Matematik, KTH
Svante Linusson

SF2740, Graph Theory
HT 2015

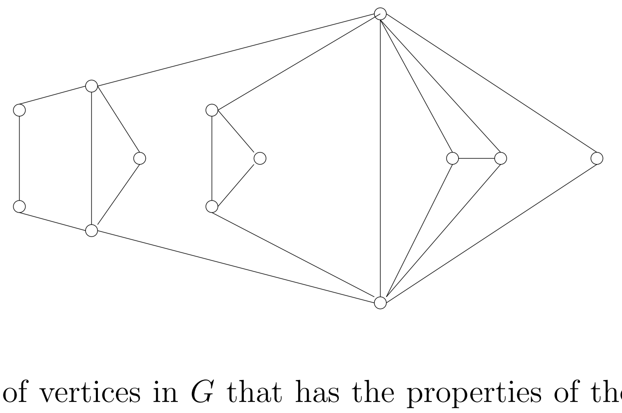
First set of hand in problems

Be sure to write solutions with clear arguments that are easy to follow. You should try to have a level of details so your solution would be understandable to other students. Staple your solution together in the top left corner and write down your solutions in order. Write your name in the top right corner.

Code of conduct(Hederskodex): It is assumed that:
-you shall solve the problems on your own (or in cooperation with one or two fellow students) and write down your own solution. If you cooperate with someone you must mention that persons name for each problem.
- you must not use other resources
- if you in spite of this are using something you have gotten from somewhere else for some reason (a friend, a book or the internet etc.) you must give a reference to the source.

Your solutions to these problems are due October 16, before class starts.

1. Let $v \in V$ be a cutvertex of a graph $G = (V, E)$. Prove that $\overline{G} \setminus v$ is connected.
2. Let $G = (V, E)$ be a (simple) graph. Prove that G has a bipartite subgraph H, with $V(G) = V(H)$ such that
$$d_H(v) \geq \frac{1}{2} d_G(v), \quad \text{for all } v \in V.$$
3. Let G be a bipartite graph with vertex sets A and B. Assume that G has a perfect matching of A. Prove that there is a vertex $a \in A$ such that every edge incident to a is part of a matching of A.
4. Let $G = (V, E)$ be a $\ell + 1$-regular graph. Assume further that $|V|$ is even and that $\lambda(G) \geq \ell$. Prove that G has a 1-factor.
5. Recall from the 4th lecture the Gallai-Edmond's decomposition of a graph G. See the extra material for notation.
a) Determine the sets $A(G), D(G), C(G)$ for the graph G below (motivate your claim!).



b) Find a subset of vertices in G that has the properties of the set S as described in Theorem 2.2.3 in Diestel, (4th ed).

Lycka till! / Svante