EP2200 Queueing theory and teletraffic systems

Lecture 7

M/M/m/m/C – Engset loss system

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Markov queuing systems

- Systems considered until now:
  - M/M/1
  - M/M/1/K
  - M/M/m/m (Erlang loss system)
  - M/M/m (Erlang wait system)

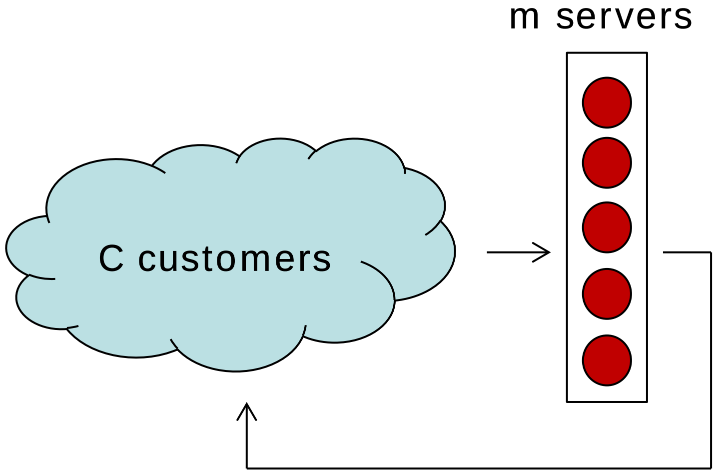
$\lambda_i = \lambda$
State independent Poisson arrival process:
to model independent requests from a large user population

- What happens if the population is small?

- Finite population system:
  - M/M/m/m/C – Engset loss system
  - General case with buffer on the recitation

M/M/m/m/C

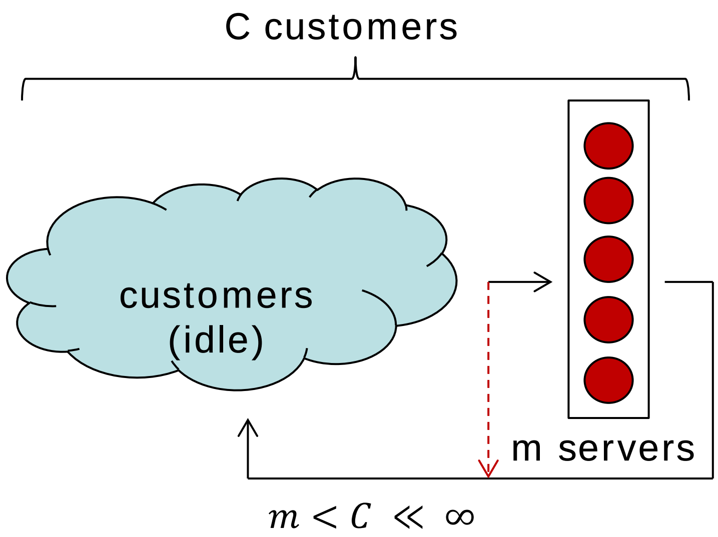
- System definition
  - C customers
  - m servers
  - no buffer
  - Exponential service time (μ)
  - Arrival process should reflect the finite population: customer does not generate new request while under service
    the intensity of new request arrivals depends on the number of requests under service



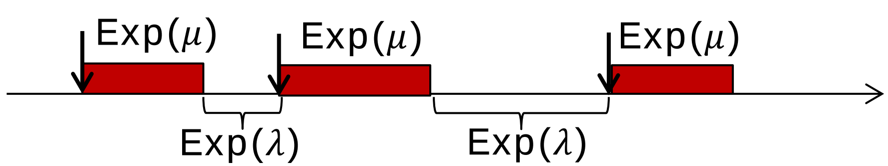
$$m < C \ll \infty$$

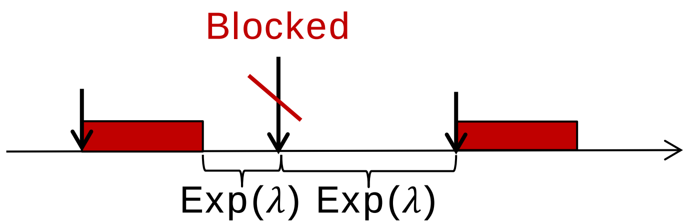
(Example: you do not start new phone calls when you already talk to someone)

M/M/m/m/C



- Modeling finite population
- Single customer behavior:
  - Thinking (idle) time after served request: Exp(λ) (Virtamo notes: γ)
  - Request service time: Exp(μ)
  - Blocked request: does not try again, moves to idle state





M/M/m/m/C

- State transition diagram (Markov chain)
- State probabilities in steady state:
  - Engset distribution:

$$p_k = \frac{\binom{C}{k}\left(\frac{\lambda}{\mu}\right)^k}{\sum_{i=0}^{m}\binom{C}{i}\left(\frac{\lambda}{\mu}\right)^i}$$

- Probability that the arriving node finds the system in state k: PASTA does not hold!

$$a_k = \frac{\lambda_k p_k}{\sum_{i=0}^{m}\lambda_i p_i}$$

- Time blocking =part of the time the system is in blocking state=p_m
- Call blocking = P(arriving request gets blocked)=a_m

M/M/m/m/C

- Performance
  - Offered traffic: $$\lambda^* = \sum_{i=0}^{m} (C-i)\lambda p_i$$
  - Effective traffic: $$\lambda_{eff} = \sum_{i=0}^{m-1} (C-i)\lambda p_i$$
  - Average number of requests under service: $$N = N_s = \lambda_{eff} / \mu$$

- When should we consider a system as finite population system?
  - Rule of tumb: *C* < *10m*

Summary

- Markovian systems:
  - Poisson arrival (may be state dependent)
  - Exponential service time (may be state dependent)
  - One or many servers
  - No buffer, limited or infinite buffer space

- Next step to extend Markovian systems
  - Non-exponential service
  - Systems still possible to model with a Markov chain
  - Erlang and Hyper-Exponential service times

- But first we will look at Markovian queuing networks