Recitation 7: - Multiple server: 7.6 (see earlier notes)
- Finite population: 8.4.

Problem 8.4

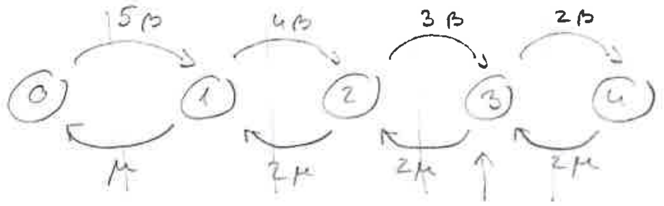
- 2 servers
- 2 queuing positions
- 5 sources

$\Big\}$ M/M/2/4/5

Idle: $Exp(\beta)$

Service $Exp(\mu)$

a) State transition diagram (Markov chain)



2 under service
1 waiting
2 idle

b) Mean waiting time : $p \Rightarrow N_q \Rightarrow W$

- State probabilities: $\mu = \beta = 1$

- $p_0 \cdot 5\beta = p_1 \mu$ $\qquad p_1 = \frac{5\beta}{\mu} p_0 = 5 p_0$

$p_1 \cdot 4\beta = p_2 \cdot 2\mu$ $\qquad p_2 = \frac{4}{2}\frac{\beta}{\mu} \cdot p_1 = \frac{5 \cdot 4}{2} p_0 = 10 p_0$

$p_2 \cdot 3\beta = p_3 \cdot 2\mu$ $\qquad p_3 = \frac{3}{2}\frac{\beta}{\mu} p_2 = \frac{5 \cdot 4 \cdot 3}{2 \cdot 2} p_0 = 15 p_0$

$p_3 \cdot 2\beta = p_4 \cdot 2\mu$ $\qquad p_4 = p_3 = \frac{5 \cdot 4 \cdot 3}{2 \cdot 2} p_0 = 15 p_0$

$$p = \left\{ \overset{0}{\frac{1}{46}}, \overset{1}{\frac{5}{46}}, \overset{2}{\frac{10}{46}}, \overset{3}{\frac{15}{46}}, \overset{4}{\frac{15}{46}} \right\}$$

$$N_q = 1 \cdot p_3 + 2 \cdot p_4 = \frac{45}{46}$$

$$\lambda_{eff} = \sum_{i=0}^{3}{}^{!} \underbrace{(5-i)\beta}_{\lambda_i} \cdot p_i = 5 \cdot p_0 + 4 \cdot p_1 + \dots + 2 \cdot p_3 = \frac{85}{46}$$

$$W = \frac{N_q}{\lambda_{eff}} = \frac{45}{85}$$

c) Time blocking = part of time in blocking state $= p_4 = \frac{15}{46} \approx 0.32$

d) Call blocking = P(arriving call is blocked) $= a_4 = \frac{\lambda_4 p_4}{\sum_{i=0}^{4}{}^{!} \lambda_i p_i} \approx 0.15$

e) P(served without waiting) $= a_0 + a_1 = \frac{\lambda_0 p_0 + \lambda_1 p_1}{\sum_{i=0}^{3} \lambda_i p_i} \approx$

f) P(served after waiting) $= a_2 + a_3 = \frac{\lambda_2 p_2 + \lambda_3 p_3}{\sum_{i=0}^{3} \lambda_i p_i}$

Recitation 7

Poisson: $P_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$

Multiple server systems: 7.6.

Finite customers: 8.4, + borrowed question from 8.6 e

have to divide to condition prob. (wait: states 2, 3)

g) $P(W > 1s \mid \text{waits}_{2,3}) = P(W>1 \mid \text{arrives in state } 2) \cdot P(\text{arrive in } 2 \mid \text{out of } 2,3) + P(W>1 \mid \text{arrives in } 3) \cdot P(\text{arrives in } 3 \mid \text{out of } 2,3)$

· Arrives to state 2: has to wait to 1 finished service: $Exp(2\cdot\mu) = Exp2$

$$P(W>1 \mid \text{State } 2) \overset{CCDF}{=} e^{-2\mu t}\Big|_{t=1} = \underline{\underline{e^{-2}}}$$

Arrives to state 3: — " — 2 finished service: Erlang-2(2μ)

or:

$$P(W>1 \mid \text{state } 3) = P(\text{0 or 1 service in the Poisson}(2\mu) \text{ service process}) =$$

$$= \underbrace{e^{-2\mu t}}_{\text{0 finished service}} + \underbrace{2\mu t e^{-2\mu t}}_{\text{1 finished}} = (1+2)e^{-2} = \underline{\underline{3e^{-2}}}$$

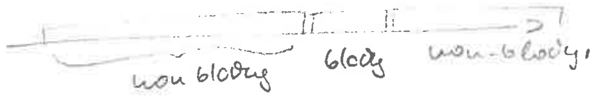
$$P(\text{arrives in } 2 \mid \text{out of } 2,3) = \frac{P(\text{arrives in } 2)}{P(\text{arrives in } 2 \text{ or } 3)} = \frac{3\beta p_2}{3\beta p_2 + 2\beta p_3} = \frac{30}{60} = \frac{1}{2}$$

$$P(\text{arrives in } 3 \mid \text{out of } 2,3) = \frac{2\beta p_3}{3\beta p_2 + 2\beta p_3} = \frac{30}{60} = \frac{1}{2}$$

$$P(W>1s \mid \text{waiting}) = \frac{1}{2}\left[e^{-2} + 3e^{-2}\right] = \underline{\underline{2e^{-2}}} \approx 0.27$$

h) Mean non-blocking time: $\bar{\tau}_{nb}$

We know:



$$P(\text{blocking state}) = p_4 = \frac{15}{46}$$

$$\bar{\tau}_b = \frac{1}{2\mu} = \frac{1}{2} \qquad (\tau_b \sim Exp(2\mu))$$

$$P(\text{blocking}) = \frac{\bar{\tau}_b}{\bar{\tau}_{nb} + \bar{\tau}_b}$$

$$\frac{15}{46} = \frac{\frac{1}{2}}{\bar{\tau}_{nb} + \frac{1}{2}} = \frac{1}{2\bar{\tau}_{nb} + 1}$$

$$2\bar{\tau}_{nb} + 1 = \frac{46}{15}, \qquad \bar{\tau}_{nb} = \left(\frac{46}{15} - 1\right)\cdot\frac{1}{2} = \frac{31}{30}$$