KTH, Matematik, Serguei Shimorin

SF2705 Fourier Analysis

Homework assignement for the Lecture 1

1. Prove that two following groups are isomorphic: the group $\mathbb{R}$ of all real numbers with addition as a group operation and the group $\mathbb{R}_+$ of positive real numbers with multiplication as a group operation.

2. The group of characters of a group G is called the dual group to G. It is denoted by $\hat{G}$. The mapping $e_n \mapsto n$ gives an isomorphism between the dual group to S^1, $(S^1)^\wedge$ and the group of integers $\mathbb{Z}$. Prove that the group $\mathbb{Z}^\wedge$ dual to $\mathbb{Z}$ is isomorphic to S^1.

3. Assume that $(e_n)_{n\geq 1}$ is a unit-perpendicular family in an inner product space, i.e. $(e_n, e_m) = \delta_{mn}$. Prove that $\|e_i - e_j\| = \sqrt{2}$ for any $i \neq j$.