac{1186}{450}} \frac{593}{225} = 1$

$p_0 = \frac{225}{\cancel{578}\ 593}, \quad p_1 = \frac{240}{\cancel{289}\ \cancel{578}\ 593}, \quad p_3 = \frac{128}{\cancel{578}\ 593} = 0.215 = P_{loss} \Rightarrow \lambda_{loss} = \lambda_3 p_3 = \underline{\underline{0.67/\text{min}}}$

d) T for all not dropped customers – can not use Little.

$T_1 = \frac{x_1}{1-\rho_1}$

$T_2 = \frac{x_2}{1-\rho_2}$

$T_3 = x_3$

$T = T_1 + P(\text{to node } 2)T_2 + P(\text{to node } 3)T_3$

$= T_1 + \frac{\lambda\beta}{\lambda\beta + \lambda(1-\beta)(1-p_{loss})} T_2 + \frac{\lambda(1-\beta)(1-p_{loss})}{\lambda\beta + \lambda(1-\beta)(1-p_{loss})}$

Note, if even blocked customers count $\tilde{T} = \frac{\bar{N}_1 + \bar{N}_2 + \bar{N}_3}{\lambda}$

0/0

e.)

$P(\text{entire system empty}) = \prod_{i=1}^{3} P(\text{node } i \text{ is empty}) =$

$(1-\rho_1)(1-\rho_2)\cdot p_{30}$

$P(\text{one single customer is in the system}) =$

$p_{11}\, p_{20}\, p_{30} + p_{10}\, p_{21}\, p_{30} + p_{10}\, p_{20}\, p_{31}.$

$P(\text{an arriving customer is served without delay}) =$

$P(\text{selects node } 2)\, P(\text{node } 1 \text{ empty})\, P(\text{node } 2 \text{ empty}) +$

$P(\text{selects node } 3)\, P(\text{node } 1 \text{ empty})\, P(\text{node } 3 \text{ empty}) =$

$(1-\rho_1)\cdot\left[\beta(1-\rho_2) + (1-\beta)\, p_{30}\right].$