R3

2 Poisson Process

2.1. Done in lecture

2.2. Done in lecture

2.3. Done in lecture

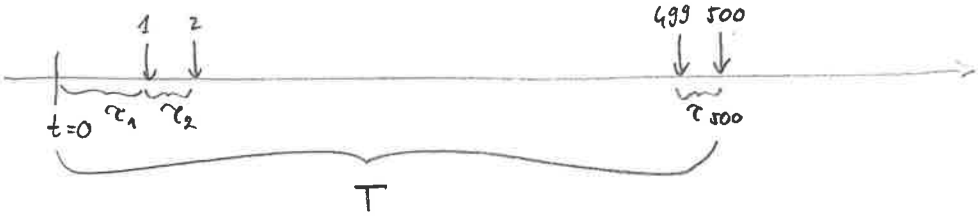
2.4

Poisson arrival process, $\lambda = 1000$ packets/sec.

$p_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$, interarrival $\tau \sim Exp(\lambda)$, $E[\tau] = \frac{1}{\lambda}$, $Var[\tau] = \frac{1}{\lambda^2}$

a) $P(\text{no arrival in } 10^{-3} s) = p_0(10^{-3}) = e^{-10^3 \cdot 10^{-3}} = \boxed{e^{-1}} = 0.36$

b)



$E[T] = ?$ $Var[T] = ?$

$T = \sum_{i=1}^{500} \tau_i$, $E[\tau] = \frac{1}{\lambda}$ $(= 1ms)$: sum of iid r.v. sum of Exp $\Rightarrow$ $\boxed{\text{Erlang - 500}}$

$E[T] = \sum_1^{500} E[\tau] = 500 ms$

$Var[T] = \sum_1^{500} Var[\tau] = 500 \cdot 10^{-6} s^2$

2.6

ALOHA

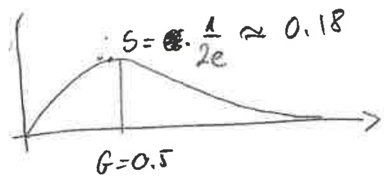
- packets of constant transmission time, T
- packets + retransmitted packets: Poisson (g), g: intensity packets/second, $G = gT$
- Throughput: (define as useful tx time)

Throughput: (time of useful transmission) $S = gT \cdot P(\text{success})$

When is this packet successful?

$= gT P(\text{no packets in } 2T)$

$= gT \cdot e^{-g 2T} = Ge^{-2G}$



2.7

PASTA $\boxed{\text{Poisson arrivals}}$

$a_k(t)$: arriving customer finds state k

$P_k(t)$: state probability

want to prove: $a_k(t) = P_k(t)$

$a_k(t) = P(s(t) = k \mid \text{arrival at } t) = \frac{P(s(t) = k, \text{arrival at } t)}{P(\text{arrival at } t)} = \frac{P(\text{arrival at } t \mid s(t) = k) P(s(t) = k)}{P(\text{arrival at } t)}$

$\stackrel{\text{Poisson}}{=} \frac{P(\text{arrival at } t) P(s(t) = k)}{P(\text{arrival at } t)} = P(s(t) = k) = P_k(t)$. $t \to \infty$ $a_k = P_k$.

3. Balance eq., etc.

3.1 } Solve at home (previous recitation, lectures)
3.2 }

3.3 Solve at home. My solution on the last page.

In class: 3.4 -

3.4. $C = 4.8$ kbit/s

Arrival: Poisson(λ), $\lambda = 10$ mssg/sec.

$p_1 = 0.5$ Type 1: $L_1 \sim Exp$, $\bar{L}_1 = 300$ bits $\Rightarrow T_1 \sim Exp(\mu_1)$ $\frac{1}{\mu_1} = \frac{1}{16}$ sec.

$p_2 = 0.5$ Type 2: $L_2 \sim Exp$ $\bar{L}_2 = 150$ bits $\Rightarrow T_2 \sim Exp(\mu_2)$ $\frac{1}{\mu_2} = \frac{1}{32}$ sec.

a) $E[T] = 0.5\, E[T_1] + 0.5\, E[T_2] = \frac{3}{64}$ sec.

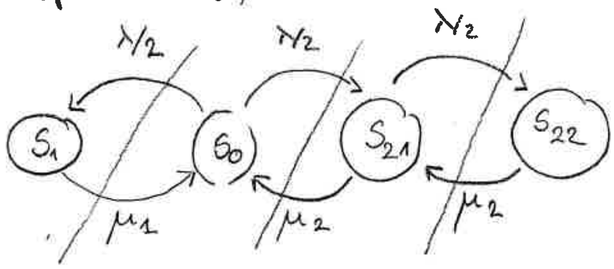
b) Let us draw a Markov-chain of this system!

States: Empty: S_0

1 Type 1: S_1

1 Type 2: S_{21}

2 Type 2: S_{22}



Blocked packets do not afect the state!

(3) Balance eq. - con't

(3.4) con't

Balance equations

let us keep p_0

$p_1 \mu_1 = p_0 \frac{\lambda}{2}$ $\quad p_1 \cdot 16 = p_0 \cdot 5$ $\Rightarrow$ $p_1 = p_0 \cdot \frac{5}{16}$

$p_0 \cdot \frac{\lambda}{2} = p_{21} \mu_2$ $\quad p_0 \cdot 5 = p_{21} \cdot 32$ $\quad p_{21} = p_0 \cdot \frac{5}{32}$

$p_{21} \frac{\lambda}{2} = p_{22} \mu_2$ $\quad p_{21} \cdot 5 = p_{22} \cdot 32$ $\quad p_{22} = p_{21} \cdot \frac{5}{32} = p_0 \cdot \frac{25}{(32)^2}$

$p_0 + p_1 + p_{21} + p_{22} = 1$ $\Rightarrow$ $p_0 \left(1 + \frac{5}{16} + \frac{5}{32} + \frac{25}{(32)^2}\right) = 1$

$p_0 \left(\frac{32^2 + 5 \cdot \cancel{2} \cdot 32 + 5 \cdot 32 + 25}{32^2} \right) = 1$

$p_0 = \frac{1024}{1529} \approx 0.670$

$p_1 \approx 0.105$

$p_{21} \approx 0.016$

$p_{22} \approx 0.209$

b) $E[T_1 \mid \underbrace{\text{accepted}}_{\text{State 0 only}}] = E[\text{transmission time}] = \frac{1}{\mu_1} \approx 62.5\,\text{ms}.$

$E[T_2 \mid \underbrace{\text{accepted}}_{S_0, S_1}] = E[T_2 \mid S_0]\, P[S_0 \mid \text{accepted}] +$
$E[T_2 \mid S_1]\, P[S_1 \mid \text{accepted}] =$

$E[\text{transmission time}] \cdot \frac{p_0}{p_0 + p_1} +$

$E[\text{waiting + transmission time}] \cdot \frac{p_1}{p_0 + p_1} =$

$\frac{1}{p_0 + p_1} \left[p_0 \cdot \frac{1}{16} + p_1 \cdot \frac{2}{16} \right] \approx 35.5\,\text{ms}.$

$\boxed{\text{Pasta! } P(\text{arrives to state } S_0) = p_0}$

c) P(message arrives an is dropped)

$P(\text{dropped} \mid \text{Type 1}) = 1 - P_0 \approx 0.33$

$P(\text{dropped} \mid \text{Type 2}) = P_2 + P_3 \approx 0.225$

3.3

Holding time, T: $f(t) = \mu e^{-\mu t}$ $\quad E[T] = \int_0^\infty t\, f(t)\, dt = \int_0^\infty \mu t e^{-\mu t}\, dt = \frac{1}{\mu}$

Arrival Process $\quad P[N_c = k \mid T = t] = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \qquad E[N_c \mid t] = \lambda t$

$$P(N_c = k] = \int_0^\infty P[N_c = k \mid T = t]\, f(t)\, dt$$

$$\underline{\underline{E[N_c]}} = \sum_{k=0}^\infty k \cdot P[N_c = k] =$$

$$\sum_{k=0}^\infty k \cdot \int_0^\infty P[N_c = k \mid T = t] f(t)\, dt =$$

(finite)

$$\int_0^\infty \underbrace{\sum_{k=0}^\infty k\, P[N_c = k \mid T = t]}_{E[N_c \mid t] = \lambda t} f(t)\, dt =$$

$$\lambda \underbrace{\int_0^\infty t f(t)\, dt}_{E[T] = \frac{1}{\mu}} = \underline{\underline{\frac{\lambda}{\mu}}}$$