Summary

Recitation 6

M/M/m/m Erlang loss systems

$a = \frac{\lambda}{\mu}$ (sometimes noted as g!)

$$P(\text{blocking}) = p_m = \frac{a^m/m!}{\sum_{1}^{m} a^i/i!} = E_m(a) = B(m,a)$$ $\boxed{\text{Erlang-B form}}$

$$T = \bar{x} = \frac{1}{\mu}, \quad N = N_S = \lambda_{eff}\bar{x} = (1-p_m)a$$

M/M/m Erlang wait system

Stability: $\frac{a}{m} < 1$

$$P(\text{wait}) = D_m(a) = C(m,a)$$ $\boxed{\text{Erlang-C form}}$

$$D_m(a) = \frac{m E_m(a)}{m - a(1 - E_m(a))}$$

$$N_S = \lambda \bar{x} = a$$

$$N_q = \sum_{k=m+1}^{\infty} (k-m) p_k = \cdots = D_m(a) \frac{a}{m-a}$$ $\Rightarrow N, T \ldots$

~~[illegible]~~

Recitation plan

6.5, 6.6. (Do 6.2, 6.3, 6.4 at home!)

(6.5)

$\lambda = 180$ calls/hour $= 3$ calls/min.
$\bar{x} = 110$ sec. $= \frac{110}{60}$ min
$\Big\}\ a = \lambda \bar{x} = 5.5$

$m = 10$

a) $P(\text{blocking}) = B(10, 5.5) \overset{\text{Tables}}{\approx} 0.029$

b) Calls rejected in an hour $= \lambda_b \cdot 60\text{min} = P(\text{block})\,\lambda \cdot 60 = 5.27$ calls/hour

c) Effective Load per server $= \frac{\lambda_{eff}\bar{x}}{m} = 0.55\,(1 - P(\text{block}))$

Load per server $= \frac{\lambda \bar{x}}{m} = 0.55$

d) $P(\text{block}) < 0.02$ $\quad \lambda_{max} = ?$

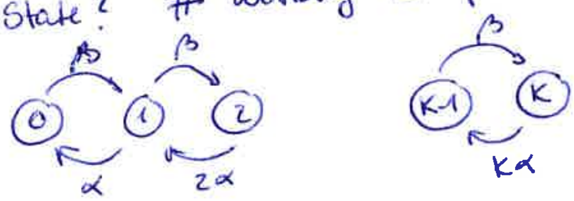
$B(10, a) < 0.02 \overset{\text{Tables}}{\Rightarrow} a < 5.05$

$\lambda < \frac{a}{\bar{x}} = 2.75$

(6.6.)

Break down $\sim \text{Exp}(\alpha)$
Repair $\sim \text{Exp}(\beta)$
K computers, 1 repairman

State? # working computers



$\Rightarrow$ M/M/K/K queue! $\quad a = \frac{\beta}{\alpha}$

a) $p_i = \dfrac{a^i/i!}{\sum_0^K a^k/k!}$

b) Failure rate: $\bar{\alpha} = \dfrac{\text{Number of failures in } T}{T} = \dfrac{\sum_{i=0}^{K} i\alpha\, p_i T}{T} = \sum_{i=0}^{K} i \cdot \alpha\, p_i$

c) Repairman busy: $1 - p_K$

d) All computers broken: p_0

e) $N = ?$ K computers work in average

Average number of working computers: $\sum_{i=1}^{N} i p_i \geq K$ ...

Recitation plan:

7.1, 7.6, ~~Ex 7~~. (All other problems + Ex 7 at home)

7.1

$\lambda' = 100$ customers/hour

A: M/M/1

$\lambda = \frac{\lambda'}{3}$

$\bar{x} = 45$ sec.

$T = \frac{\bar{N}}{\lambda}$, $\bar{N} = \frac{\rho}{1-\rho}$

$\rho = \lambda \bar{x}\Big|_{min} = \frac{\overset{25}{\cancel{100}}}{\cancel{3}\cdot 60} \cdot \frac{\cancel{3}}{4} = \frac{5}{12}$

$\bar{N} = \frac{\frac{5}{12}}{\frac{7}{12}} = \frac{5}{7}$

$T = \frac{5}{7} \cdot \frac{1}{100/3\cdot 60} = \frac{\cancel{5}}{7} \cdot \frac{\overset{3}{\cancel{3 \cdot 60}}}{\underset{2}{\cancel{100}}} = \frac{9}{7}$ min $= 1.28$ min.

B: M/M/3

$\lambda = \lambda' = \frac{100}{60} = \frac{5}{3}$ cust/min

$\bar{x} = 50$ sec. $= \frac{5}{6}$ cust/min.

$a = \frac{25}{18} \approx 1.38$

$\bar{W} = \frac{1}{m\mu - \lambda} P(wait)$, $T = \bar{W} + \bar{x}$

$E_m(a) = E_3(1.38) \overset{\text{Table}}{=} 0.119$

$P(wait) = P_3(a) = \frac{3 \cdot E_3(a)}{3 - a(1 - E_3(a))} = \dots = 0.201$

$T = \frac{1}{3 \cdot \frac{6}{5} - \frac{5}{3}} \cdot 0.201 + \frac{5}{6} = 0.937$ min

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7.6

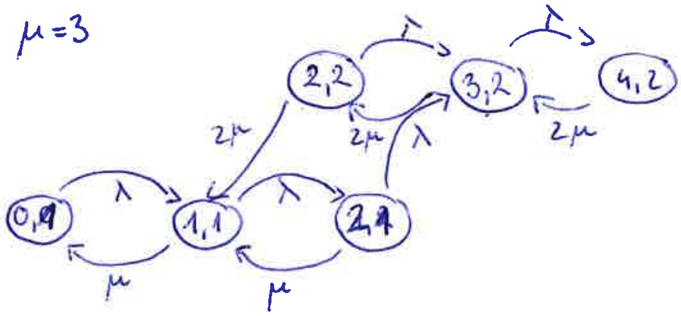
$\lambda = 3$

$\mu = 3$

State: {# customers, # servers}

(Will be solved at the next recitation!)

a)



b) at home

c) $\bar{N}_q = 0 p_{01} + 0 p_{11} + 1 \cdot p_{21} + 0 \cdot p_{22} + \sum_{i=3}^{\infty} (i-2) p_{i,2}$

$\bar{N}_s = 0 p_{01} + 1 \cdot p_{11} + 1 \cdot p_{21} + 2 p_{22} + \sum_{i=3}^{\infty} 2 \cdot p_{i,2}$

d) $f_W(t)$, customer arriving to state $(i,2)$ $i \geq 3$

$\Rightarrow$ $i-2$ services have to be completed, time between comp. services: Exp(2μ)

$\Rightarrow$ Erlang$_{i-2}(2\mu)$

e) Customer arrives to $S_{1,1}$: Service: if there is a new arrival or the service completes $\Rightarrow$ Exp$(\lambda + \mu)$