

## Conditional Distribution

⑥

$$P(X_1, X_2) = N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}\right)$$

Goal: want to find  $p(x_1|x_2)$

$$P(X_1, X_2) = p(x_1|x_2)p(x_2)$$

$$P(X_2) = N(\mu_2, \Sigma_{22})$$

$$P(X_1, X_2) \propto \exp\left(-\frac{1}{2}\begin{pmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \end{pmatrix}^T \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}^{-1} \begin{pmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \end{pmatrix}\right)$$

① we want to factor out

$$p(x_2) \propto \exp\left(-\frac{1}{2}(x_2 - \mu_2)^T \Sigma_{22}^{-1} (x_2 - \mu_2)\right)$$

from this exponent

② - If we can re-write the covariance in such a manner so that it factorises, i.e. becomes block diagonal.

- How do we invert the covariance matrix to keep  $\Sigma_{22}$  isolated?