EP2200
Queueing theory and teletraffic systems

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Lecture 1

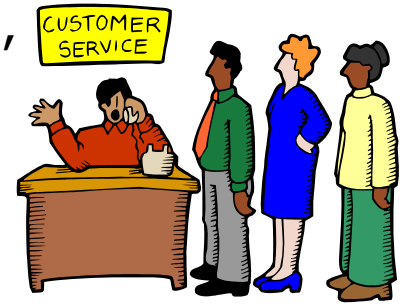
"If you want to model networks
Or a complex data flow
A queue's the key to help you see
All the things you need to know."

(Leonard Kleinrock, Ode to a Queue from IETF RFC 1121)

What is queuing theory? What are teletraffic systems?

Queuing theory

- Mathematical tool to describe resource sharing systems, e.g., telecommunication networks, computer systems
  - Requests arrive dynamically
  - Request may form a queue to wait for service
- Applied probability theory, stochastic processes



Teletraffic systems

- Systems with telecommunication traffic (data networks, telephone networks)
- Are designed and evaluated using queuing theory

Why do we need a whole theory for that?

What is queuing theory?
What are teletraffic systems?

Streaming:

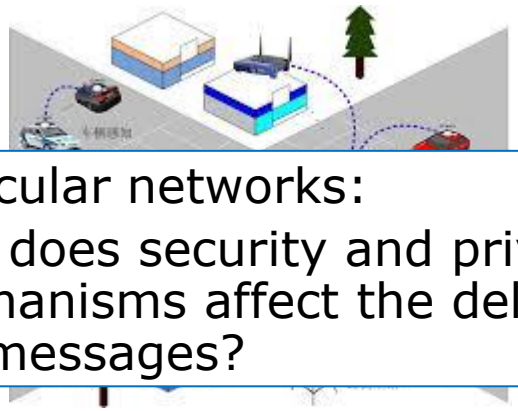
How long should the system pre-fetch to ensure continuous streaming?

Cloud computing:

What limits the performance? The server? The network? My device?

Software Defined Networking:

Will increased time of routing decisions degrade network performance?



Smart grid:

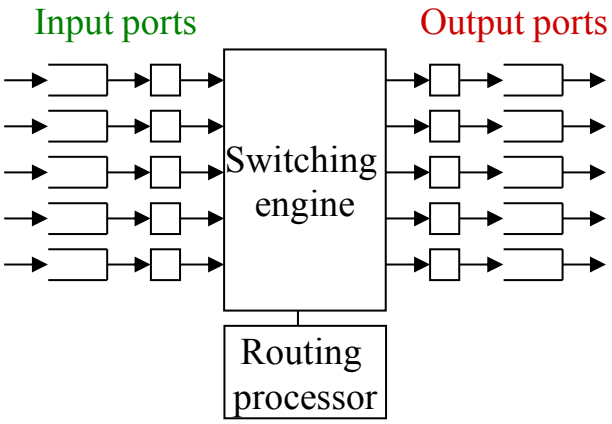
Can the network delay ensure efficient control? Would secure routing paths compromise performance?

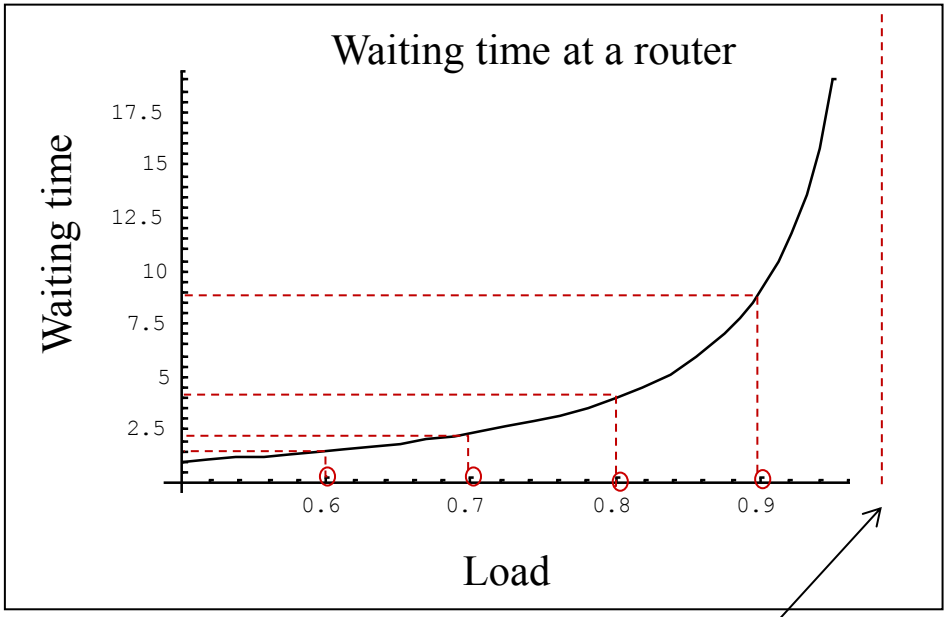
Vehicular networks:

How does security and privacy mechanisms affect the delay of the messages?

Why do we need teletraffic theory?

- Waiting delays at output buffers in network routers – depending on the traffic to be served?
- How will the delay change if the number of packets arriving within a second increases?



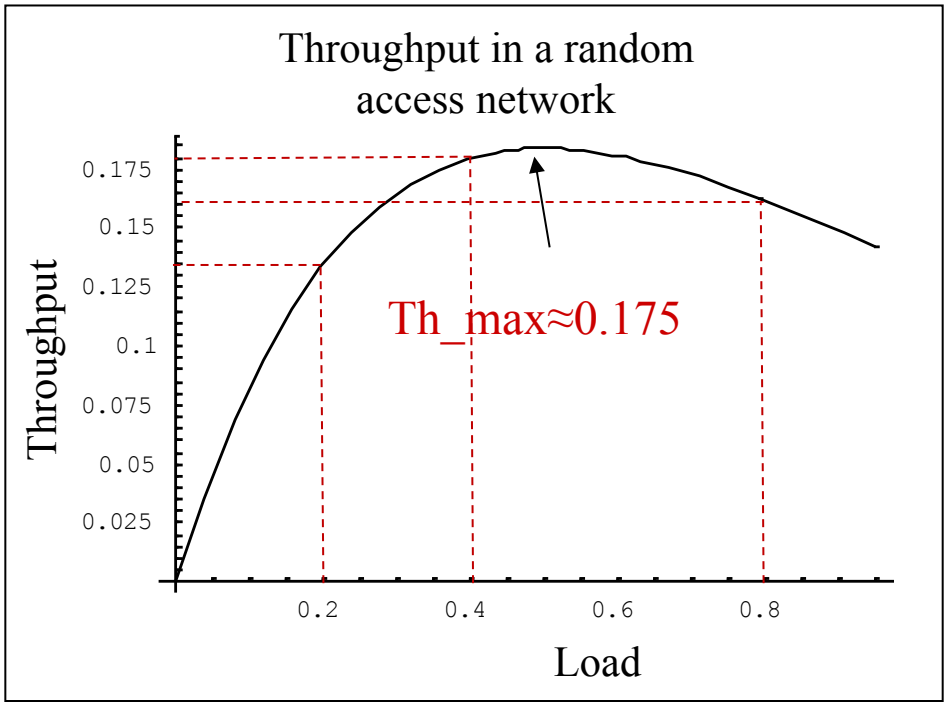


$$Load = \frac{Transmitted\ data\ per\ time\ unit}{Link\ capacity}$$

Load=1 could be served if packets would not arrive randomly

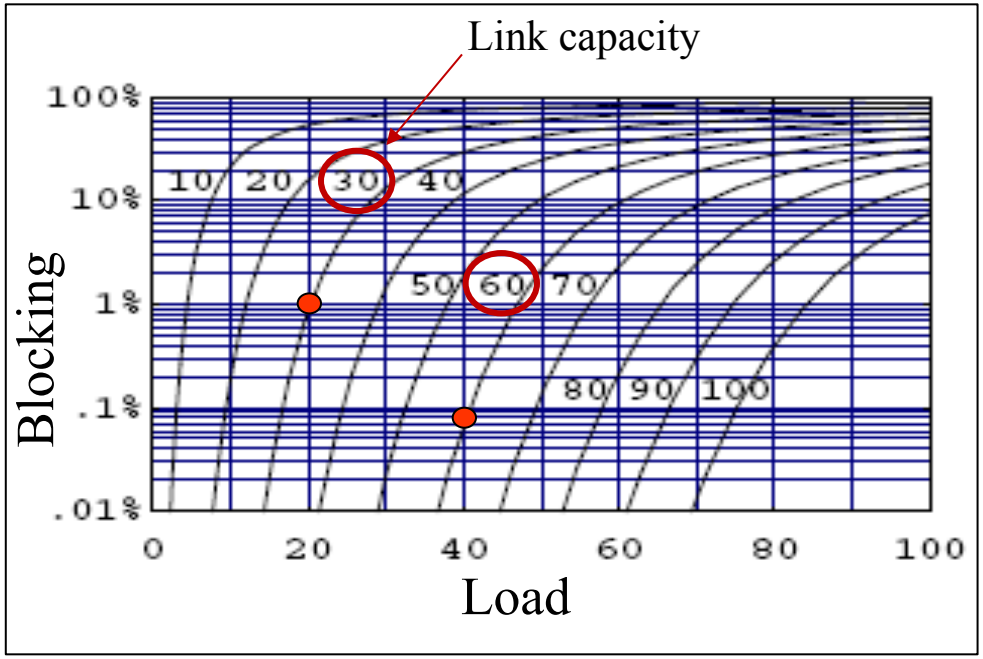
Why do we need teletraffic theory?

- Throughput (useful transmissions) in a wireless network with random access
- If transmissions may collide, how would the throughput change if the number of packets to be sent doubles? What is the effect of packet collisions?

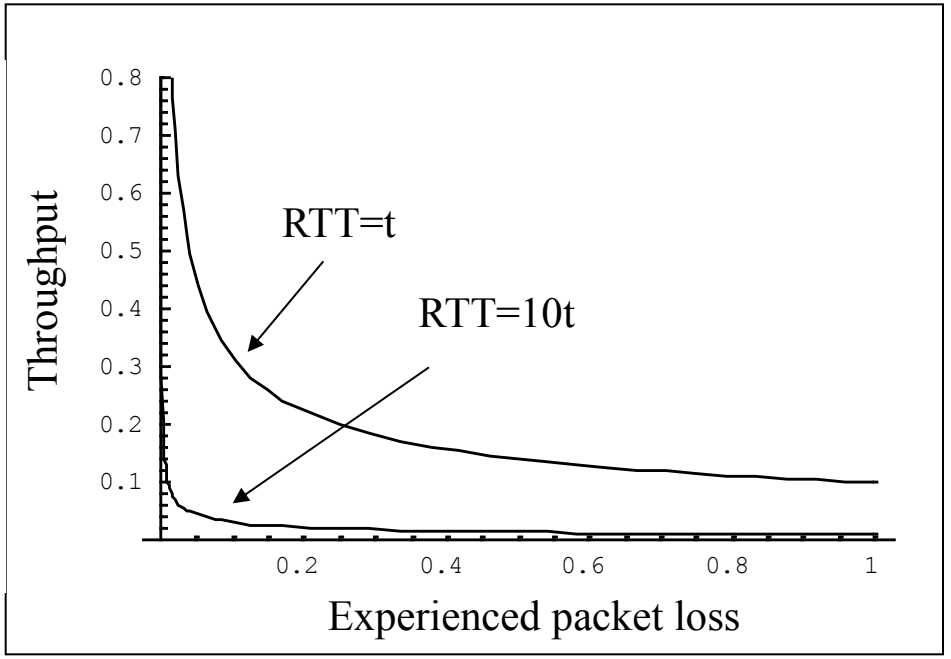


Why do we need teletraffic theory?

Call blocking probability
in a telephone network vs. load
Erlang (Danish, 1878-1929)



TCP throughput vs. packet loss



- Teletraffic systems are non-linear, and mathematical tools are needed to predict their performance and to assist system or protocol design.

Course objectives

- Basic theory
  - understand the theoretical background of queuing systems, apply the theory for systems not considered in class
- Applications
  - find appropriate queuing models of simple problems, derive performance metrics
- Basis for modeling more complex problems
  - advanced courses on performance evaluation
  - master thesis project
  - industry (telecommunication engineer)
- Prerequisites
  - mathematics, statistics, probability theory, stochastic systems
  - communication networks, computer systems

Learning Outcomes

- Discuss and apply the theory of continuous time Markov-processes to describe complex stochastic systems.
- Discuss, derive and apply the theory of Markovian queuing systems, and some of the simpler non-Markovian queuing systems.
- Discuss and apply the theory of non-Markovial queuing systems.
- Discuss and apply main theoretic results for the modeling of queuing networks.
- Analyze communication, networking or computer engineering related problems with the tools of Markov-processes and queuing models.

Course organization

- Course responsible, lectures and recitations
  - Viktoria Fodor <vfodor@kth.se>

- Course web page
  - KTH Social EP2200 (https://www.kth.se/social/course/EP2200/)
  - Course reading material, home assignments, project, messages, updated schedule and course information
    - Your responsibility to stay up to date!
  - Useful resources: applets, calculators
  - Useful links: on-line books
  - Links to probability theory basics

Course material

- All course material on line
  - Lecture notes by Jorma Virtamo, HUT, and Philippe Nain, INRIA
    - Used with their permission
  - Excerpts from L. Kleinrock, *Queueing Systems*
  - Problem set with outlines of solutions
  - Old exam problems with full solutions
  - Erlang tables
  - Formula sheet, Laplace transforms

- Printed course material: STEX (from Friday)

- No text book needed!
  - If you would like a book, then you can get one on your own
    - Ng Chee Hock, *Queueing Modeling Fundamentals*, Wiley, 1998. (simple)
    - L. Kleinrock, *Queueing Systems, Volume 1: Theory*, Wiley, 1975 (well known, engineers)
    - D. Gross, C. M. Harris, *Fundamentals of Queueing Theory*, Wiley, 1998 (difficult)
  - Beware, the notations might differ

Course organization

- 12 lectures – cover the theoretical part
- 12 recitations – applications of theory
- Two home assignments and a project (1.5 ECTS, compulsory, pass/fail)
- Deadlines, information on the web
- Home assignments (beginning and middle of the course)
  - numerical exercises and proofs
  - individual submission, only handwritten version
  - you need 75% satisfactory solution to pass this moment
  - Submit in class or at the STEX office
- Small project (end of the course)
  - computer exercise (matlab, C, java, simulation platform…)
  - +5 points for outstanding projects (upper 10%)

Exam

- There is a written exam to pass the course, 5 hours
  - Consists of five problems of 10 points each
    - Passing grade usually 20 ± 3 points
  - Allowed aid is the Beta mathematical handbook (or similar) and simple calculator. Probability theory and queuing theory books are not allowed!
  - The sheet of queuing theory formulas will be provided, also Erlang tables and Laplace transforms, if needed (same as in the course binder and on the web)
- Possibility to complementary oral exam if you miss E by 2-3 points (Fx)
  - Complement to E
- Registration is mandatory for all the exams
  - At least two weeks prior to the exam
- Students from previous years: contact STEX (stex@ee.kth.se) if you are not sure what to do

- Questions: e-mail or via KTH Social, including asking for meeting

PhD students in brief

- Lectures and recitations as for all students
- Submission of home assignments as all students
- Exam as all students (P/F, at least grade B level to pass)

- Additional weekly seminars on advanced material
- Different project

- Please say after class for a brief discussion

Step 0

- Queuing theory is applied probability theory
- Students need to be able to work with basic probability theory tools

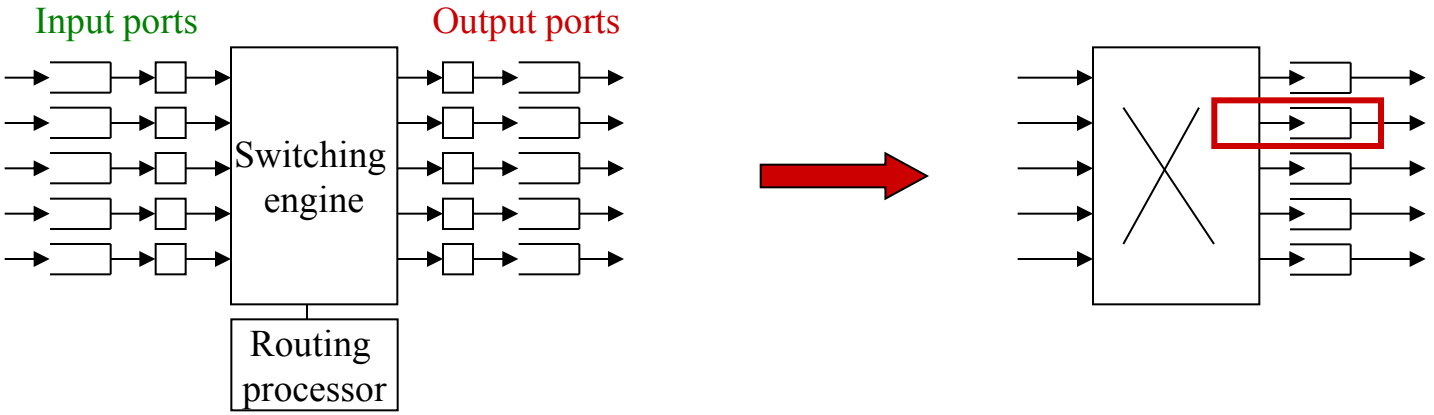
- Short summary in Virtamo notes, chapters 1-4
- Suggested video lectures
- First recitation is dedicated to probability theory overview
- Early home assignment with only probability theory problems

Lecture 1
Queuing systems - introduction

- Teletraffic examples and the performance triangle
- The queuing model
  - Block diagram
  - System parameters
  - Performance measures
- Stochastic processes recall

Example

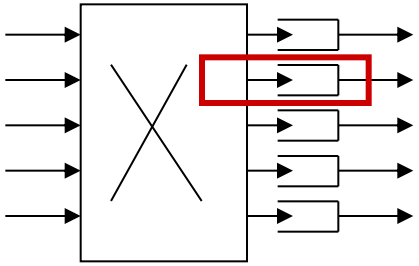
- Packet transmission at a large IP router



- We simplify modeling
  - typically the switching engine is very fast
  - the transmission at the output buffers limits the packet forwarding performance
  - we do not model the switching engine, only the output buffers

Example

- Packet transmission at the output link of a large IP router – packets arrive randomly and wait for free output link

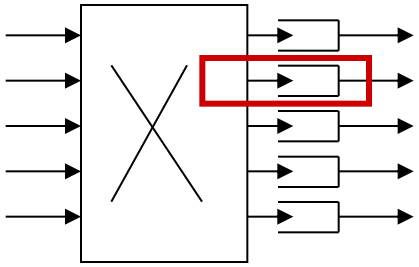


- Performance:
  - Waiting time in the buffer
  - Number of packets waiting
  - Probability of buffer overflow and packet loss

- Depends on:
  - How many packets arrive in a time period (packet/sec)
  - How long is the transmission time (packet out of the buffer)
    - Link capacity (bit/s)
    - Packet size (bits)

Example

- Packet transmission at the output link of a large IP router - packets wait for free output link



- Performance:
  - Number of packets waiting
  - Waiting time in the buffer
  - Probability of buffer owerflow

- Depends on:
  - How many packets arrive
  - Packet size

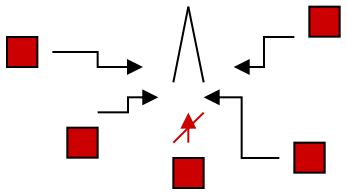
  → Service demand

  - Link capacity

  → Server capacity

Example

- Voice calls in a GSM cell – calls arrive randomly and occupy a "channel". Call blocked if all channels busy.



- Performance
  - Probability of blocking a call
  - Utilization of the channels

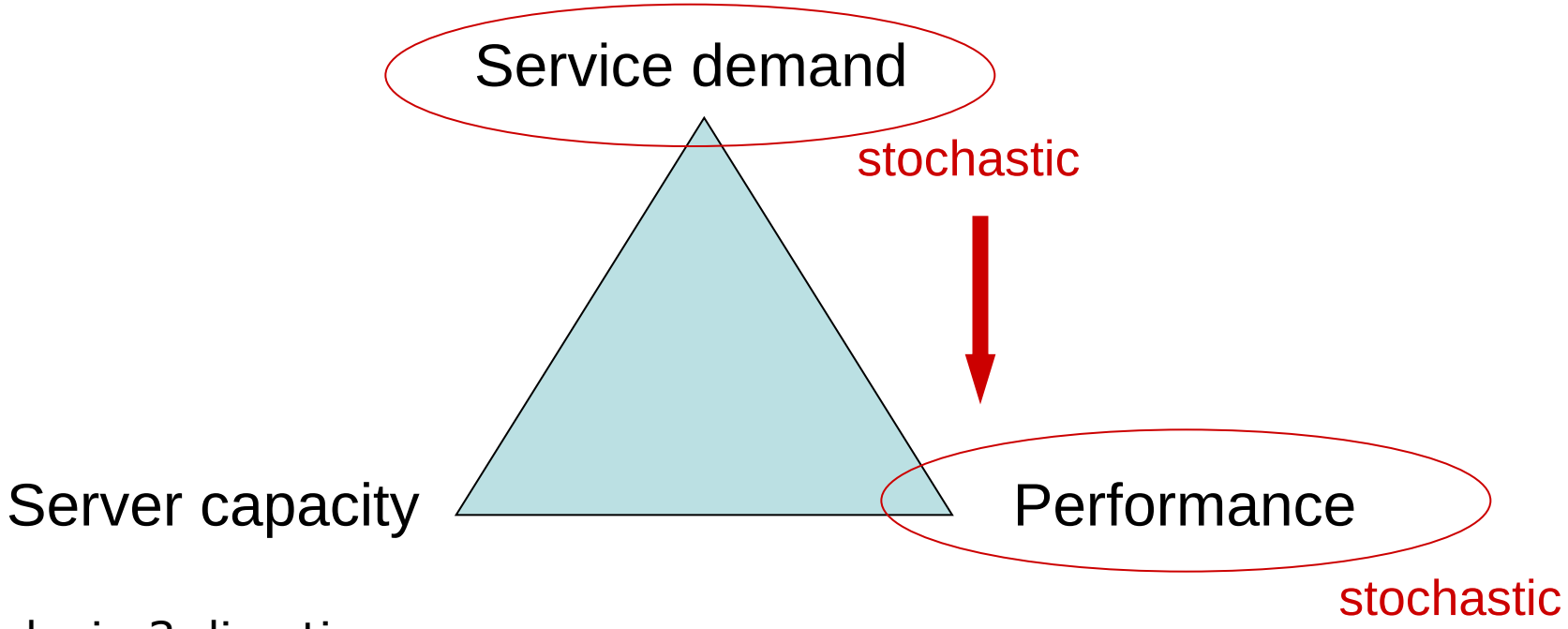
- Depends on:
  - How many calls arrive
  - Length of a conversation

  } → Service demand

  - Cell capacity (number of voice channels) → Server capacity

Performance of queuing systems

- The triangular relationship in queuing



- Works in 3 directions
  - Given service demand and server capacity → achievable performance
  - Given server capacity and required performance → acceptable demand
  - Given demand and required performance → required server capacity

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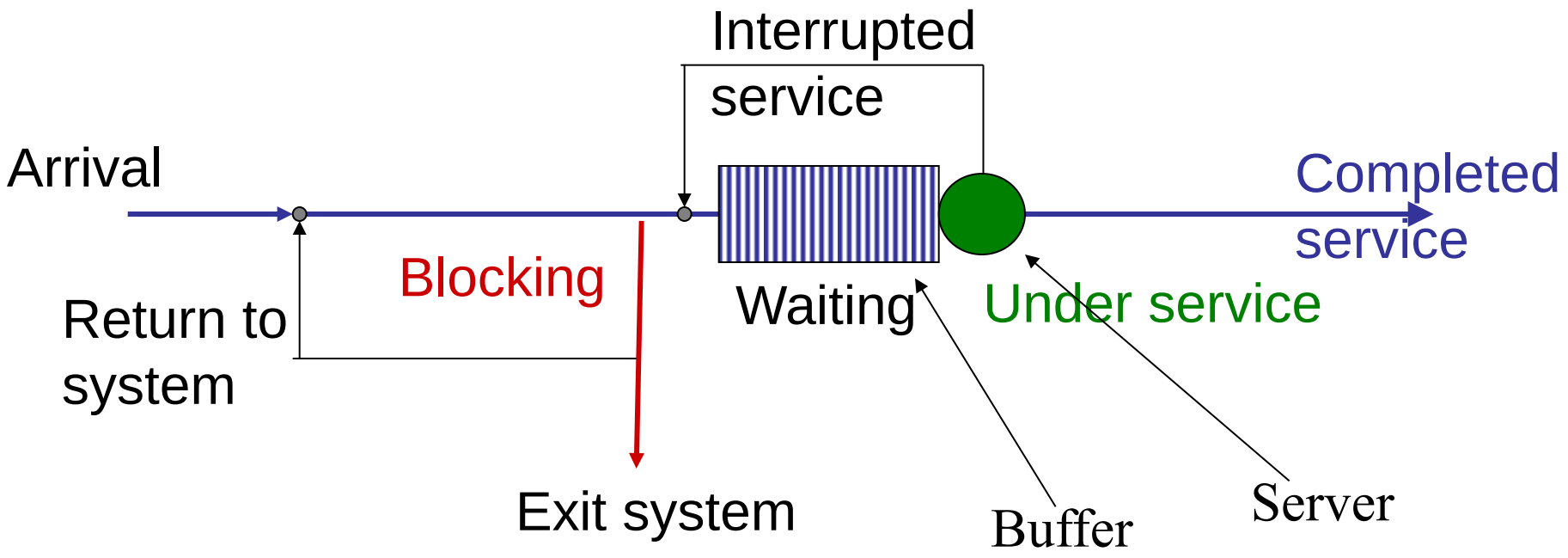
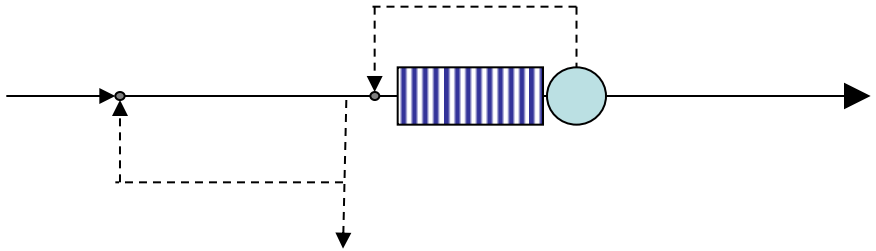
Block diagram of a queuing system

- Queuing system: abstract model of a resource sharing system
  - buffer and server(s)
- Customers: arrive, wait, get served and leave the queuing system
  - customers can get blocked, service can be interrupted



Description of queuing systems

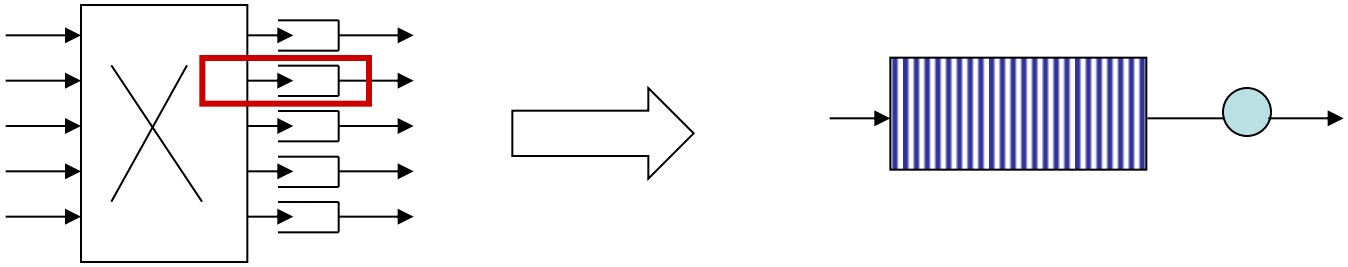
- System parameters (related to server capacity)
  - Number of servers (customers served in parallel)
  - Buffer capacity
    - Infinite: enough waiting room for all customers
    - Finite: customers might be blocked
  - Order of service (FIFO, random, priority)
- Service demand (stochastic)
  - Arrival process: How do the customers arrive to the system – given by a stochastic process
  - Service process: How long service time does a customer demand – given by a probability distribution



Customer:
- IP packet
- Phone call

Examples in details

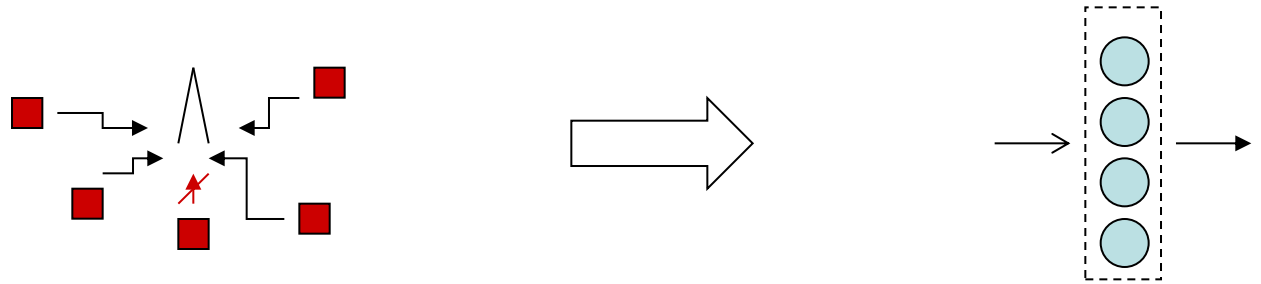
- Packet transmission at the output link of a large IP router



- Number of servers: 1
- Buffer capacity: max. number of IP packets
- Order of service: FIFO
- Arrivals: IP packet multiplexed at the output buffer
- Services: transmission of one IP packet
  (service time = transmission time = packet length / link transmission rate)

Examples in details

- Voice calls in a GSM cell
  - channels for parallel calls, each call occupies a channel
  - if all channels are busy the call is blocked



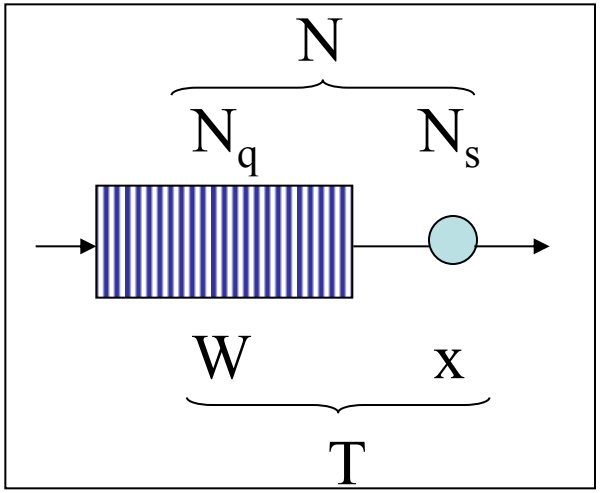
- Number of servers: number of parallel channels
- Buffer capacity: no buffer
- Order of service: does not apply
- Arrivals: call attempts in the GSM cell
- Service: the phone call (service time = length of the phone call)

Group work

1. Service at a bank, with “queue numbers” and several clerks
2. Cashiers at the supermarket, customers select a queue randomly and wait there in a queue
3. Several terminals transmitting in a WLAN

- Draw the block diagram of the queuing systems
- Describe the model: arrivals, service, number of servers, buffer capacity, order of service

Performance measures



- Number of customers in the system (N)
  - Number of customers in the queue (N_q)
  - Number of customers in the server (N_s)
- System time (T)
  - Waiting time of a customer (W)
  - Service time of a customer (x)
- Probability of blocking (blocked customers / all arrivals)
- Utilization of a server (time server occupied / all considered time)

- Transient measures
  - how will the system state change in the near future?
- Stationary measures
  - how does the system behave on the long run?
  - average measures
  - often considered in this course

Lecture 1
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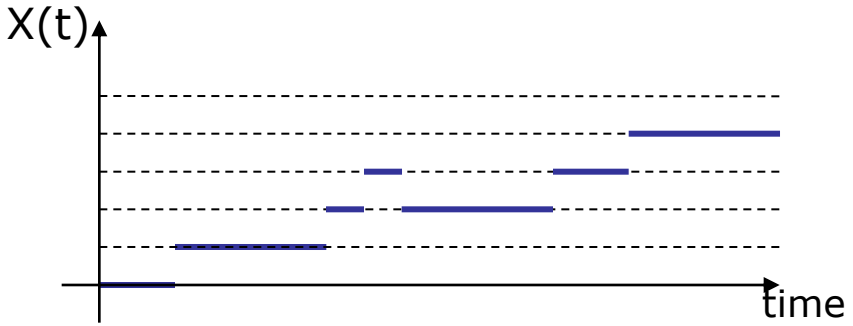
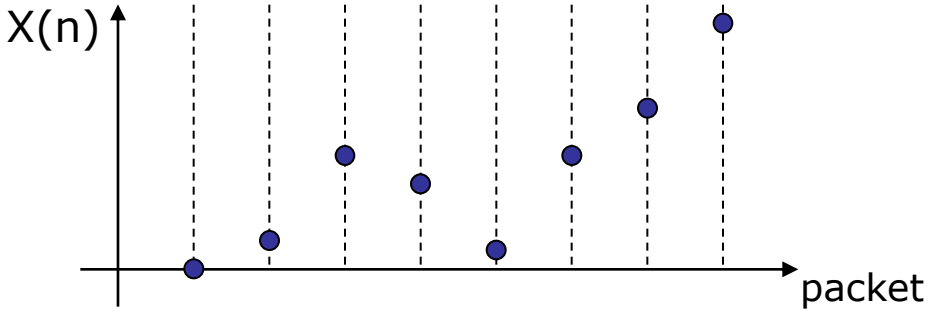
- Stochastic processes recall

Stochastic process

- Stochastic process
  - –A system that evolves – changes its state - in time in a random way
  - –Family of random variables
  - –Variables indexed by a time parameter
    - Continuous time: *X(t),* a random variable for each value of *t*
    - Discrete time: *X(n),* a random variable for each step *n=0,1,…*
  - –State space: the set of possible values of r.v. *X(t) (or X(n))*
    - Continuous or discrete state



- Number of packets waiting:
  - – Discrete space
  - – Continuous time



- Waiting time of consecutive packets:
  - – Discrete time
  - – Continuous space

Stochastic process - statistics

- We are interested in quantities, like:
  - time dependent (transient) state probabilities (statistics over many realizations, an *ensemble* of realizations, *ensemble average*):

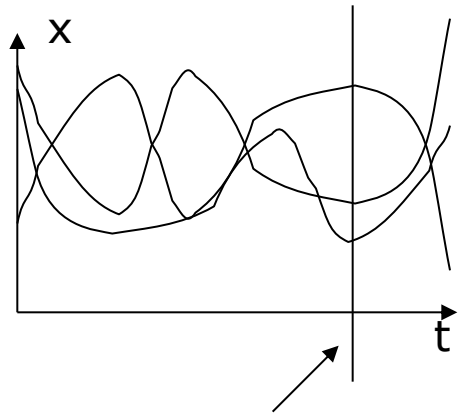
$$f_x(t) = P(X(t) = x), \;\; F_x(t) = P(X(t) \le x)$$

  - n^{th} order statistics – joint distribution over n samples

$$F_{x_1,\ldots,x_n}(t_1,\ldots,t_n) = P(X(t_1) \le x_1,\ldots, X(t_n) \le x_n)$$

  - limiting (or stationary) state probabilities (if exist) :

$$f_x = \lim_{t\to\infty} P(X(t) = x), \quad F_x = \lim_{t\to\infty} P\{X(t) \le x\}$$



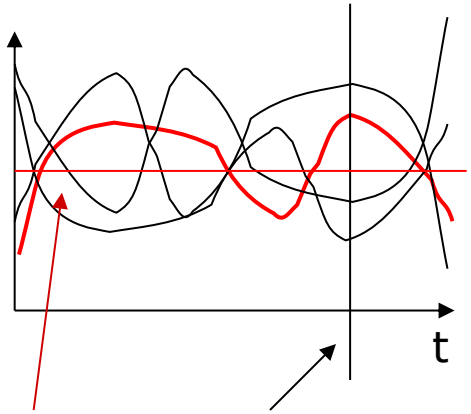
Stochastic process - terminology

- The stochastic process is:
  - stationary, if all nth order statistics are unchanged by a shift in time:

$$F_x(t+\tau) = F_x(t), \quad \forall t$$

$$F_{x_1,\ldots,x_n}(t_1+\tau,\ldots,t_n+\tau) = F_{x_1,\ldots,x_n}(t_1,\ldots,t_n), \quad \forall n, \quad \forall t_1,\ldots,t_n$$

  - ergodic, if the ensemble average is equal to the time average of a single realization
  - consequence: if a process ergodic, then the statistics of the process can be determined from a single (infinitely long) realization and vice versa



Stochastic process

- Example on stationary versus ergodic
- Consider a source, that generates the following sequences with the same probability (state space A,B,E):
  - ABABABAB…
  - BABABABA…
  - EEEEEEEE…
- Is this source stationary?

- Is this source ergodic?

Summary

Today:

- Queuing systems - definition and parameters
- Stochastic processes

Next lecture:

- Poisson processes and Markov-chains, the theoretical background to analyze queuing systems

Recitation:

- Probability theory and transforms (Have a quick look at Virtamo 1-4 before class)
  - Definition of probability of events
  - Conditional probability, law of total probability, Bayes formula, independent events
  - Random variables, distribution functions
    - Bernoulli, Binomial, Geometric, **Poisson**
    - Uniform, **Exponential, Erlang-k**
  - Z and Laplace transforms