Prof. Dr. Tino Weinkauf

Introduction to Visualization and Computer Graphics

Grids and Interpolation

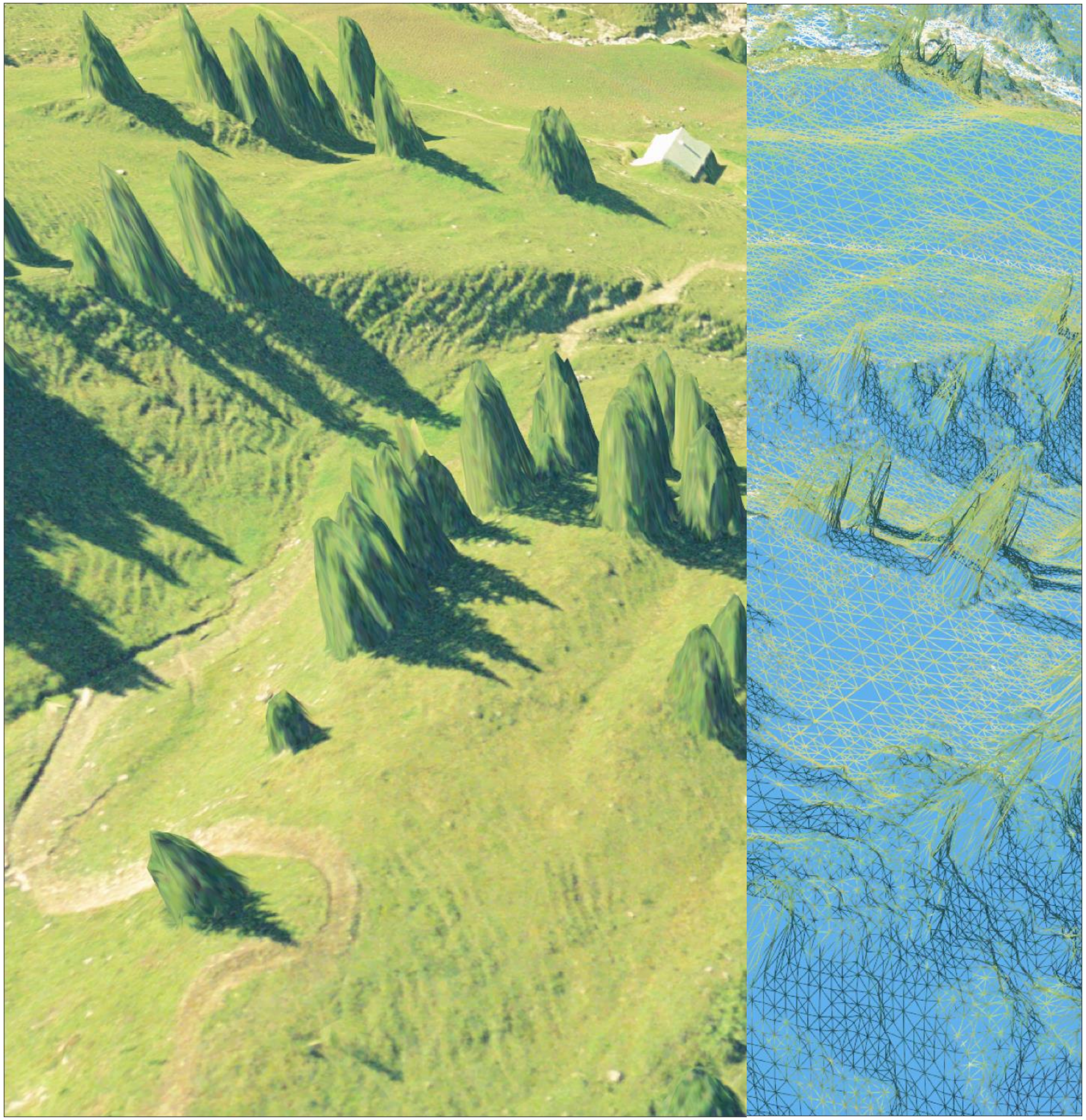
- No lecture next Tuesday!

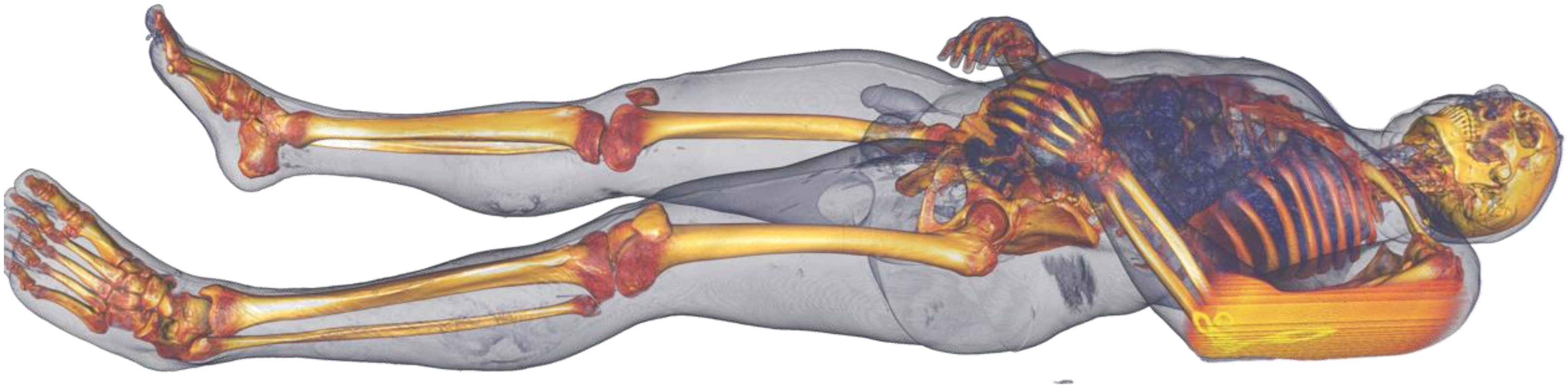
Introduction to Visualization and Computer Graphics
DH2320, Fall 2015

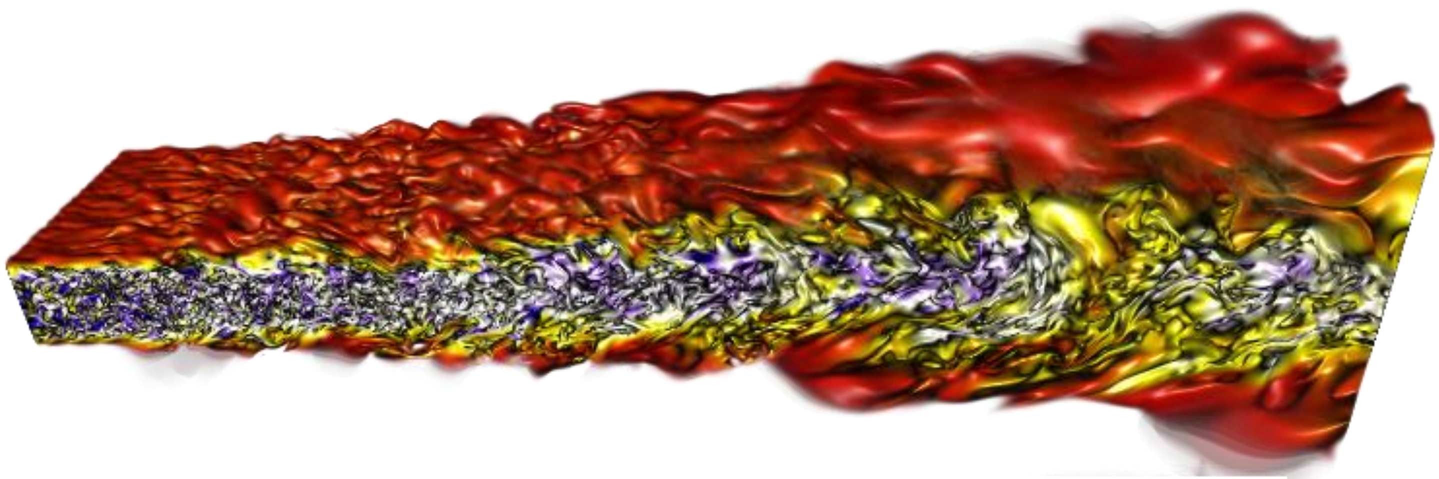
Prof. Dr. Tino Weinkauf

Grids and Interpolation

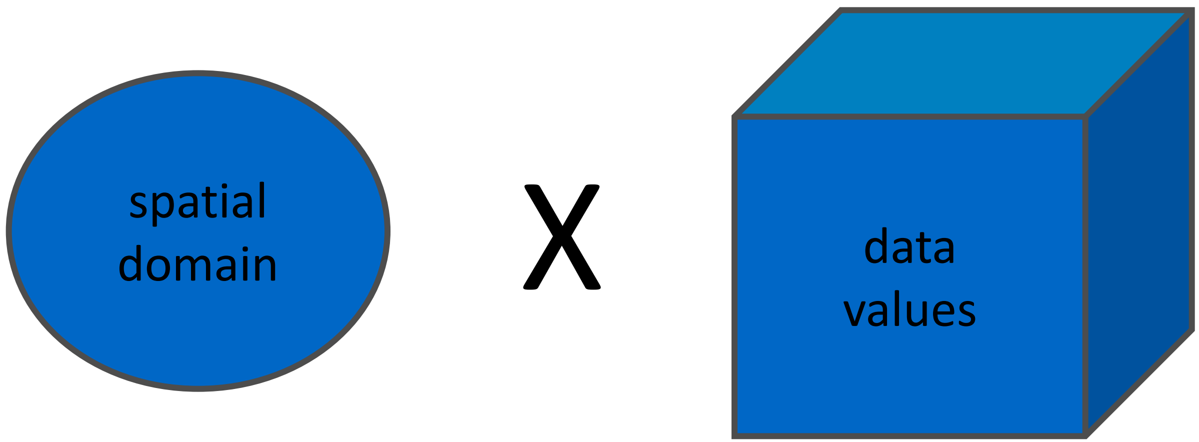
Structured Grids
Unstructured Grids







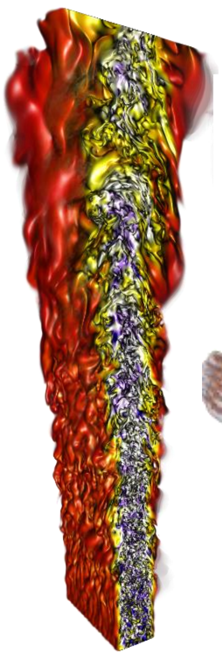
R^n R^m

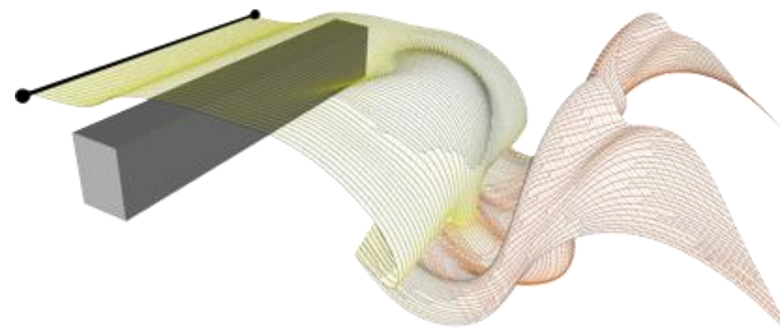


independent variables *dependent variables*

- In most cases, the visualization data represent a **continuous real object**, e.g., an oscillating membrane, a velocity field around a body, an organ, human tissue, etc.
  - This object lives in an n-dimensional space - the **domain**
- Usually, the data is only given at a finite set of locations, or **samples**, in space and/or time
  - Remember imaging processes like numerical simulation and CT-scanning, note similarity to pixel images
- We call this a **discrete structure**, or a **discrete representation** of a continuous object

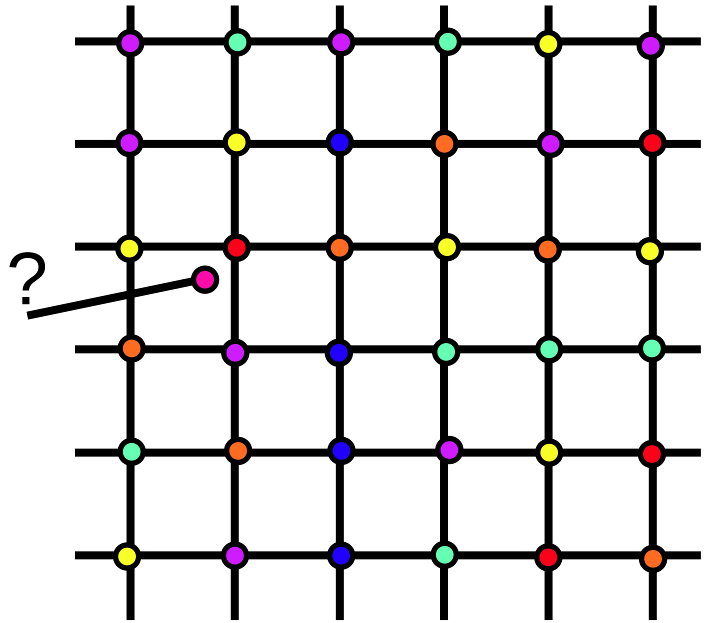
- Discrete representations
  - We usually deal with the reconstruction of a continuous real object from a given discrete representation



- Discrete structures consist of point samples
- Often, we build **grids/meshes** that connect neighboring samples

- Discrete representations
  - We usually deal with the reconstruction of a continuous real object from a given discrete representation



- Discrete structures consist of point samples
- Often, we build **grids/meshes** that connect neighboring samples

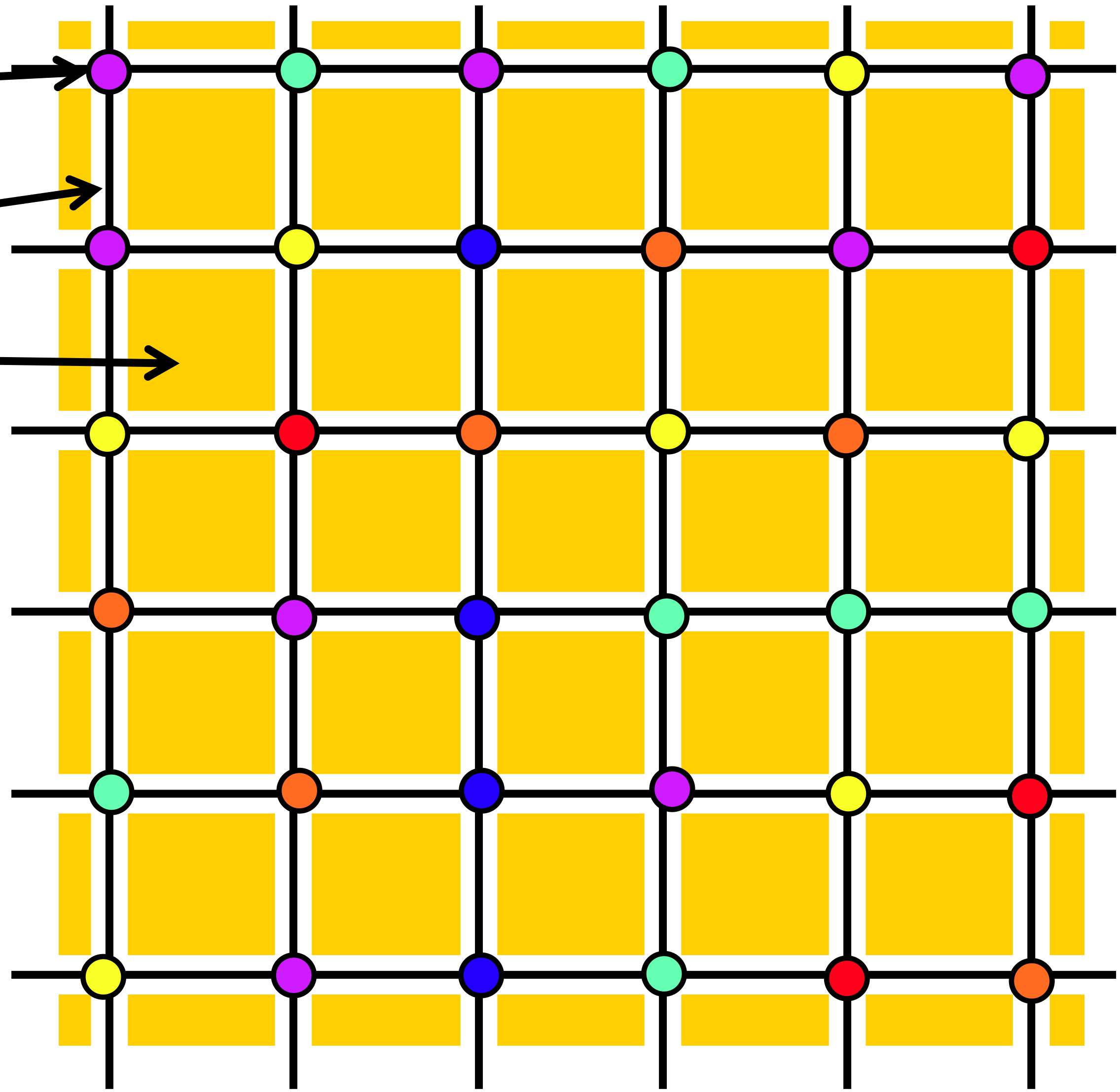
- Grid terminology

0D: grid vertex
(grid point)

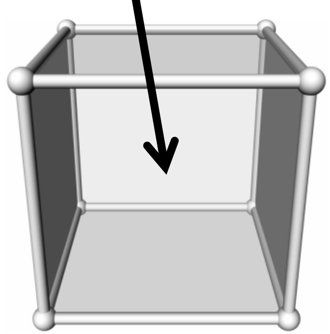
1D: grid line

2D: grid face

3D: grid voxel



grid vertices
grid lines
grid faces



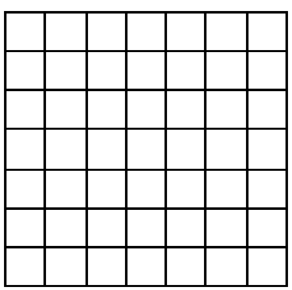
grid cell: largest-dimensional element in a grid
2D: grid face
3D: grid voxel

Data Connectivity

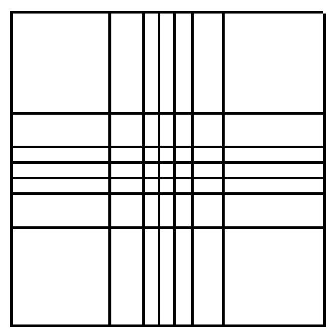
- There are different types of grids:
- **Structured grids**
  connectivity is implicitly given.
  - **Block-structured grids**
    combination of several structured grids
- **Unstructured grids**
  connectivity is explicitly given.
- **Hybrid grids**
  combination of different grid types

Structured grids

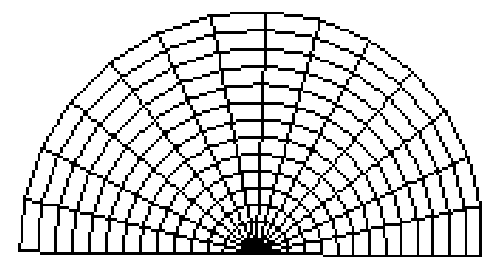
- “Structured” refers to the implicitly given connectivity between the grid vertices
- We distinguish different types of structured grids regarding the implicitly or explicitly given coordinate positions of the grid vertices



uniform grid
implicitly given coordinates



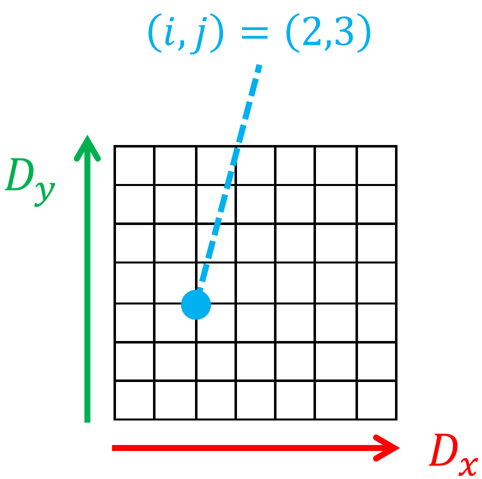
rectilinear grid
semi-implicitly given coordinates



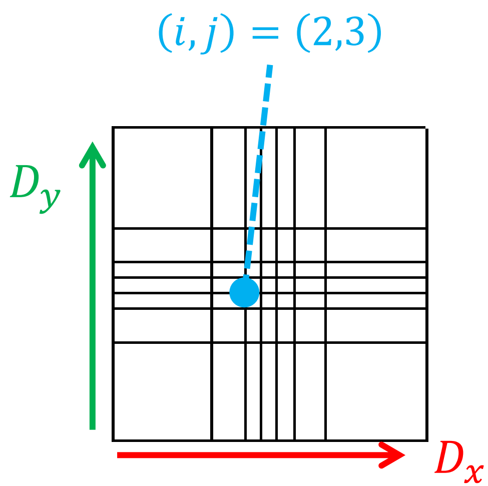
curvilinear grid
explicitly given coordinates

Structured grids

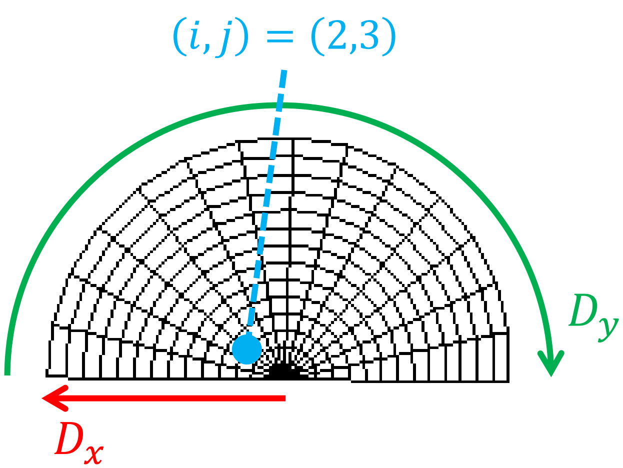
- Number of grid vertices: D_x, D_y, D_z
- We can address every grid vertex with an index tuple (i, j, k)
  - $0 \leq i < D_x$ $\quad$ $0 \leq j < D_y$ $\quad$ $0 \leq k < D_z$



uniform grid
implicitly given coordinates



rectilinear grid
semi-implicitly given coordinates



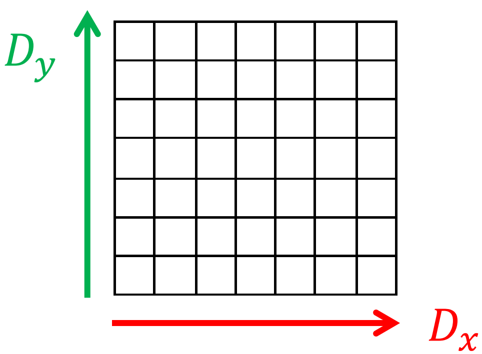
curvilinear grid
explicitly given coordinates

Structured grids

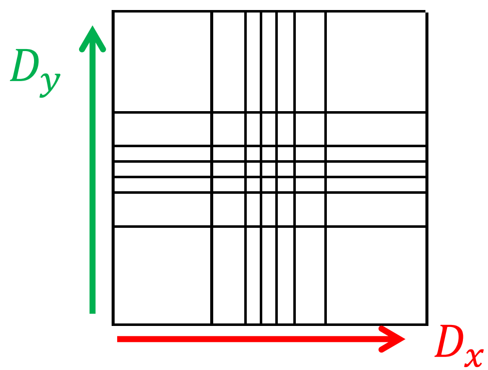
- Number of grid vertices: D_x, D_y, D_z

- We can address every **grid cell** with an index tuple (i, j, k)
  - $0 \leq i < D_x - 1$ $\quad$ $0 \leq j < D_y - 1$ $\quad$ $0 \leq k < D_z - 1$

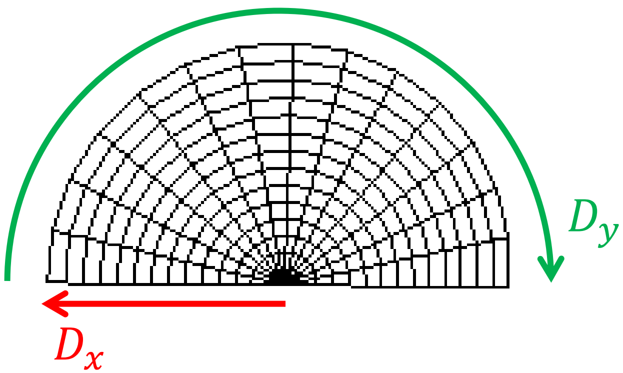
- ➔ Number of cells: $(D_x - 1) \times (D_y - 1) \times (D_z - 1)$



uniform grid
implicitly given coordinates



rectilinear grid
semi-implicitly given coordinates



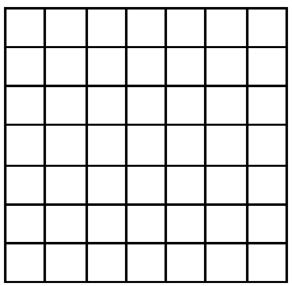
curvilinear grid
explicitly given coordinates

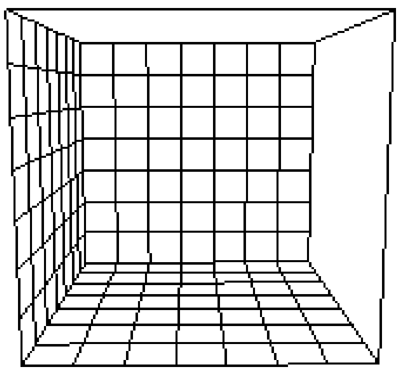
very important

- **Regular or uniform grids**

- Cells are rectangles or rectangular cuboids of the same size

- All grid lines are parallel to the axes





- To define a uniform grid, we need the following:
  - Bounding box: $(x_{min}, y_{min}, z_{min})$ - $(x_{max}, y_{max}, z_{max})$
  - Number of grid vertices in each dimension: D_x, D_y, D_z
  - ➔ Cell size: d_x, d_y, d_z

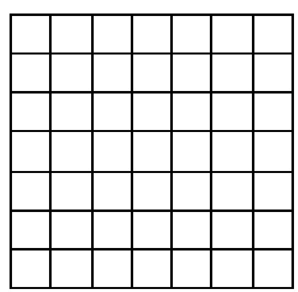
- **Regular or uniform grids**

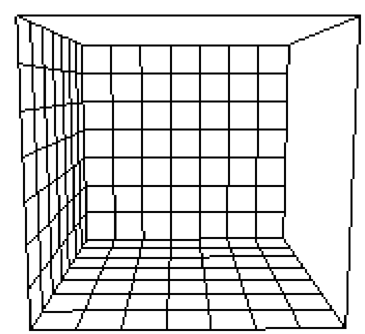
- Well suited for image data (medical applications)

- Coordinate ➔ cell is very simple and cheap
  - Global search is good enough; local search not required

- Coordinate of a grid vertex:

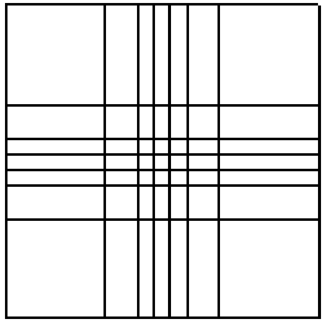
$$(i \cdot d_x, j \cdot d_y, k \cdot d_z)$$



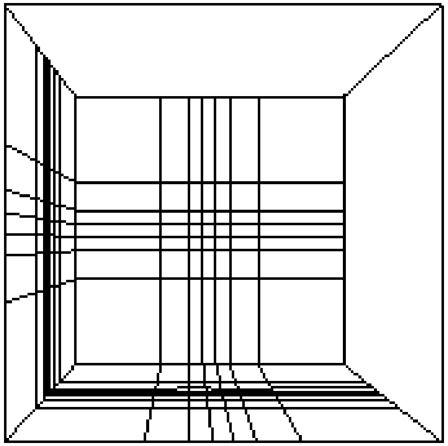


- **Cartesian grid**
- Special case of a uniform grid: $d_x = d_y = d_z$
- Consists of squares (2D), cubes (3D)

- **Rectilinear grids**



- Cells are rectangles of *different* sizes
- All grid lines are parallel to the axes
- Vertex locations are inferred from positions of grid lines for each dimension:
  - XLoc = {0.0, 1.5, 2.0, 5.0, …}
  - YLoc = {-1.0, 0.3, 1.0, 2.0, …}
  - ZLoc = {3.0, 3.5, 3.6, 4.1, …}



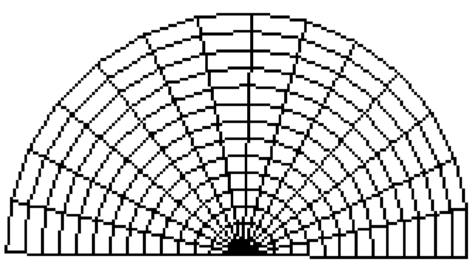
- Coordinate ➔ cell still quite simple

- **Curvilinear grids**

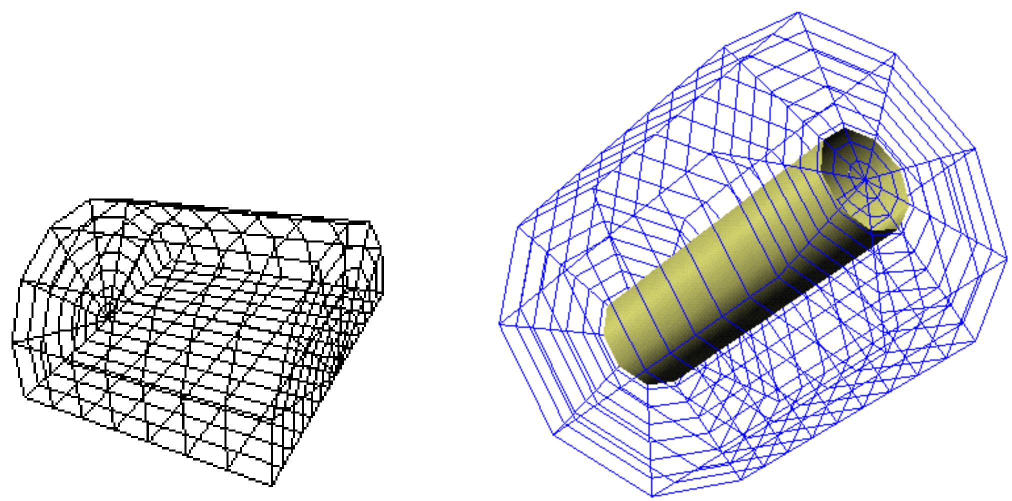
- Vertex locations are explicitly given
  - XYZLoc = {(0.0, -1.0, 3.0), (1.5, 0.3, 3.5), (2.0, 1.0, 3.6), …}

- Cells are quadrilaterals or cuboids

- Grid lines are not (necessarily) parallel to the axes



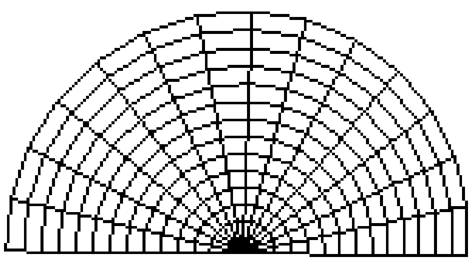
2D curvilinear grid



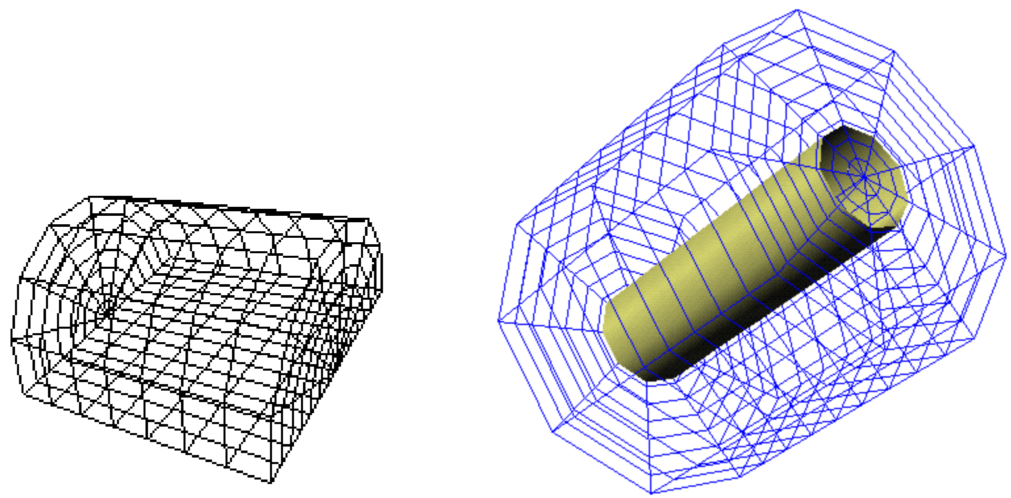
3D curvilinear grids

- **Curvilinear grids**

- Coordinate ➔ cell:
  - **Local search** within last cell or its immediate neighbors
  - **Global search** via quadtree/octree



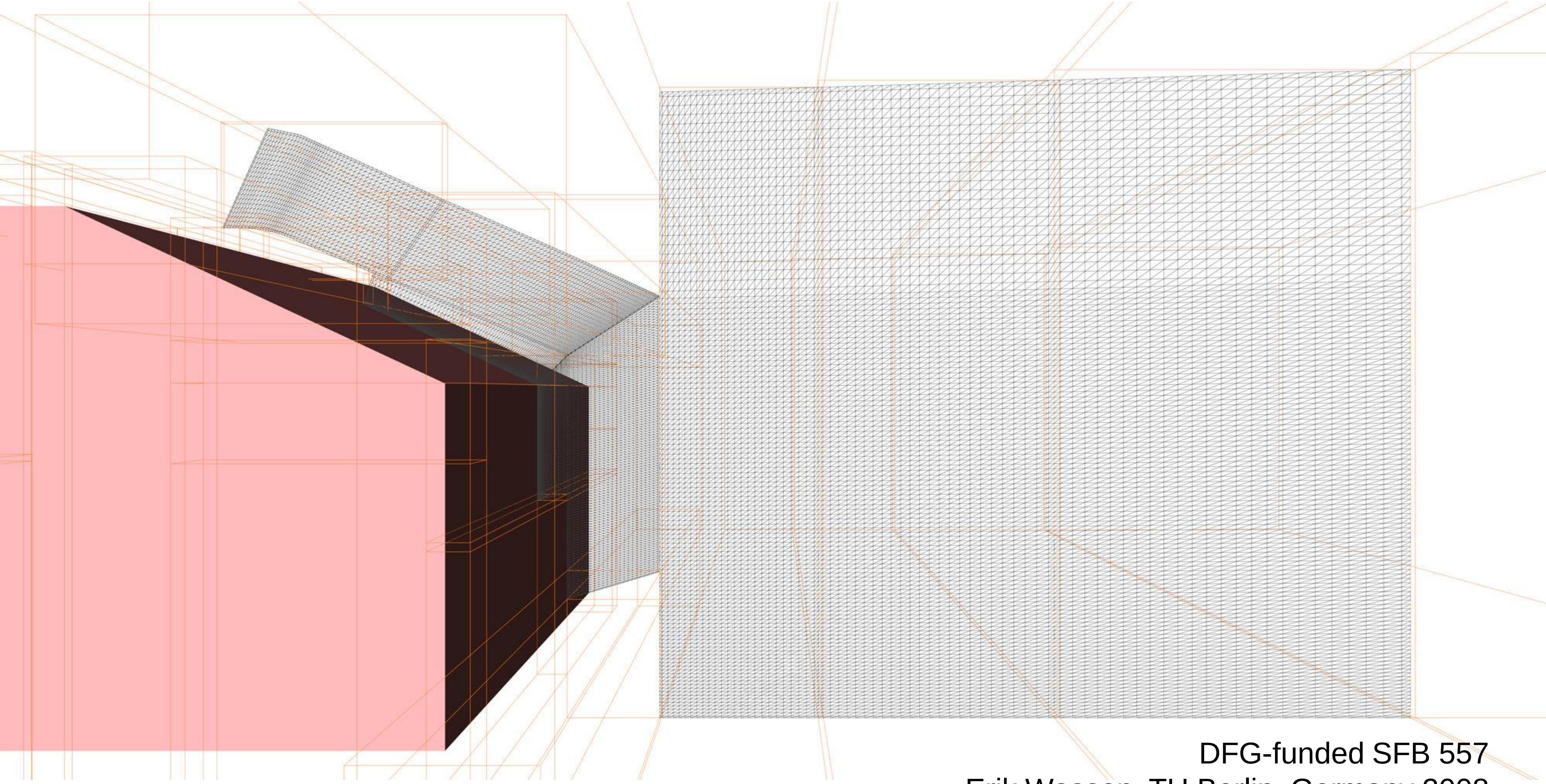
2D curvilinear grid



3D curvilinear grids

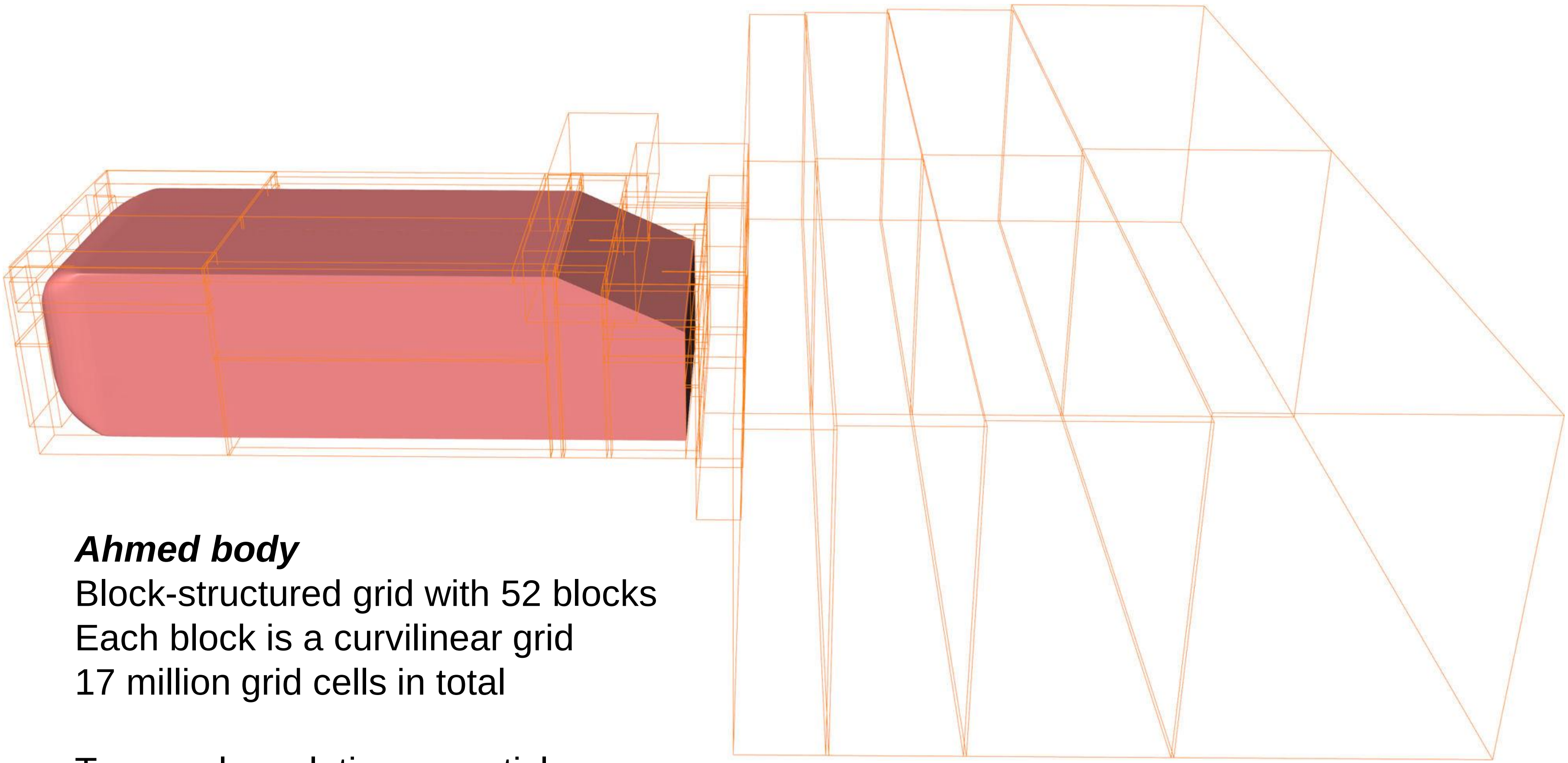
- **Block-structured grids**

- combination of several structured grids



DFG-funded SFB 557
Erik Wassen, TU Berlin, Germany 2008

- Demands on data storage, an example:

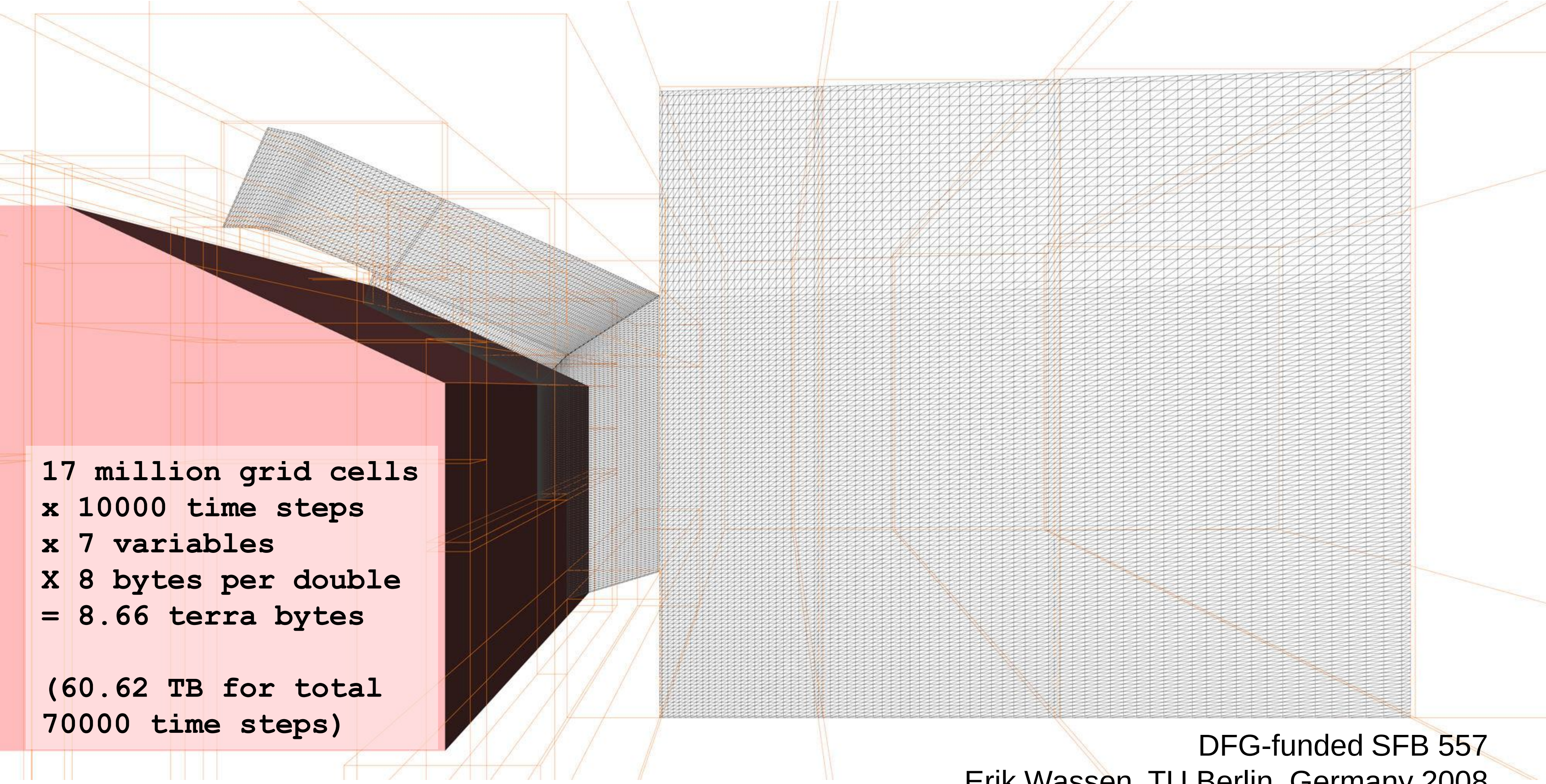


Ahmed body
Block-structured grid with 52 blocks
Each block is a curvilinear grid
17 million grid cells in total

Temporal resolution: a particle needs 10000 time steps from front to back of the Ahmed body

DFG-funded SFB 557
Erik Wassen, TU Berlin, Germany 2008

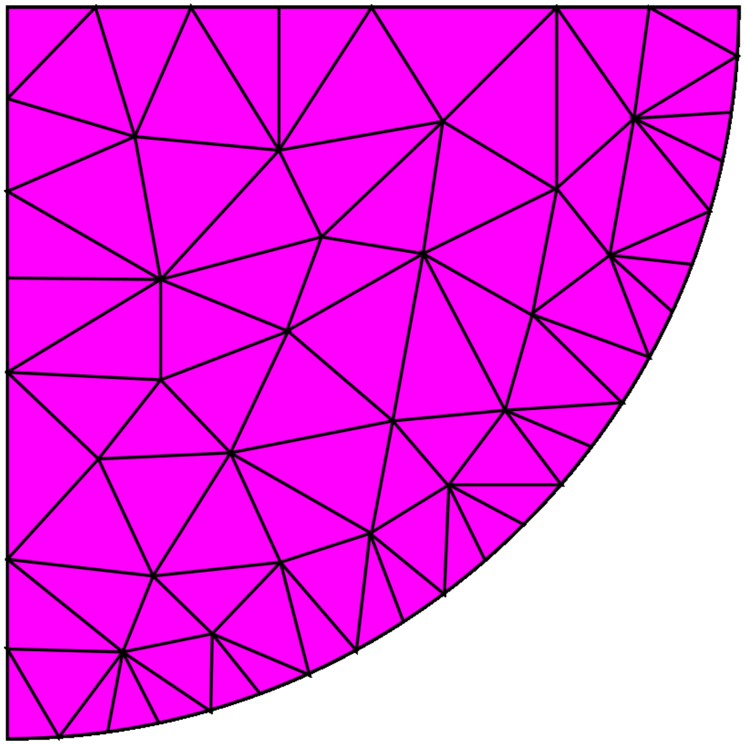
- Demands on data storage, an example:



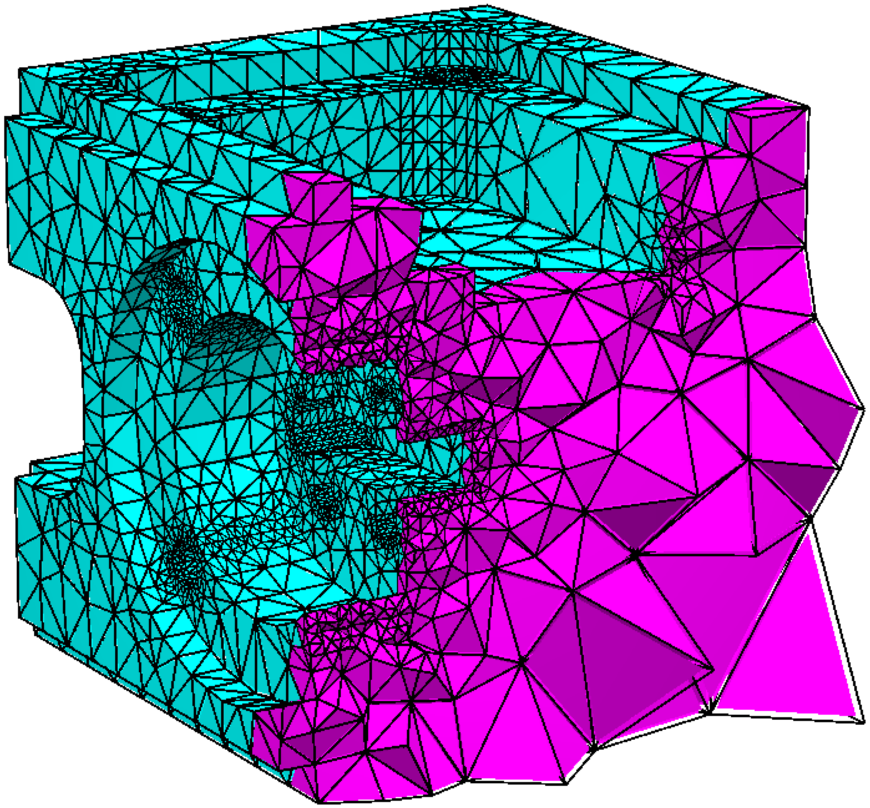
DFG-funded SFB 557
Erik Wassen, TU Berlin, Germany 2008

➔ Do not save every time step, not every variable, and not every block.

- **Unstructured grids**



2D unstructured grid
consisting of triangles



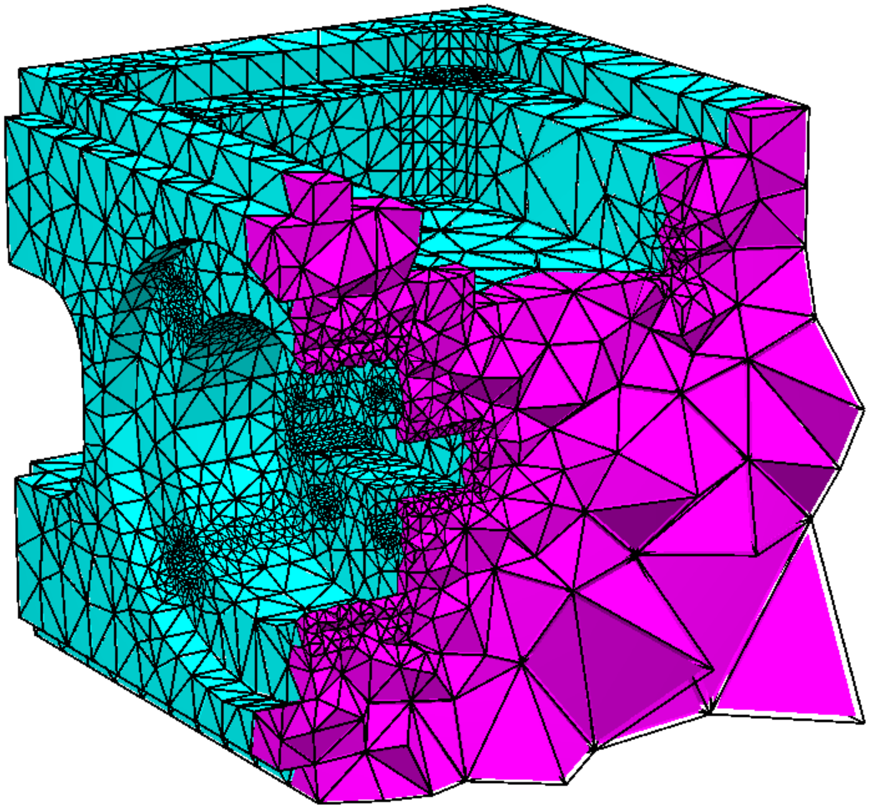
3D unstructured grid
consisting of tetrahedra
(from TetGen user manual)

- **Unstructured grids**

- Vertex locations and connectivity explicitly given

- Linear interpolation within a triangle/tetrahedron using barycentric coordinates

- Coordinate ➔ triangle/tetra:
  - **Local search** within last triangle/tetra or its immediate neighbors
  - **Global search** via quadtree/octree



3D unstructured grid consisting of tetrahedra (from TetGen user manual)

How to store unstructured grids? Different requirements:

- Efficient storage
  - bytes per face / bytes per vertex
- Efficient access
  - of face / vertex properties (e.g., position)
- Efficient traversal
  - e.g., neighboring face, 1-ring of a vertex,...
- Requirements are competing

Face set

- Store faces
  - 3 positions
  - no connectivity ("match positions")
- Example: STL
  - very simple structure (too simple, unpractical!)
  - easily portable

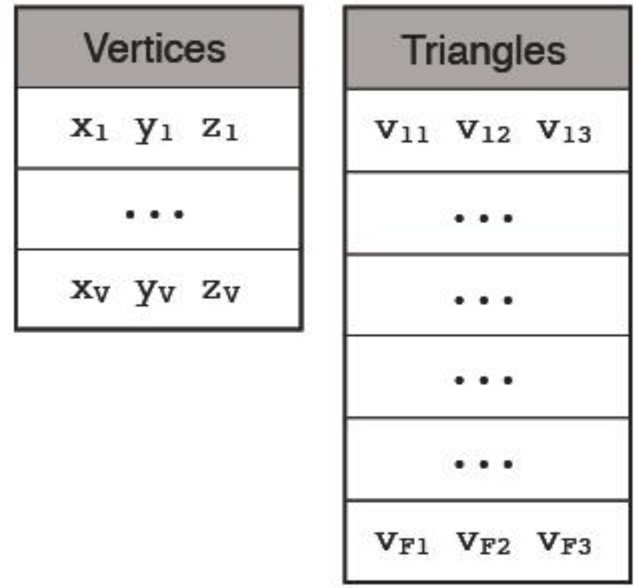
Triangles		
x_{11} y_{11} z_{11}	x_{12} y_{12} z_{12}	x_{13} y_{13} z_{13}
x_{21} y_{21} z_{21}	x_{22} y_{22} z_{22}	x_{23} y_{23} z_{23}
...	...	...
x_{F1} y_{F1} z_{F1}	x_{F2} y_{F2} z_{F2}	x_{F3} y_{F3} z_{F3}

36 B/f = 72 B/v

no connectivity!

Shared vertex

- vertex table stores positions
- triangle table stores indices into vertices
- No explicit connectivity
- Examples: OFF, OBJ, PLY
  - Quite simple and efficient
  - Enables efficient operations on *static* meshes

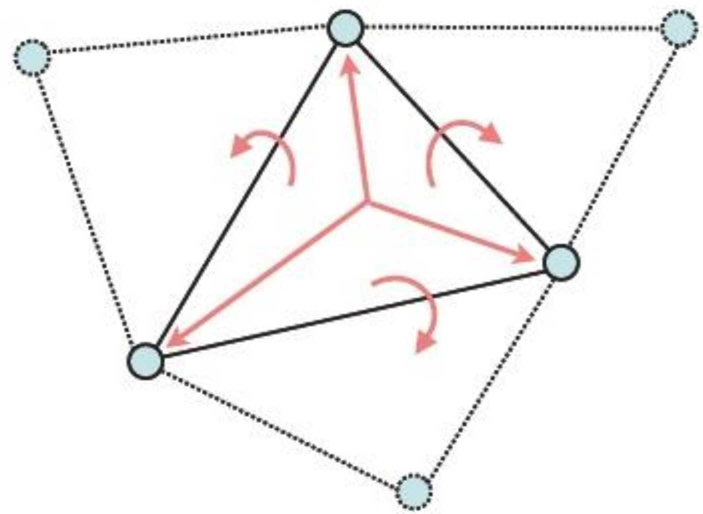


12 B/v + 12 B/f = 36 B/v

no neighborhood info

Face-based connectivity

- vertices store
  - position
  - face reference
- faces store
  - 3 vertex references
  - references to 3 neighboring faces

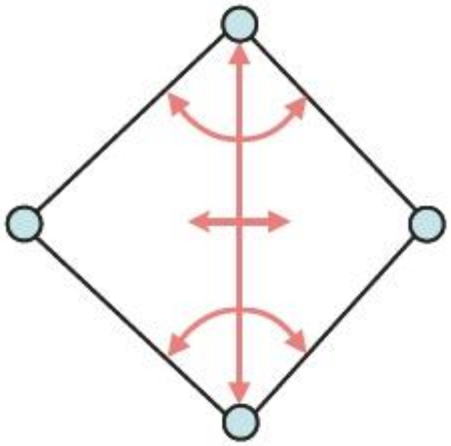


64 B/v

no edges!

Edge-based connectivity

- vertex stores
  - position
  - reference to 1 edge
- edge stores references to
  - 2 vertices
  - 2 faces
  - 4 edges
- face stores
  - reference to 1 edge

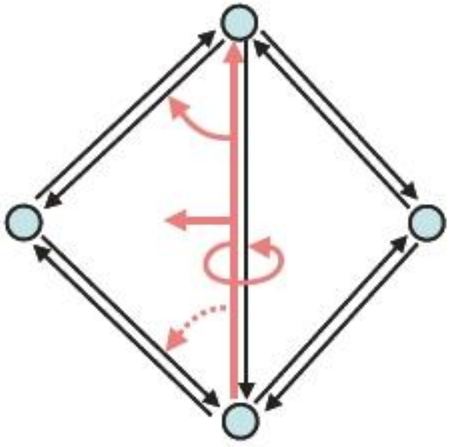


120 B/v

edge orientation?

Half-edge based connectivity

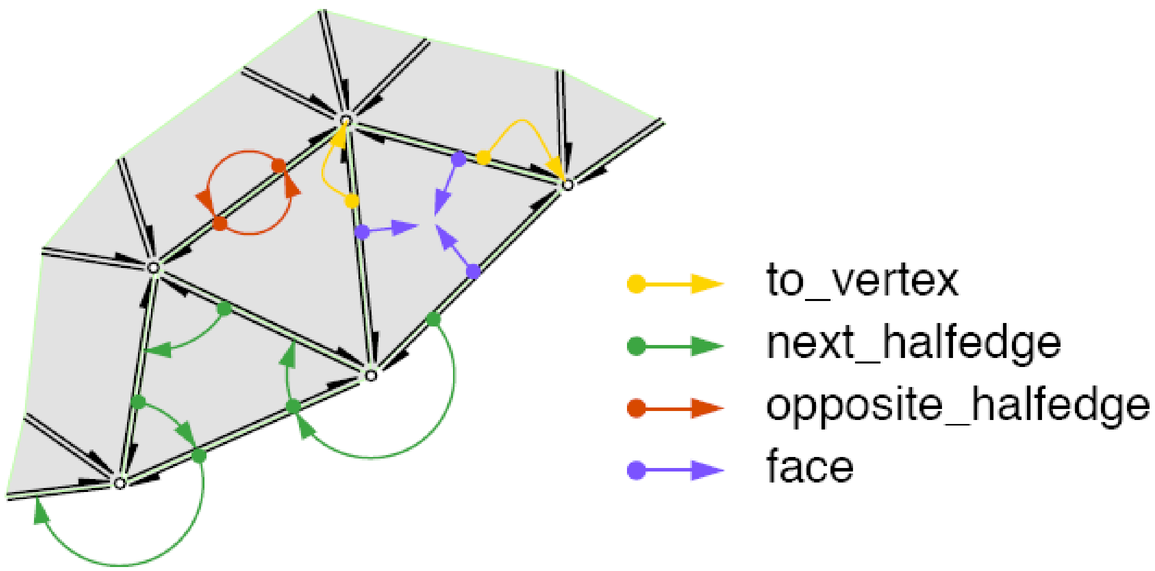
- vertex stores
  - position
  - reference to 1 half-edge
- half-edge stored references to
  - 1 vertex
  - 1 face
  - 1, 2, or 3 half-edges
- face stores
  - reference to 1 half-edge



96 to 144 B/v

no case distinctions during traversal

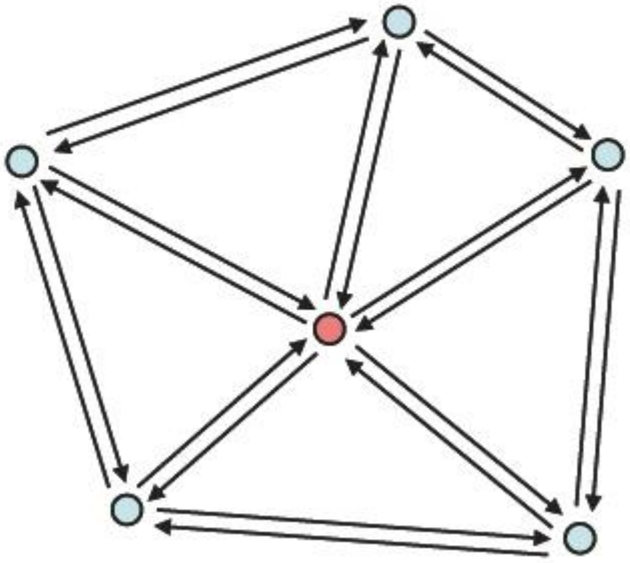
- **Half-edge based connectivity**



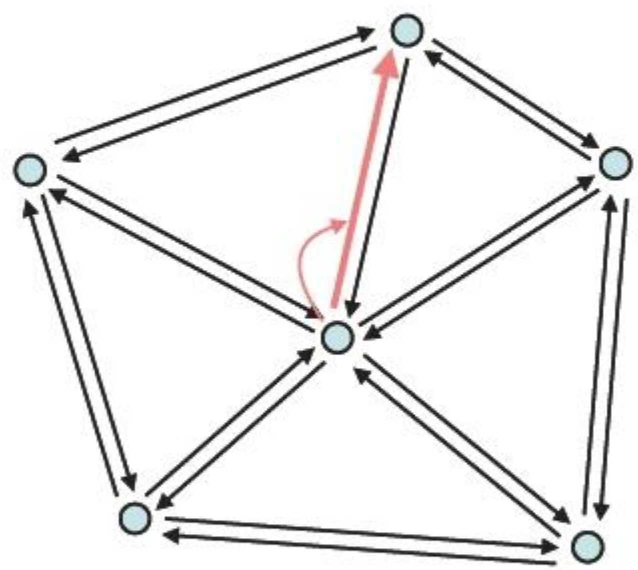
Half-edge based connectivity: Traversal

- Building blocks
  - Vertex to (outgoing) halfedge
  - half-edge to next (previous) halfedge
  - half-edge to neighboring half-edge
  - half-edge to face
  - half-edge to start (end) vertex
- Example: Traverse around vertex (1-ring)
  - enumerate vertices/faces/half-edges

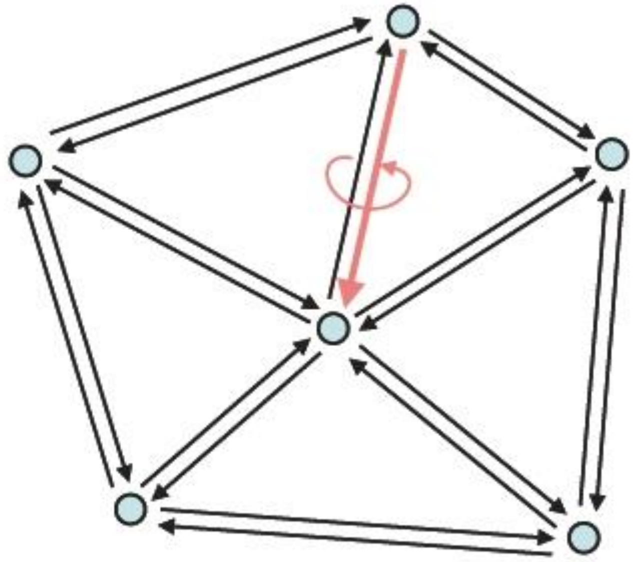
- Start at vertex



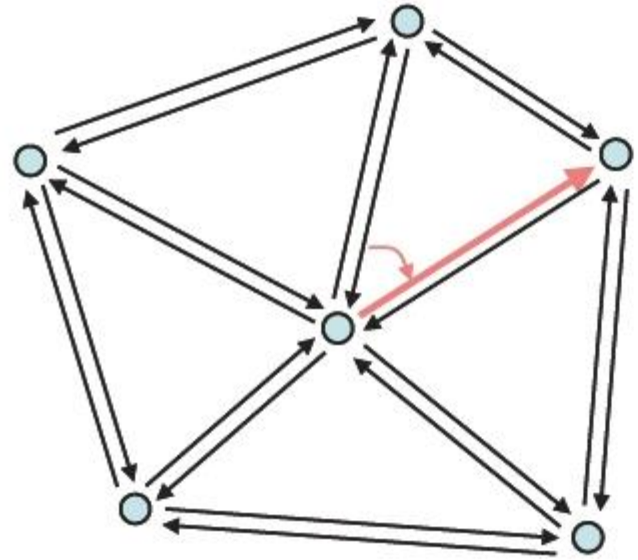
- Start at vertex
- Outgoing halfedge



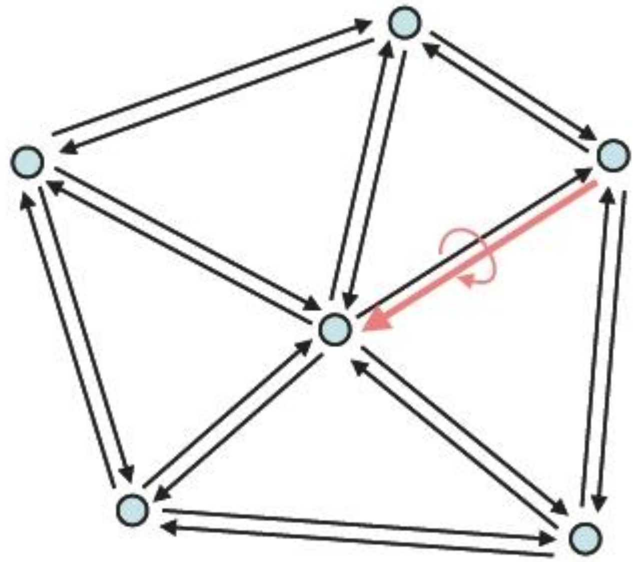
- Start at vertex
- Outgoing halfedge
- Opposite halfedge



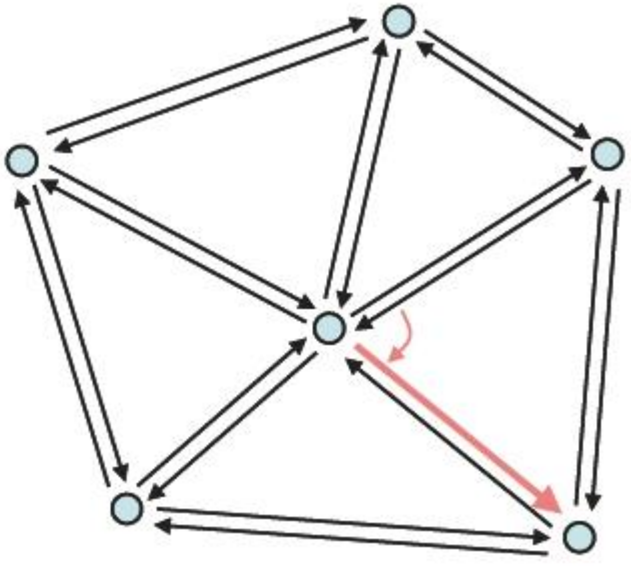
- Start at vertex
- Outgoing halfedge
- Opposite halfedge
- Next half-egde



- Start at vertex
- Outgoing halfedge
- Opposite halfedge
- Next half-egde
- Opposite ...

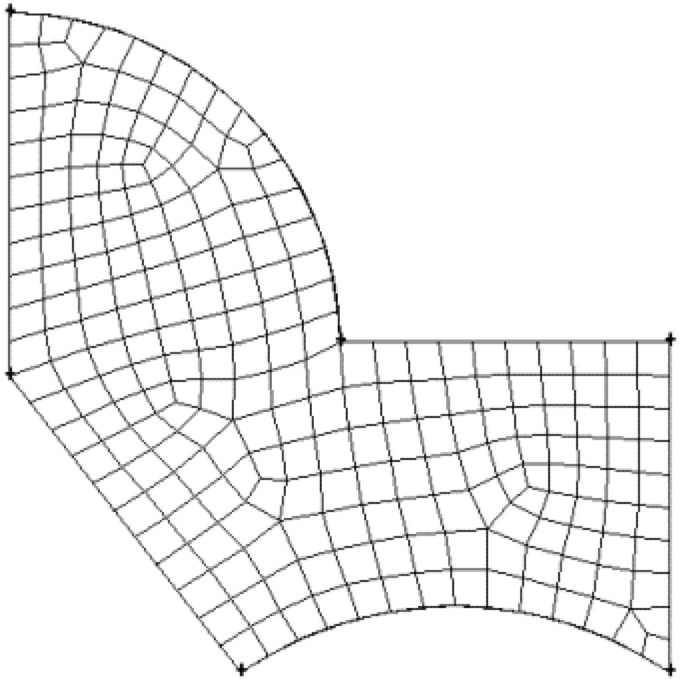


- Start at vertex
- Outgoing halfedge
- Opposite halfedge
- Next half-egde
- Opposite ...
- Next ...
- ...



- CGAL
    - www.cgal.org
    - Computational geometry
    - Free for non-commercial use
- Open Mesh
    - www.openmesh.org
    - Mesh processing
    - Free, LGPL license
- gmu (gmu-lite)
    - proprietary, directed edges

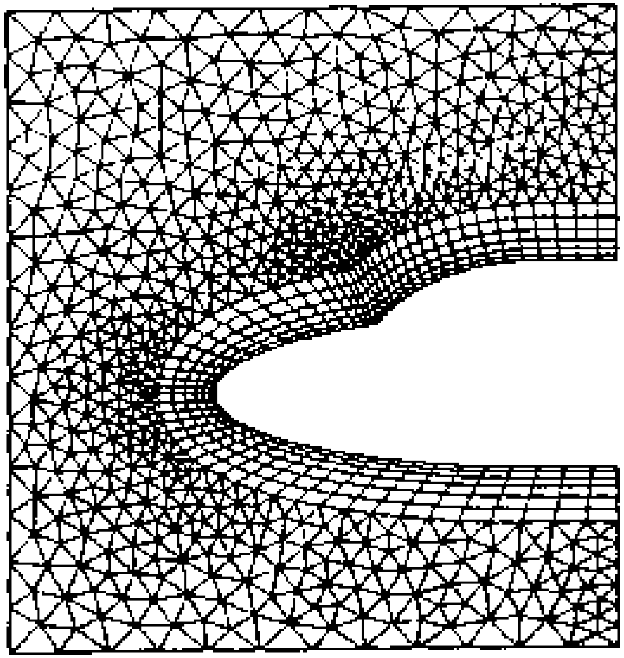
- **Unstructured grids**



2D unstructured grid
consisting of quads

Source: https://www.sharcnet.ca/Software/Gambit/html/modeling_guide/mg0303.htm

- **Hybrid grids**
- combination of different grid types



2D hybrid grid

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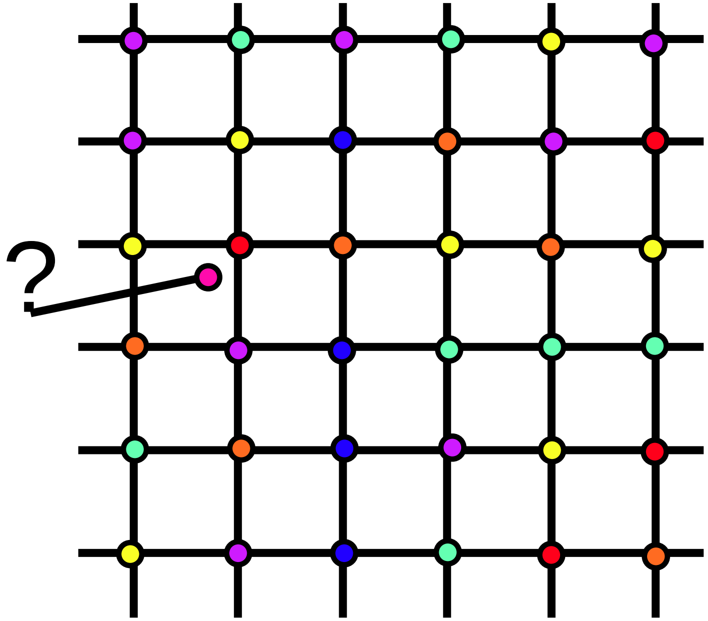
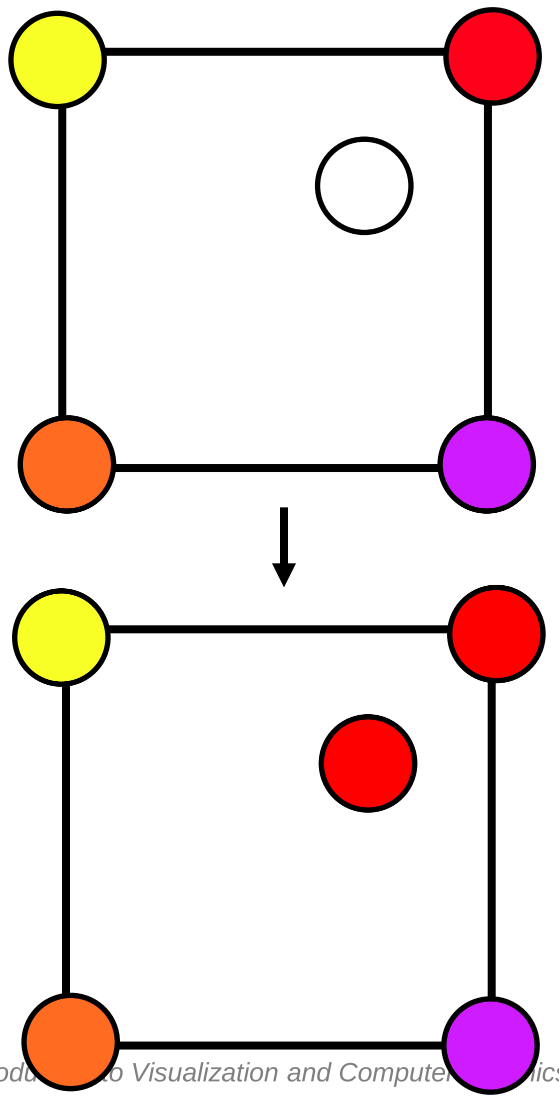
Grids and Interpolation

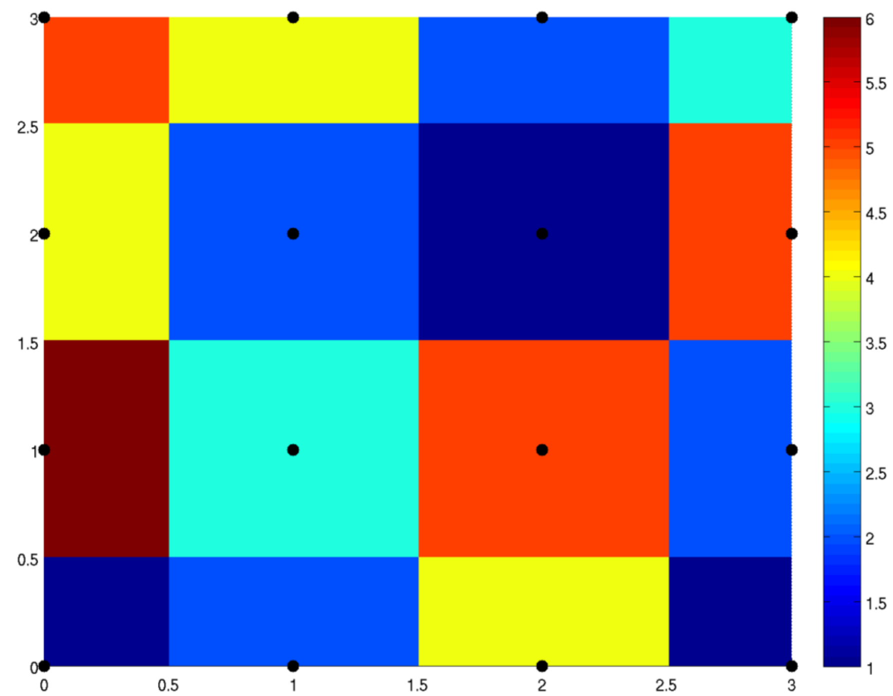
Linear, Bilinear, Trilinear Interpolation in Structured Grids
Gradients
Linear Interpolation in Unstructured Grids

- A grid consists of a finite number of **samples**
  - The continuous signal is known only at a few points (**data points**)
  - In general, data is needed in between these points
- By **interpolation** we obtain a representation that matches the function at the data points
  - **Reconstruction** at any other point possible

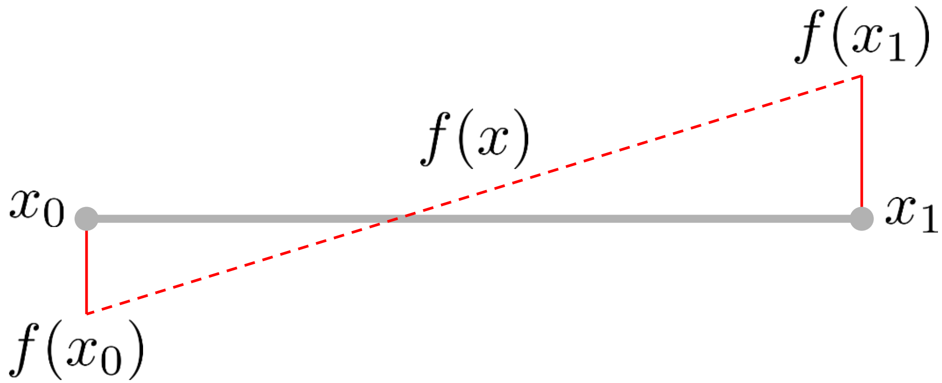


- Simplest approach: **Nearest-Neighbor Interpolation**
  - Assign the value of the nearest grid point to the sample.





- **Linear Interpolation** (in 1D domain)
  - Domain points x, scalar function $f(x)$



General:

$$f(x) = \frac{x_1 - x}{x_1 - x_0} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1) \qquad x \in [x_0, x_1]$$

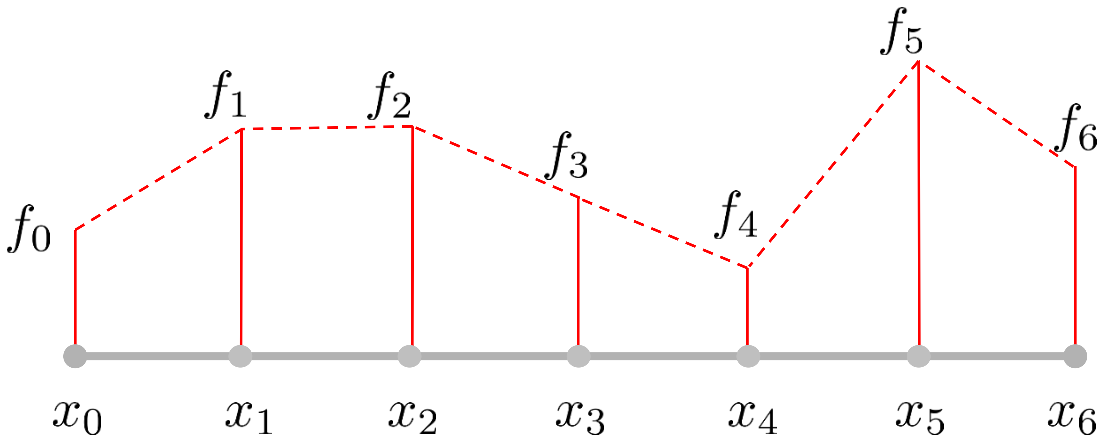
Special Case:

$$f(x) = (1 - x)\ f(0) + x\, f(1) \qquad\qquad x \in [0, 1]$$

$$= \begin{bmatrix} (1 - x) & x \end{bmatrix} \begin{pmatrix} f(0) \\ f(1) \end{pmatrix} = \begin{bmatrix} 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} f(0) \\ f(1) \end{pmatrix}$$

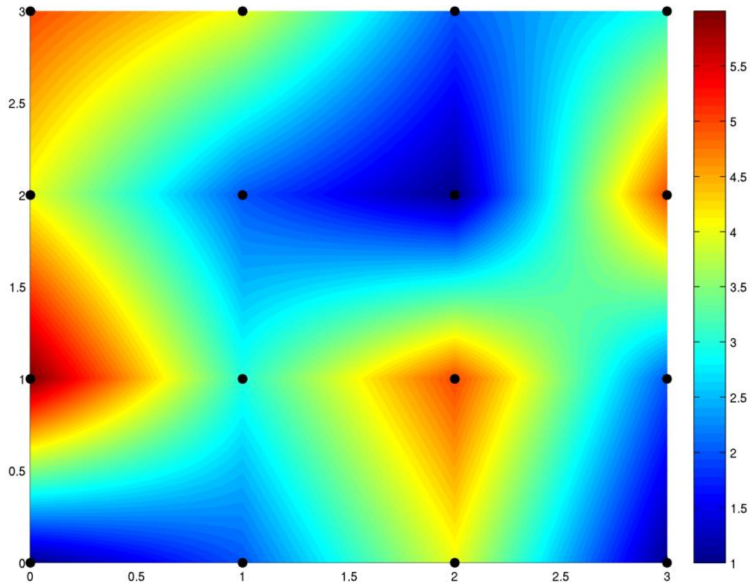
Basis Coefficients

- **Linear Interpolation** (in 1D domain)
  - Sample values $f_i := f(x_i)$

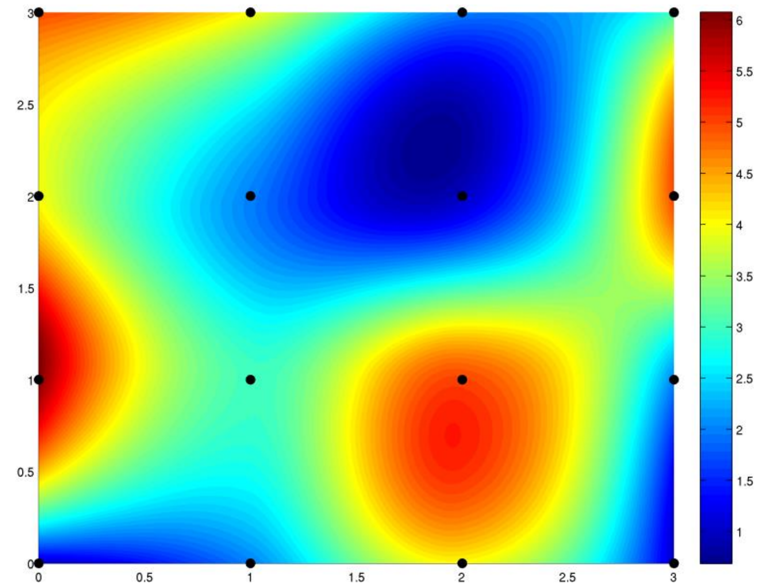


- C^0 Continuity (discontinuous first derivative)
  - Use higher order interpolation for smoother transition, e.g., **cubic** interpolation

- **Interpolation in 2D, 3D, 4D, …**



Bi-Linear



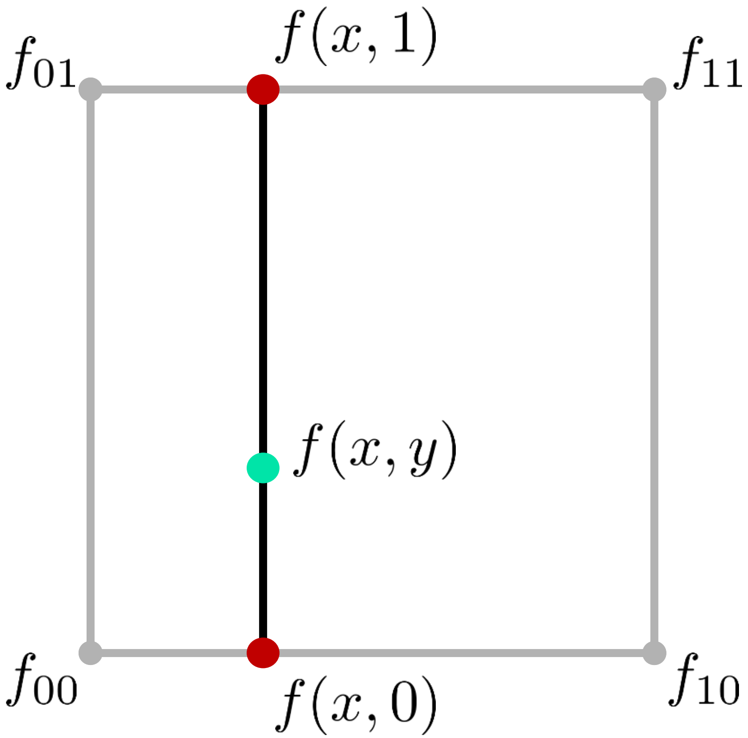
Bi-Cubic

- **Tensor Product Interpolation**
  - Perform linear / cubic … interpolation in each x,y,z … direction **separately**

very important

- Bilinear Interpolation

2D, “bi-linear”

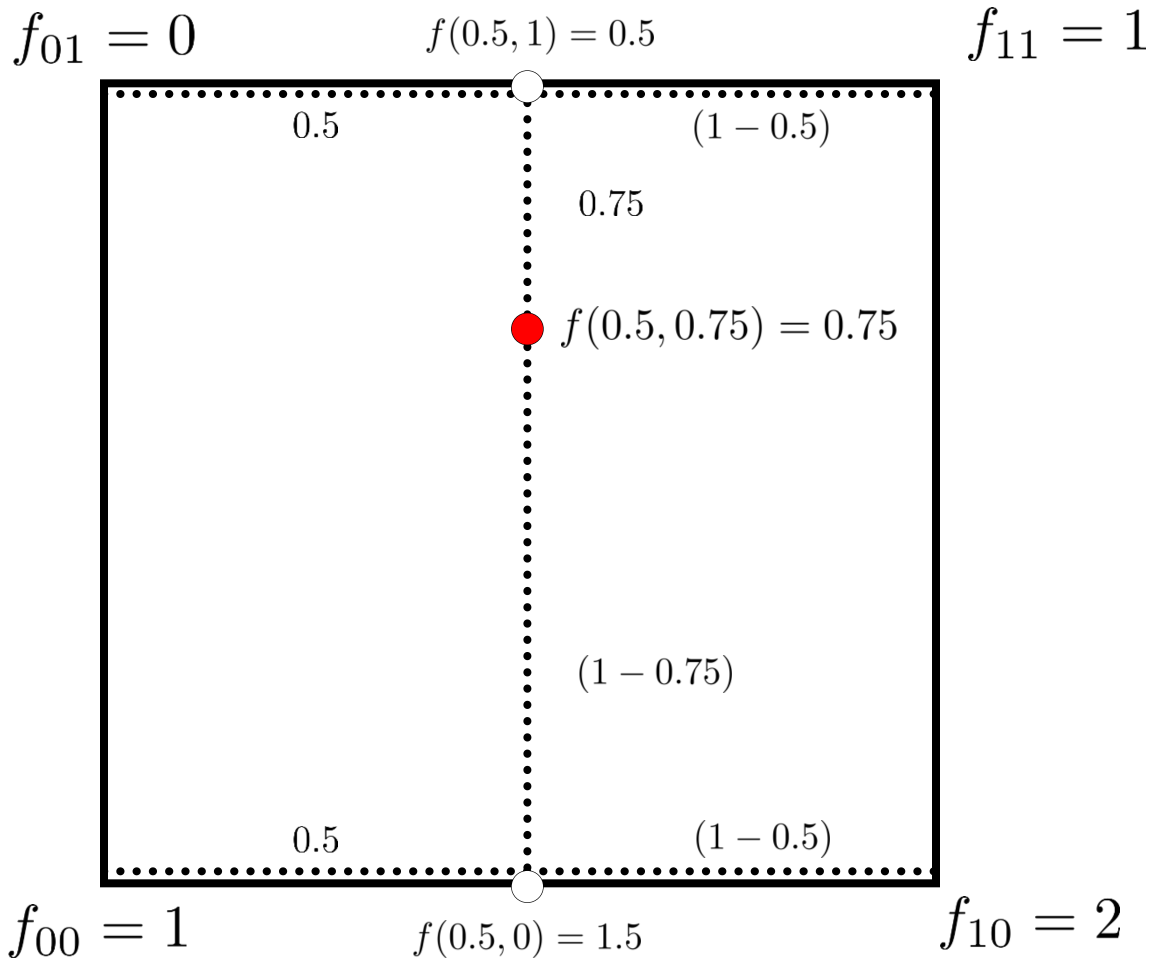


$$\begin{aligned} f(x,y) = &(1-x)\,(1-y)\,f_{00} + x\,(1-y)\,f_{10} + \\ &(1-x)\,y\,f_{01} + x\,y\,f_{11} \\ = &(1-y)\,((1-x)\,f_{00} + x\,f_{10}) + \\ &y\,((1-x)\,f_{01} + x\,f_{11}) \end{aligned}$$

“interpolate twice in x direction
and then once in y direction”

- **Example: Bi-linear interpolation in a 2D cell**
  - Repeated linear interpolation

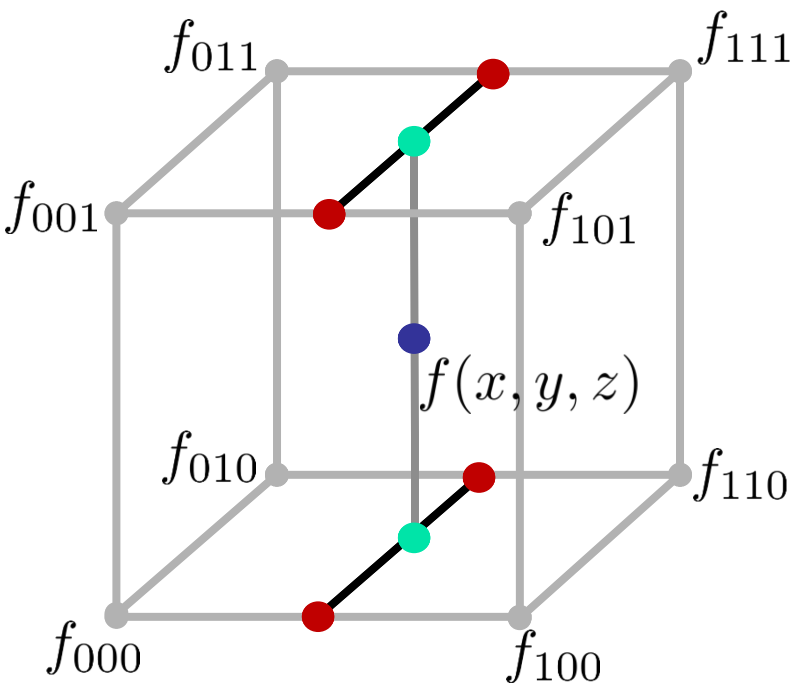
$$f(0.5, 0.75) = ?$$



- Trilinear Interpolation

3D, “tri-linear”

$$f(x,y,z) = \sum_{k=0}^{p} \sum_{j=0}^{m} \sum_{i=0}^{n} b_i(x) b_j(y) b_k(z) \, f_{ijk}$$



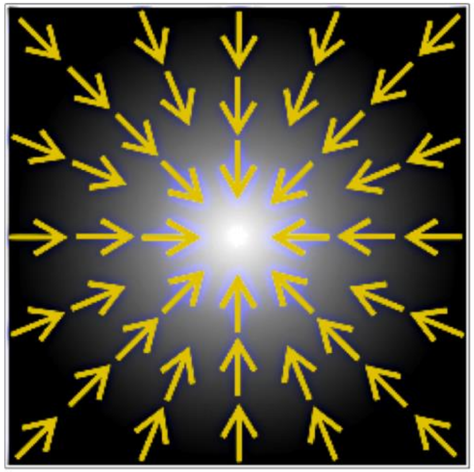
“interpolate four times in x direction, twice in y direction, and once in z direction”

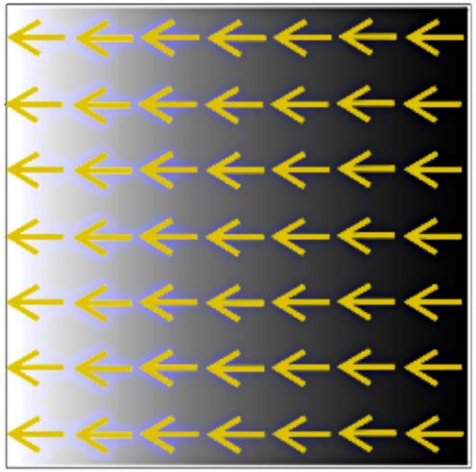
- **Function Derivative Estimation**
  - Called **Gradients** for multidimensional functions
  - Have a lot of important applications (e.g., normal for volume rendering, critical point classification for vector field topology …)

$$\nabla f(x,y,z) = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} f(x,y,z) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}$$

"vector of partial derivatives"

  - Describes direction of steepest ascend





- Two ways to estimate gradients:
  - Direct derivation of interpolation formula
  - Finite differences schemes

- **Field Function Derivatives**, Bi-Linear

$$f(x,y) = \begin{bmatrix} (1-x) & x \end{bmatrix} \begin{bmatrix} f_{00} & f_{01} \\ f_{10} & f_{11} \end{bmatrix} \begin{bmatrix} (1-y) \\ y \end{bmatrix} \longrightarrow$$

derive this interpolation formula

$$\frac{\partial f(x,y)}{\partial x} = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} f_{00} & f_{01} \\ f_{10} & f_{11} \end{bmatrix} \begin{bmatrix} (1-y) \\ y \end{bmatrix}$$

$$= (f_{10} - f_{00})(1-y) + (f_{11} - f_{01})\,y$$

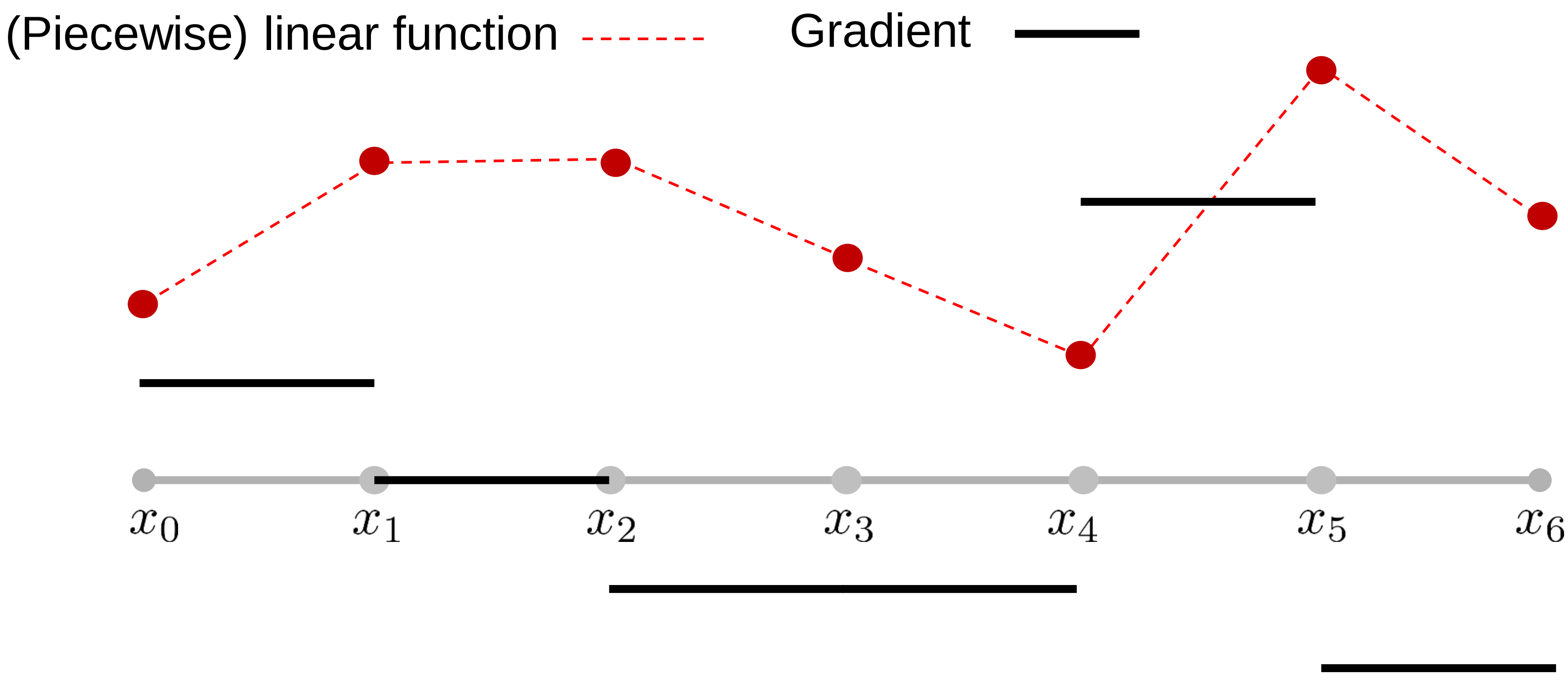
“constant in x direction“

$$\frac{\partial f(x,y)}{\partial y} = \begin{bmatrix} (1-x) & x \end{bmatrix} \begin{bmatrix} f_{00} & f_{01} \\ f_{10} & f_{11} \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= (f_{01} - f_{00})(1-x) + (f_{11} - f_{10})\,x$$

“constant in y direction“

- Problem of exact linear function differentiation: discontinuous gradients



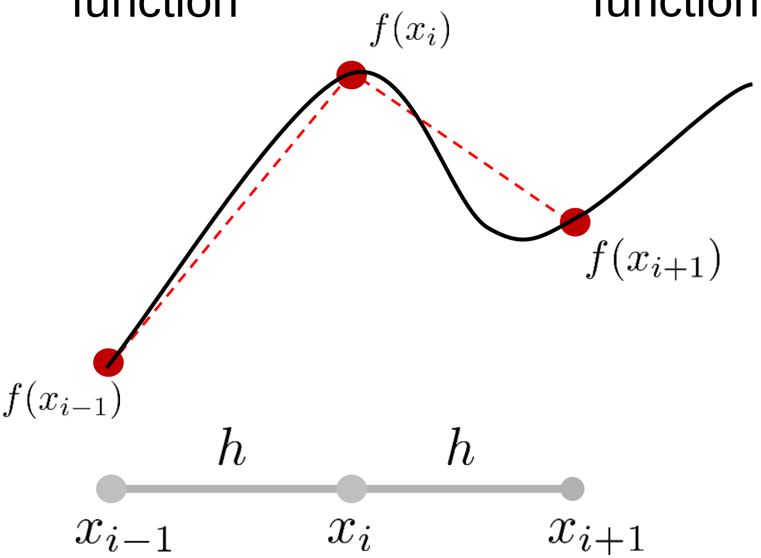
- Solution:
  - Use higher order interpolation scheme (cubic)
  - Use **finite difference estimation**

Finite Differences Schemes

- Apply Taylor series expansion around samples

Continuous function

Linear function



$f(x_{i+1})$

Taylor expansion

$$f(x_i + h) = f(x_i) + h\,\frac{\mathrm{d}f(x_i)}{\mathrm{d}x} + \frac{h^2}{2}\,\frac{\mathrm{d}^2 f(x_i)}{\mathrm{d}x^2} + O(h^3)$$

$$\rightarrow \quad \frac{\mathrm{d}f(x_i)}{\mathrm{d}x} \approx \frac{f(x_{i+1}) - f(x_i)}{h}$$

Forward difference

$$\rightarrow \quad \frac{\mathrm{d}f(x_i)}{\mathrm{d}x} \approx \frac{f(x_i) - f(x_{i-1})}{h}$$

Backward difference

- **Finite Differences Schemes**

$$f(x_{i+1}) = f(x_i) + h\,\frac{\mathrm{d}f(x_i)}{\mathrm{d}x} + \frac{h^2}{2}\,\frac{\mathrm{d}^2 f(x_i)}{\mathrm{d}x^2} + O(h^3)$$

$$f(x_{i-1}) = f(x_i) - h\,\frac{\mathrm{d}f(x_i)}{\mathrm{d}x} + \frac{h^2}{2}\,\frac{\mathrm{d}^2 f(x_i)}{\mathrm{d}x^2} + O(h^3)$$

Difference

$$\longrightarrow \quad (f(x_{i+1}) - f(x_i)) - (f(x_{i-1}) - f(x_i)) = 2h\,\frac{\mathrm{d}f(x_i)}{\mathrm{d}x} + O(h^3)$$

$$\longrightarrow \quad \frac{\mathrm{d}f(x_i)}{\mathrm{d}x} \approx \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$

Central difference

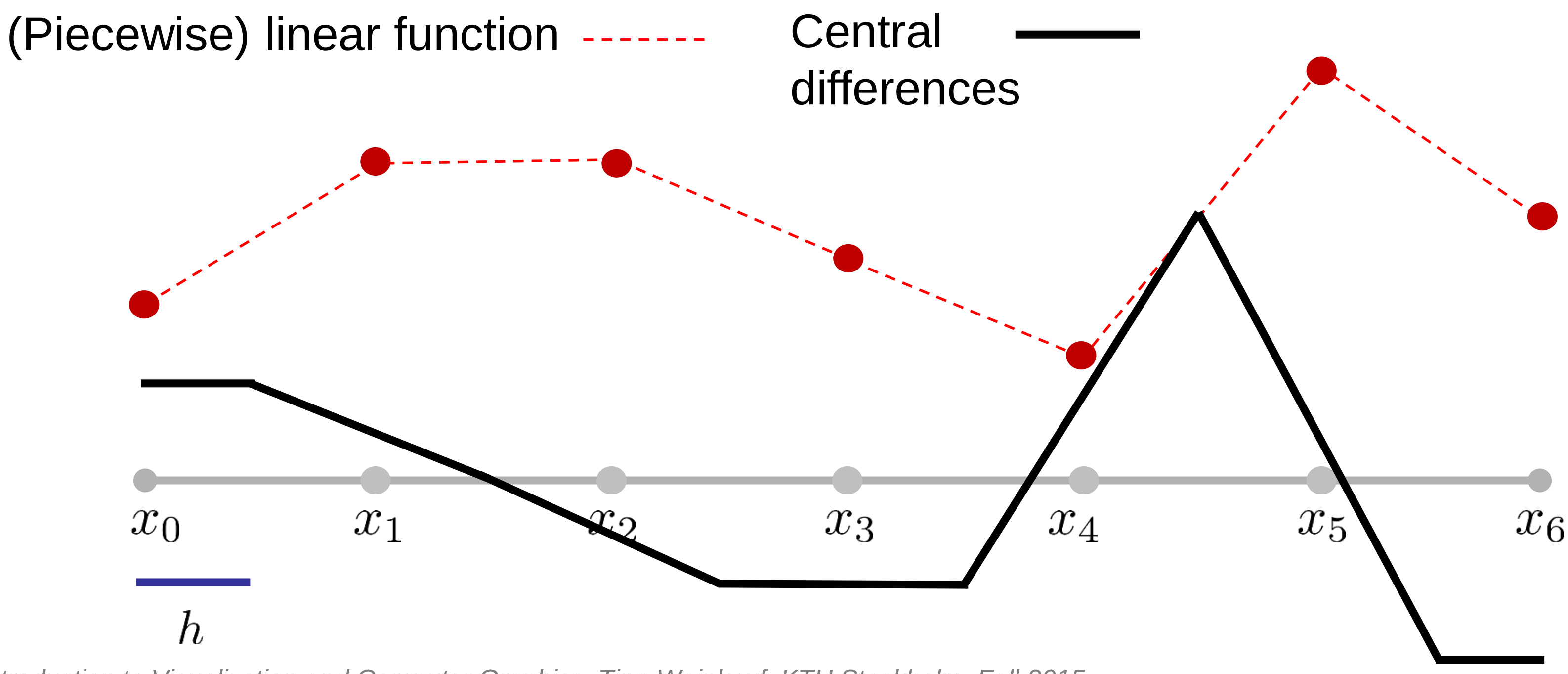
- Central differences have higher approximation order than forward / backward differences

- **Finite Differences Schemes,** Higher order derivatives

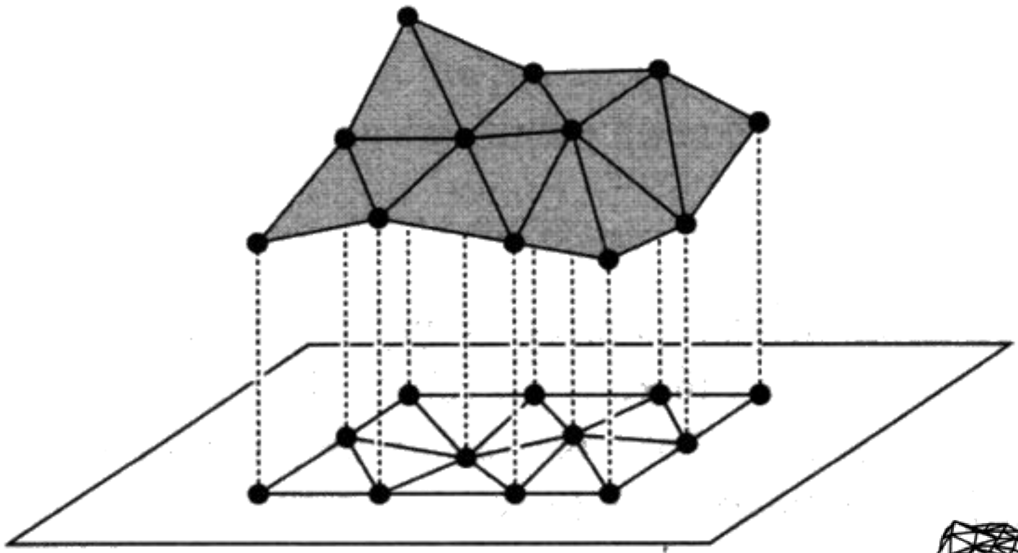
$$\frac{\mathrm{d}^2 f(x_i)}{\mathrm{d}x^2} \approx \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$

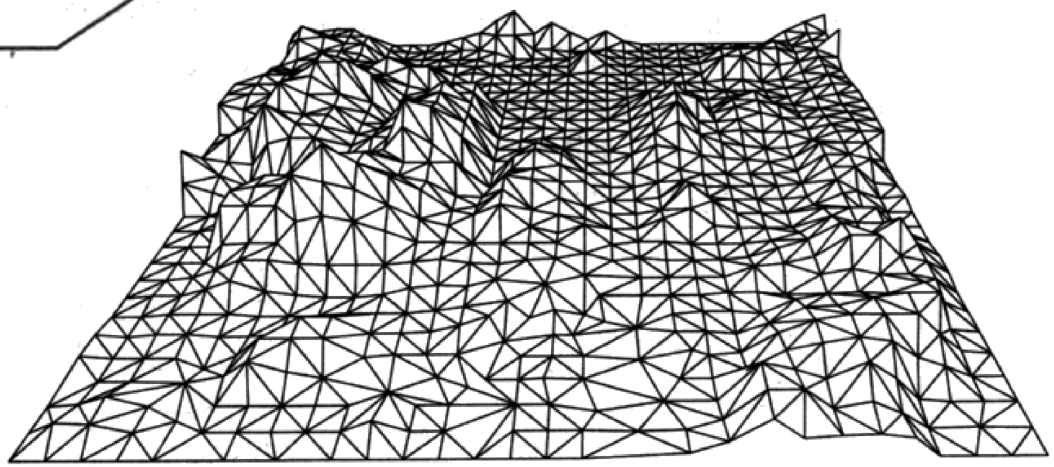
$$\frac{\partial^2 f(x_i, y_j)}{\partial xy} \approx \frac{f(x_{i+1}, y_{j+1}) - f(x_{i+1}, y_{j-1}) - f(x_{i-1}, y_{j+1}) + f(x_{i-1}, y_{j-1})}{4\, h_x\, h_y}$$

- 1D Example, linear interpolation

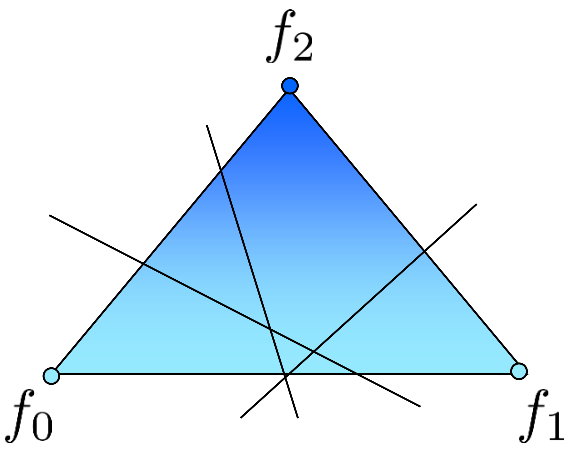


- **Piecewise Linear Interpolation in Triangle Meshes**





- **Linear Interpolation in a Triangle**
  - There is exactly one linear function that satisfies the interpolation constraint



- A linear function can be written as

$$f(x,y) = a + b\,x + c\,y$$

  - Polynomial can be obtained by

    solving the linear system

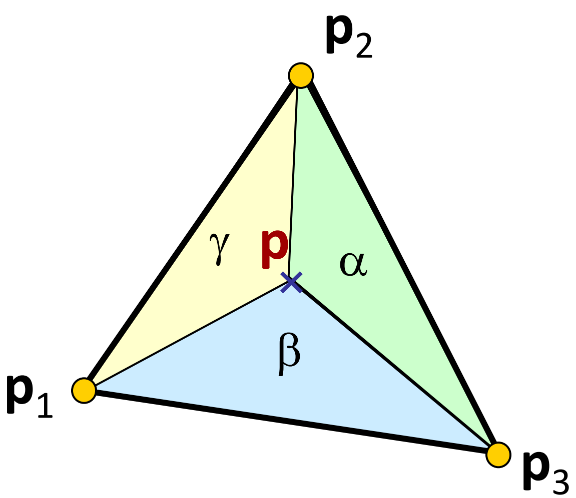
$$\begin{bmatrix} 1 & x_0 & y_0 \\ 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix}$$

- Linear in x and y
  - Interpolated values along any ray in the plane spanned by the triangle are linear along that ray

- Barycentric Coordinates:
  - Planar case:
    Barycentric combinations of 3 points

$$\mathbf{p} = \alpha \mathbf{p}_1 + \beta \mathbf{p}_2 + \gamma \mathbf{p}_3, \text{with} : \alpha + \beta + \gamma = 1$$

$$\gamma = 1 - \alpha - \beta$$

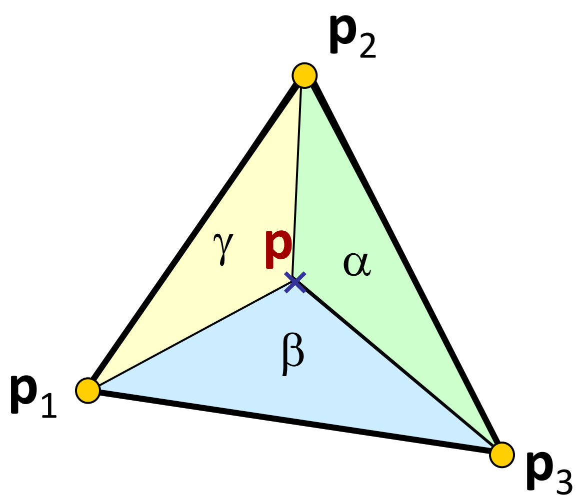


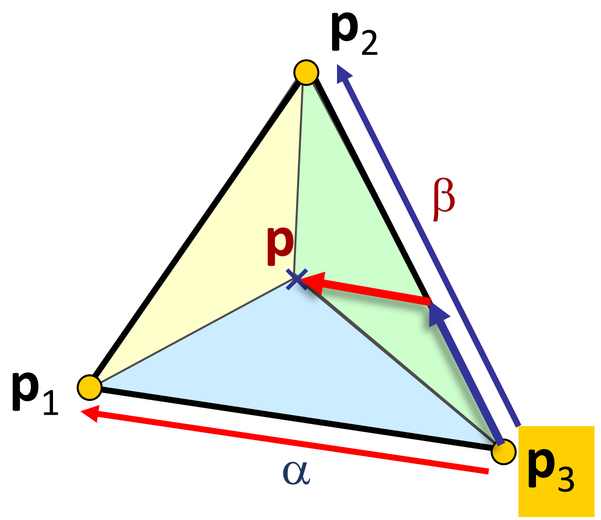
  - Area formulation:

$$\alpha = \frac{area(\Delta(\mathbf{p}_2, \mathbf{p}_3, \mathbf{p}))}{area(\Delta(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3))}, \beta = \frac{area(\Delta(\mathbf{p}_1, \mathbf{p}_3, \mathbf{p}))}{area(\Delta(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3))}, \gamma = \frac{area(\Delta(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}))}{area(\Delta(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3))}$$

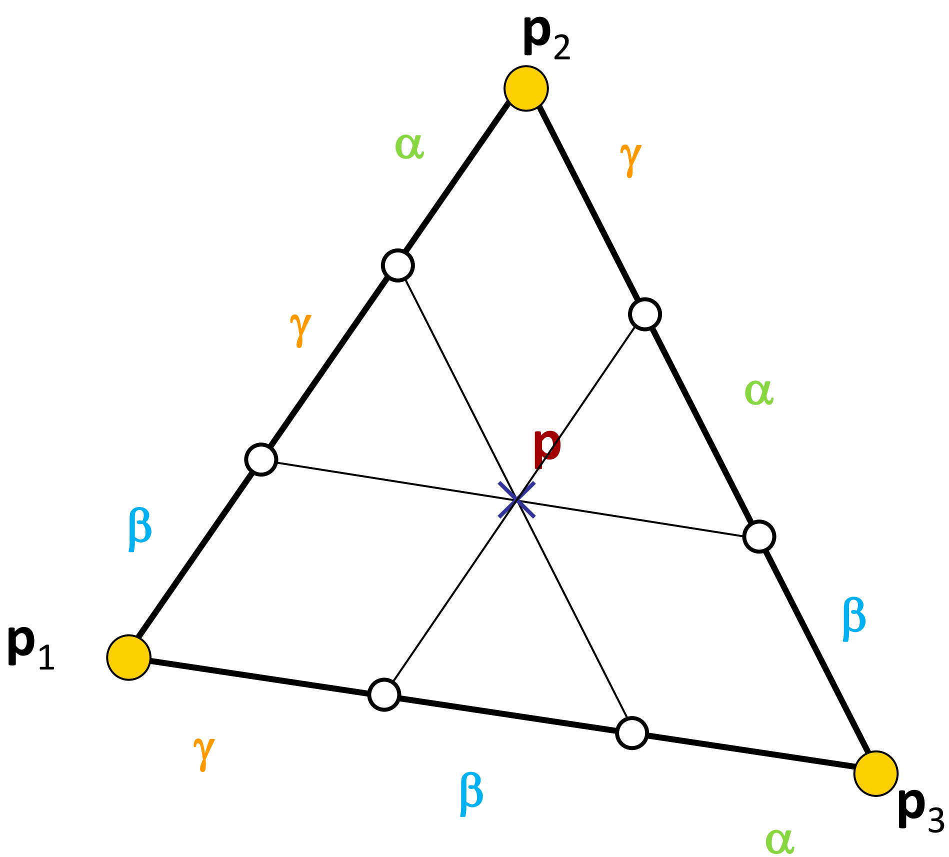
- Barycentric Coordinates:
  - Linear formulation:

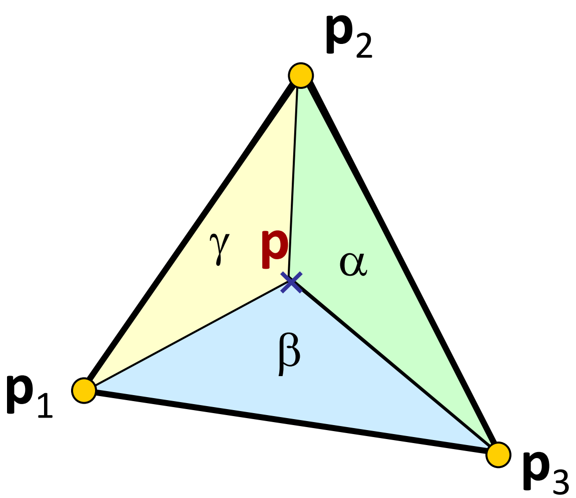
$$\begin{aligned}\mathbf{p} &= \alpha\mathbf{p}_1 + \beta\,\mathbf{p}_2 + \gamma\,\mathbf{p}_3\\ &= \alpha\mathbf{p}_1 + \beta\,\mathbf{p}_2 + (1-\alpha-\beta)\,\mathbf{p}_3\\ &= \alpha\mathbf{p}_1 + \beta\,\mathbf{p}_2 + \mathbf{p}_3 - \alpha\,\mathbf{p}_3 - \beta\,\mathbf{p}_3\\ &= \boxed{\mathbf{p}_3} + \underline{\alpha\left(\mathbf{p}_1 - \mathbf{p}_3\right)} + \underline{\beta\left(\mathbf{p}_2 - \mathbf{p}_3\right)}\end{aligned}$$





$$\mathbf{p} = \alpha\mathbf{p}_1 + \beta\,\mathbf{p}_2 + \gamma\,\mathbf{p}_3, \text{with} : \alpha + \beta + \gamma = 1$$





- **Barycentric Interpolation in a Triangle**

very important

  - The linear function of a triangle can be computed at any point as

    $$f(x,y) = \alpha_0(x,y) f_0 + \alpha_1(x,y) f_1 + \alpha_2(x,y) f_2$$

    with $\alpha_0 + \alpha_1 + \alpha_2 = 1$ (**Barycentric Coordinates**)

  - This also holds for the coordinate $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ of the triangle:

    $$\mathbf{x} = \alpha_0\,\mathbf{x}_0 + \alpha_1\,\mathbf{x}_1 + \alpha_2\,\mathbf{x}_2$$

    → Can be used to solve for unknown coefficients α_i :

$$\begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

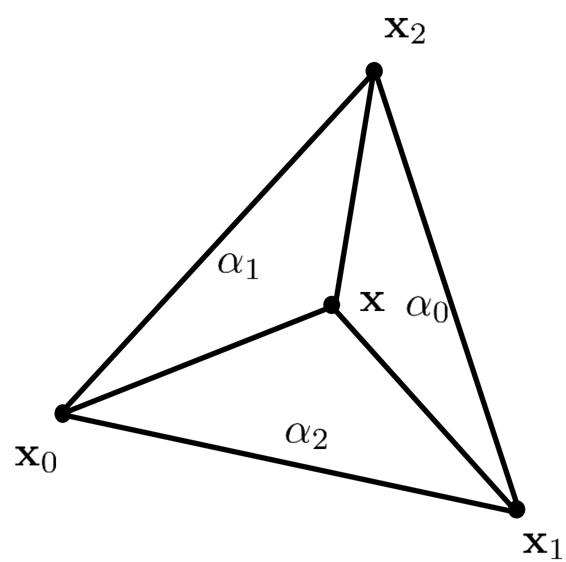
• Barycentric Interpolation in a Triangle

- Solution of $\begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ (e.g. Cramer's rule):

$$\alpha_0 = \frac{1}{2A} \det\left(\begin{bmatrix} x & x_1 & x_2 \\ y & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix} \right) \qquad \alpha_0 = \frac{\mathrm{Area}([\mathbf{x}, \mathbf{x}_1, \mathbf{x}_2])}{\mathrm{Area}([\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2])}$$

$$\alpha_1 = \frac{1}{2A} \det\left(\begin{bmatrix} x_0 & x & x_2 \\ y_0 & y & y_2 \\ 1 & 1 & 1 \end{bmatrix} \right) \qquad \alpha_1 = \frac{\mathrm{Area}([\mathbf{x}_0, \mathbf{x}, \mathbf{x}_2])}{\mathrm{Area}([\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2])}$$

$$\alpha_2 = \frac{1}{2A} \det\left(\begin{bmatrix} x_0 & x_1 & x \\ y_0 & y_1 & y \\ 1 & 1 & 1 \end{bmatrix} \right) \qquad \alpha_2 = \frac{\mathrm{Area}([\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}])}{\mathrm{Area}([\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2])}$$



with

$$A = \frac{1}{2} \det\left(\begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix} \right)$$

Inside triangle criteria

$$0 \leq \alpha_0,\ \alpha_1,\ \alpha_2 \leq 1$$

- **Barycentric Interpolation in a Tetrahedron**
- Analogous to the triangle case

Gradient of a linearly interpolated function in a triangle/tetrahedron

- **Constant!**