KTH, Matematik, Serguei Shimorin

SF2705 Fourier Analysis
Homework assignement for the Lecture 4

1. (1.4.6) Prove the formula for the Féjer kernel

$$F_n(x) = \frac{1}{n}(D_0 + D_1 + \cdots + D_{n-1}) = \frac{(\sin \pi n x)^2}{n(\sin \pi x)^2}.$$

2. (1.4.8) Prove that for any $f \in C(S^1)$ and any $r \in (0, 1)$

$$\sum_{n=-\infty}^{\infty} \hat{f}(n) r^{|n|} e_n(x) = \int_0^1 \frac{1 - r^2}{1 - 2r\cos(2\pi(x - y)) + r^2} f(y)\, dy.$$

Hint: use integral formula for $\hat{f}(n)$ and then sum up untegrals.

3. (1.4.9) Use previous exercise and prove that for $f \in C(S^1)$

$$lim_{r \to 1-} \left\| f - \sum_{n=-\infty}^{\infty} \hat{f}(n) r^{|n|} e_n \right\|_\infty = 0.$$

4. (1.5.7) For any $f \in L^1(S^1)$ and real y, let f_y be defined as $f_y(x) = f(x + y)$. Prove that $\lim_{y \to 0} \|f - f_y\|_1 = 0$.

Hint: prove this first for functions from $C(S^1)$.