EP2200 Queueing theory and teletraffic systems 2016.

Home assignment problems

1. (Poisson process)
Very high speed networks, such as optical networks, try to collect several packets, to transmit them together as a block. This larger block size helps collision avoidance and makes scheduling easier. Assume that packets are generated according to a Poisson process, with a rate of 1000 packets per second.

a) You design a system, where 50 packets are transmitted together in a block. How long time, in average, the first packet of a block needs to wait to be transmitted?
b) To make sure that the first packet of a block does not need to wait too long, you change the system as follows. Once the first packet of a block arrives, the system waits for 10ms, and then all the packets that arrived are transmitted together in a block. Give the probability that *k* packets are transmitted in a block, and the average number of packets in a block.

2. (Markov chains)
A radio device (e.g., the radio interface of a small sensor) has four states:

State 1: it is in stand-by, and consumes very little energy
State 2: it listens to the radio channel for new packets to be received
State 3: it receives a packet
State 4: it transmits a packet.

The state transition intensity matrix, Q is as follows:

$$Q = \begin{bmatrix} ? & 1 & 0 & 1 \\ 1 & ? & 3 & 0 \\ 0 & 3 & ? & 0 \\ 0 & 3 & 0 & ? \end{bmatrix}$$

a) Fill in the diagonal, and draw the Markov chain.
b) Interpret the Q matrix, by describing how the radio is controlled, e.g., for how long time it stays in stand-by, how long are the packets that are transmitted or received, etc.
c) Calculate the probability that the radio is in stand-by.

3. (Little's result)
Little's result, $N=\lambda T$ is very useful, if it is complicated to calculate N or T. It is also useful, if you can observe two of the three parameters, and would like to estimate the third one. Little's result holds for any "black boxes" users enter to and then leave from. Answer the following questions with the use of Little's result.

a) You design a peer to peer (P2P) service for Spotify, to stream songs to many users. P2P service means, that a song is not streamed from a server, but collected from others listening to it. The P2P service is efficient if there are at least 5 people listening to the song at the same time. Consider a song that is 3 minutes long, and assume, that it is requested by users according to a Poisson process with the same intensity

throughout the day. How many requests are needed per day to make sure that the P2P solution is efficient?

b) A famous art exhibition is coming to town, and everyone would like to see it. The organizers estimate that people will spend 40 minutes on average to look at the paintings. To keep the exhibition area safe and the exhibition enjoyable, there can be around 250 people in the building. Tickets will be sold through the web. How many tickets should be available per hour?

4. (M/M/1)

IP packets arrive to an output buffer according to a Poisson process, the average inter-arrival time is 2ms. The transmission time of the packets is Exponentially distributed, with a mean of 1ms. One packet is transmitted at a time, and infinite number of packets can wait in the buffer.

a) Give the block diagram of the queue and the Markov chain describing the system.
b) Considering the queue in steady state, give the state probabilities, the probability that the system is empty, and the average number of packets waiting for transmission.
c) Derive the expression for the probability that there are at least *n* packets in the system (one under service and *n-1* waiting).
d) Consider a packet arriving to the system. What is the probability that it has to wait until the start of its transmission for more than 5ms?

5. (M/M/m/m)

Green cellular networks try to switch the base-stations (BS) to standby, when there are no calls in the cell. The question is how large the cells should be to minimize the energy usage. At the same time the cost of building the network should not increase much.

a) Consider first a single cell that provides *m* voice channels, that is it can serve *m* calls at the same time. Calls are generated in the cell according to a Poisson process with $\lambda=1$ call per minute, the call holding times are Exponential with a mean of 5 minutes. The parameter *m* is selected to achieve a blocking probability less than 0.01. Find the required *m* using Erlang tables. Calculate the part of the time, when the cell does not serve any calls and can be switched off.
b) Assume now, that the same area is served by two cells. We can still say that the arrival process to a cell is Poissonian, but with $\lambda=0.5$. The call lengths are the same. How many voice channels are needed in the cells now to achieve a blocking probability less than 0.01? Calculate the part of the time, when a cell does not serve any calls and can be switched off.
c) Discuss what can you conclude based on the results? Which solution costs more to build? Which solution leads to lower energy consumption?