Recitation 4 - notes

Poisson process(λ) $p_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$, $E[X|t] = \lambda t$, $\tau \sim Exp(\lambda)$

Exp (λ) $f(t) = \lambda e^{-\lambda t}$, $F(t) = 1 - e^{-\lambda t}$, $E[T] = \frac{1}{\lambda}$

Markov chains, stationary solution: $p\underline{\underline{Q}} = \underline{0}$

global, local balance equations

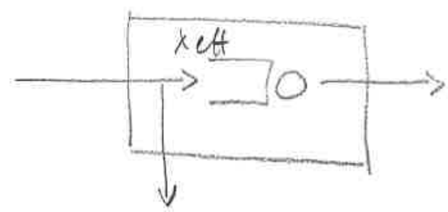
PASTA: $a_k = P(\text{state } k \mid \text{arrival}) = p_k = P(\text{state } k)$

Queuing systems: Arrival / Service / #server / system cap. / population / service order.

Little theorem: $N = \lambda T$ $(T = \bar{x} + W)$

$N_q = \lambda W$

$N_S = \lambda \bar{x}$



Little theorem for systems with blocking:

$N = \lambda_{eff} T$ T: average system time for accepted customers

$N = \lambda T'$ T': — " — , including $\emptyset$ for blocked customers.

Plan: finish 3.4

3.3

3.5

3.6: MC, rest at home

Home exercise:

complete 3.6

4.1 - 4.6(a)

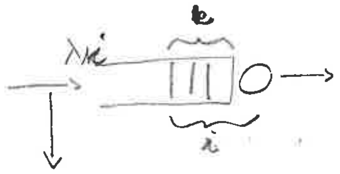
R4

3.5.

$\lambda = \frac{1}{7}$ jobs/sec., $\mu = \frac{1}{6}$ jobs/sec., single server, State: # jobs in the system.

P(do not join the queue | k) = ℓ_k, k: jobs in the <u>queue</u>

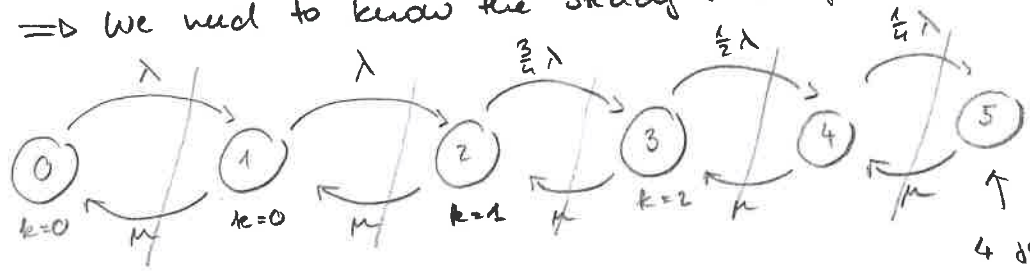
$$\ell_k = \begin{cases} k/4 & k<4 \\ 1 & k \geq 4 \end{cases} \qquad \ell_0 = 0,\ \ell_1 = \tfrac{1}{4},\ \ell_2 = \tfrac{2}{4},\ \ell_3 = \tfrac{3}{4},\ \ell_4 = \ell_5 = \dots = 1$$



a) N = mean number of customers

b) # jobs served in 100 sec. (J_{100})

⇒ We need to know the steady state probabilities.



4 jobs waiting ⇒ all new arrivals leave! ⇒ last state.

Balance equations:

$$\left.\begin{aligned} p_0\lambda &= p_1\mu \\ p_1\lambda &= p_2\mu \\ p_2\tfrac{3}{4}\lambda &= p_3\mu \\ p_3\tfrac{1}{2}\lambda &= p_4\mu \\ p_4\tfrac{1}{4}\lambda &= p_5\mu \\ \sum_{k=0}^{5} p_k &= 1 \end{aligned}\right\} \quad p_0 = 0.3\dots \quad \Rightarrow$$

a) $N = \sum_{i=0}^{5} i \cdot p_i = \dots \approx 1.43$

b) Problem: the arrival rates change with time!

⇒ long calculations!

Faster: When not in S_0, it serves jobs ⇒

$$J_{100} = \frac{(1-p_0)\cdot 100\text{ sec.}}{\bar{x}} = (1-p_0)\mu\cdot 100 \approx 11.66$$

3.6.

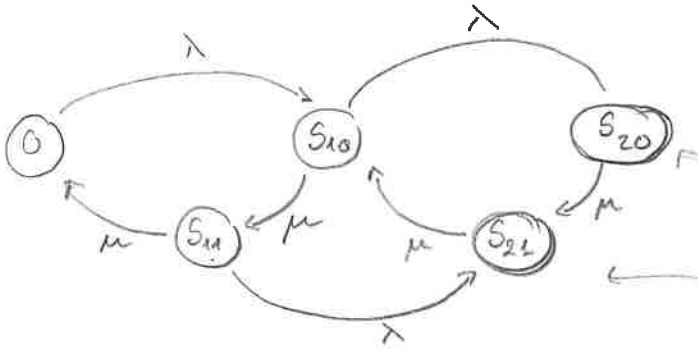
- serve 1, store 2 (altogether -> including the one served...)
- Poisson arrival
- Service: 2 task, <u>each</u> $x_i \sim Exp(\mu)$, $\frac{1}{\mu} = \frac{1}{30}$ [msec]

$p_0 = 0.6$

$T = \{$average time spent at the node$\} = ?$

=> We need arrival rate λ to answer the question, and the state probabilities. => MC

S_0: empty
S_{10}: 1 packet, error-check
S_{11}: — " — transmission
S_{20}: 2 p., one buffered, error-ch.
S_{21}: — " — , transm.

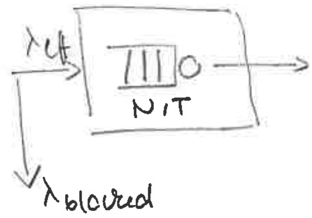


jobs arriving when the system is in these states are lost!

=> Balance eq. $(4 + \sum p_{xy} = 1)$

=> $p_0 = f(\lambda)$ => $\lambda \approx 7.63$.

$$\boxed{T = \frac{\bar{N}}{\lambda_{eff}} = \frac{0 \cdot p_0 + 1(p_{10}+p_{11}) + 2(p_{20}+p_{21})}{[1-(p_{20}+p_{21})]\lambda}}$$



Alternative solution!

Arrives when the system is in $S_0 \rightarrow T_0 = \{$own service$\} = 2 \cdot \frac{1}{\mu}$

$S_{11} \rightarrow T_{11} = \{ tx + $ own service$\} = 3 \cdot \frac{1}{\mu}$

$S_{10} \rightarrow T_{10} = \{$previous service + own service$\} = 4 \cdot \frac{1}{\mu}$

$a_0 = \frac{p_0}{1-(p_{20}+p_{21})}$

$a_{11} = \frac{p_{11}}{1-(p_{20}+p_{21})}$

$a_{10} = \frac{p_{10}}{1-(p_{20}+p_{21})}$

$$\boxed{T = a_0 T_0 + a_{11} T_{11} + a_{10} T_{10}}$$