

Lecture 3

Induction, The HD Method and
Bayesianism

Introduction

- Induction - We study the simplest scientific principle
- HD-Method - We study a more general and advanced scientific principle
- Probabilistic variant - We see how the HD-Method can be modified with probabilistic reasoning. We look at Bayesian methods

Induction

- The basic idea: We make observations and try to see a pattern in them.
- If the observations are many and all agree with the pattern we conjecture that the pattern always applies.
- There are at least two different standardized forms of the method.

Induction: A basic form

- We make observations of objects which all has property A.
- Let us assume that in all observations the objects also have property B.
- We conclude that all objects with property A also have property B.

Induction: Form 2

- This is a more general form.
- Assume that we make observations of situations of a certain type P .
- Then assume all these situations are of type Q .
- We conclude that all situations of type P also are of type Q .

Induction: Logical formulas

If we use logical formulas we can write the first form like

$$\forall x (A(x) \rightarrow B(x))$$

More general we can have some statement F and want to prove that F is always true

$$\forall x F(x)$$

We then observe instances $F(x_1), F(x_2), \dots$. If they are all true, can we conclude that $\forall x F(x)$ is true?

Does induction work?

- Yes, basically. There are however counter-examples.
- The set of observations must be chosen in a sufficiently general way.
- What is the logical basis for induction?
- One motivation for induction is the Principle of Uniformity of Nature (PUN).

PUN

- The idea is that there are *regularities* in nature
- If there are a lot of regularities to be found out there, then there is a big chance that an observed regular pattern can be an instance of a basic regularity
- If this is the case then it seems as if induction could be a logically meaningful tool for finding regularities

A critic

David Hume 1711-1776



There is no scientific ground for induction!

- Induction cannot be proved to be correct using logic.
- Induction cannot be proved using induction (circular reasoning).
- We believe in induction since it seems to work.
- But it cannot be used for scientific proofs.

A solution?

Karl Popper 1902-1994



- Popper claims that he has solved the riddle of induction.
- The solution is that we never really use induction!
- We can never verify hypothesis.
- We can only falsify them.

Can induction generate theories?

- The idea is that we can see patterns and we can generalize them into theories.
- By using the induction principle we can "prove" the theory.
- But can it be done? There are at least three objections.
- The fact (if it is a fact) that we must first have a theory before we can make observations.
- Goodman's paradox
- Underdetermination

Goodman's paradox

- It can be stated in several essentially equivalent forms. This is one:
- An object is *grue* if it has been observed and was green or has not been observed (yet) and is blue.
- All observed emeralds have been true.
- Should we conclude that all emeralds are grue?
- Another way of defining grue is that x is grue is observed before September 24th 2015 and was green or x will be blue after the same date.
- So emeralds are grue?

Underdetermination

- To each set of observations there are always different theories that fits the data.
- Take the sequence 1,2,3,4,5 (five observations)
- One hypothesis is that the sequence is 1,2,3,4,5,6,7,8,....
- Another hypothesis is that the sequence is 1,2,3,4,5,5,5,5,....
- Observation (induction) confirms both hypotheses!
- Perhaps we should chose the simplest theory (Occam's razor). But will that always give the best result?
- Goodmans paradox is an example of this problem.

So what's the problem?

- An obvious conclusion is that induction should be used with a certain measure of *common sense*.
- The problem with common sense is that it is impossible (?) to *formalize* it.
- If that is so, it seems impossible to give an algorithmic description of scientific procedure(using induction).
- A simple way of viewing this problem is that we use induction since it is successful - A pragmatic view!
- Should we be satisfied with this?

In spite of this ...

- It seems as if it is impossible not to use induction, at least in everyday situations
- But what should we do in science?
- We will describe a method that is a sort of development of the induction method.

The two methods of science

- In science we work both with deductions and observations.
- In mathematics it is almost always deductions.
- In physics we work with both methods.
- In social sciences and humanities the situation is more uncertain. But in a way observations must be used.

Is there a general scientific method?

- Experimental science has at least four different components:
- To set up hypotheses.
- To verify the hypotheses with logic.
- To evaluate the hypotheses by doing observations.
- To do experiments that generate observations.

Is there a general scientific method?

- A suggestion: It could be the Hypothetico- Deductive Method.
- It is certainly used in physics and chemistry.
- In a specialized sense it is used in mathematics.
- It seems as if it used sometimes in Social Sciences.

Carl Hempel 1905-1997



The general method

- A general method for handling observations is the Hypothetico-Deductive Method (The HD Method).
- The HD Method and the way of thinking connected to it is a central theme in scientific thinking.
- But not all researchers agree.
- Physics, astronomy, chemistry and biology seem to be the most natural areas for the method.

How it works

- Let us assume that we have a hypothesis H . We want to know if it is true or not.
- H can be a single fact or a general law.
- We have different observations E_1, E_2, \dots, E_n .
- (The observations can be generated by an experiment. They can also exist before H .)
- Does the observation *confirm* or *disconfirm* the hypothesis H ?
- The HD Method is a way to find an answer to that question.

A special case: Induction

- Goodman's problem: What hypothesis is supported by the induction?
- We first decide which hypothesis we want to test. (Goodman's problem doesn't occur.).
- A common form: H says that "All objects of type has property B".
- The observations are of the type: E1 = "Object O1 that is of type A has property B", and so on.

The HD Method used for falsification

- We have a hypothesis and want to show that it is false.
- We have a set of observations E_1, E_2, \dots, E_n .
- Assume that there is an observation E_i such that $H \Rightarrow \text{not } E_i$.
- Then E_i falsifies H .

A CS-example

- Let H be "There is an algorithmic way of solving all kind of problems"
- A logical consequence would be E: "There is an algorithmic way of solving the *Halting problem*".
- But E is false! (as Turing showed)
- So H is falsified!

An example from chemistry



Scheele



Lavoisier

Chemistry

- Great steps are taken in the 18th century.
- At the beginning of the century almost nothing is known about atoms and chemical elements. There are only two known gases: Air and carbon dioxide.
- Oxygen is discovered. (Scheele/Priestley).
- Hydrogen is discovered (Cavendish). Man It is discovered that water is composed of hydrogen and oxygen.
- Lavoisier disproves the so called phlogiston theory of combustion.

Chemistry II

- John Dalton discovers the atom.
- Berzelius describes the composition of elements.
- He creates the modern chemical notation for substances.
- Mendeleev creates the periodic table.

The Phlogiston Theory

Antoine Lavoisier



The Phlogiston Theory:
When an object is burning it
is phlogiston leaving the
object.

The Phlogiston Theory was
falsified by Lavoisier.

The falsification of The Phlogiston Theory

- Let H be The Phlogiston Theory.
- A consequence of The Phlogiston Theory must be that burning objects get lighter.
- But we can find certain metals that get heavier after burning. Let us call this observation E.
- Since $H \Rightarrow \text{not } E$, we have falsified H.

Supporting hypotheses

- It might not be possible to prove $H \Rightarrow \text{not } E$ *directly*. We might need a supporting hypothesis A such that $H \& A \Rightarrow \text{not } E$.
- A could be all our *background knowledge*. (Kuhn would call it the paradigm.)
- Eg: H = "The illness is caused by bacteria".
- A = "Penicillin kills bacteria".
- E = "The illness is not cured by penicillin".

Ad hoc hypotheses

- Supporting hypotheses should be well established and secure. Sometimes they are not:
- If $H \Rightarrow \text{not } E$ and E has been observed, someone might want to *save* H .
- This can maybe be done by assuming that the implication has the form $(H \& A \Rightarrow \text{not } E)$. Then one substitutes $A1$ for A and get $(H \& A1 \Rightarrow E)$.
- If $A1$ seems very unlikely, if considered by itself, we call $A1$ an ad hoc hypothesis.

Example: The Phlogiston Theory

- Let H = The Phlogiston Theory.
- E was the observation of a metal getting heavier after burning.
- We can argue that the implication is $H \& A \Rightarrow \text{not } E$, where A is "The phlogiston has positive weight".
- We can replace A with $A1$ = "The phlogiston in the metal has negative weight". Then $H \& A1 \Rightarrow E$!
- But how probable is $A1$?

A more critical example: Uranus and Neptune

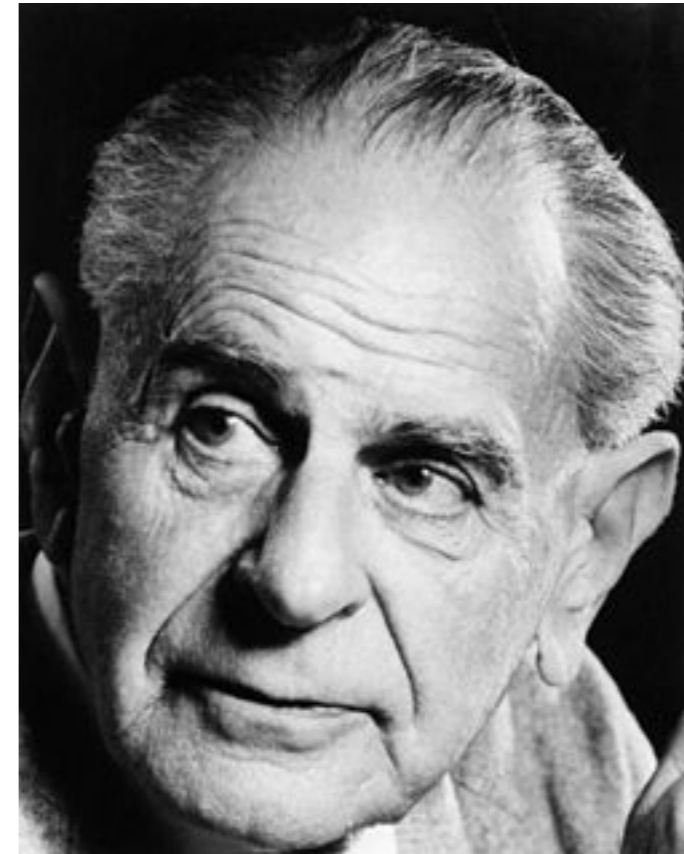
- The planet Uranus was discovered with telescope in 1781.
- In the beginning of the 19th century it was observed that Uranus didn't move in the way Newton's laws predicted.
- Call this observation E and Newton's laws H. Then we have $H \Rightarrow \text{not } E$.
- So Newton's laws were falsified!?
- But wait! The implication is really $H \& A \Rightarrow \text{not } E$ where A, amongst other things contained the statement that there are seven planets.
- But if we replace A with A* where A* says that there are unknown planets we don't get a falsification.
- and in 1846 Neptune (the eighth planet) was observed!
- So A* wasn't really an ad hoc hypothesis (or?).

The HD Method for falsification. Summary.

- We have a hypothesis and want to test if it is false.
- We use a supporting hypothesis A and deduce $H \& A \Rightarrow \text{not } E$.
- We then observe E.
- We have then falsified H.

This is what Popper believed in

- The HD-Method can be used for falsification
- But in some cases we feel that a theory can be *confirmed* by positive experiments
- Popper denied this but the logical positivists thought so
- A simple example is induction
- Now let's look at a more advanced form of induction



The HD Method used for verification

- Assume that we have a hypothesis H and observations E_1, E_2, \dots, E_n .
- When can we say that the observations confirm H ?
- One possibility is that $E_1 \& E_2 \& \dots \& E_n \Rightarrow H$. In that case H is verified.
- But let us assume that this is not the case.

Observations that confirm

- We have H and E_1, E_2, \dots, E_n .
- Assume that they are all rather improbable.
- Assume that we have a hypothesis A that we already believe is true and that $H \& A \Rightarrow E_1 \& E_2 \& \dots \& E_n$.
- Then the observations confirm H .

Arguments for and against a hypothesis

- Assume that we have observations E_1, E_2, \dots, E_n and a hypothesis H .
- Some of the observations confirm H if they together with a supporting hypothesis A_i gives $H \& A_i \Rightarrow E_i$.
- Other observations disconfirm H if they together with a supporting hypothesis B_k $H \& B_k \Rightarrow \text{not } E_k$. Observe that we don't know if B_k is true. We have not falsified H with absolute certainty.

Making a decision

- We form a type of weighted average. If the supporting hypotheses A_i are more natural than the B_k we say that H is strengthened, otherwise it is weakened.
- This works best if we can use probability theory.

A third form of the HD-Method. To choose between hypotheses.

- If we have a set of observations E_1, E_2, \dots, E_n and a hypothesis H we can try to find supporting hypotheses A_i such that $H \& A_i \Rightarrow E_i$ for all i .
- If another hypothesis H^* can do the same thing with more natural supporting hypotheses B_i (that is $H^* \& B_i \Rightarrow E_i$), then we say that H^* is a better hypothesis.

We use probability

- The previous methods were qualitative.
- We now try to do a probabilistic analysis of when observations confirm a hypothesis.
- So we have this problem: Given a hypothesis H and an observation E , when can we say that the observation confirms H ?

An important formula

Thomas Bayes 1702-1761



He found an important formula connecting different types of conditional probabilities.

This formula is the basis for so called Bayesian Statistics.

Bayes' formula

Let H be an hypothesis and e an observation.

Remember that $P(A|B) = \frac{P(A \& B)}{P(B)}$.

We assume that $P(e|H)$ and $P(e|\neg H)$ can be estimated.

We know that $P(e) = P(e|H)P(H) + p(e|\neg H)P(\neg H)$.

Then $P(H|e) = \frac{P(e|H)P(H)}{P(e)}$

$$= \frac{P(e|H)P(H)}{P(e|H)P(H) + p(e|\neg H)P(\neg H)}$$

Example: Test of medicine

- Let us assume that we have a certain medicine that is supposed to cure a disease. Call the hypothesis that the medicine works H .
- We make an observation. It is that a sick Patient gets well after been given the medicine. Call this observation E .
- Can we decide to what degree E confirms H ?

Test of medicine II

- We want to find $P(H|E)$.
- We need to estimate some probabilities in Bayes' formula.
- $P(E|H) = 1$ seems reasonable.
- $P(E|\text{not } H)$ is more complicated. Let us assume that we have the probability 0.25.
- $P(H)$ is even more complicated. Let us start with the guess $P(H)=0.5$.
- That gives us $P(H|E) = 0.8$.

Test of medicine III

- Let us now assume that we have the guess $P(H) = 0.1$.
- That gives us $P(H|E) = 0.36$.
- In both cases we find that $P(H|E) > P(H)$.
- We can use this relation to define strengthening.

A CS-example

- Assume that we have a computer running a program. We observe errors in the output. Could it be a hardware error?
- Let us call this hypothesis (hardware error) H .
- Let us assume that we have an observed error e .
- We estimate that the probability for this type of error is $P(e) = 0.1$.
- We estimate $P(e|H) = 0.4$ and $P(H) = 0.05$
- Then Bayes gives us a new estimate $P(H) = P(H|e) = 0.2$.
- This observation strengthens H .

A CS-example

- Let us look at the hypothesis not H.
- We estimate $P(\text{not H}) = 0.95$.
- We must have $P(e | \text{not H}) = 0,08$ (Why?)
- Then Bayes gives us a new estimate
- $P(\text{not H}) = P(\text{not H} | e) = 0,76$
- This observation has weakened not H.

Definition of strengthening

- We have a hypothesis H and an observation E .
- We say that E strengthens H if $P(H|E) > P(H)$.
- and we say that it weakens H if $P(H|E) < P(H)$.

Other ways of putting it

- We assume that $0 < P(E) < 1$.
- E strengthens H if $P(E|H)/P(E) > 1$, i.e.
 $P(E|H) > P(E)$.
- E weakens H if $P(E|H)/P(E) < 1$, i.e.
 $P(E|H) < P(E)$
- Or we can say it like this:
- E strengthens H if $P(E|H) > P(E| \text{not } H)$.
- E weakens H if $P(E|H) < P(E| \text{not } H)$.

Different views of probability

There are three different ways in which probability can be interpreted.

- *Axiomatic*: We postulate a set of equally probable *elementary events*. Every other events is expressed as a combination of these events.
- *Frequency*: The probability for an event is roughly the frequency with which the event will occur in repeated experiments.
- *Subjective*: We give a measure for the "probability" of events without giving a formal basis for this measure.

It seems as if the Bayesian view of verification relies on an extensive use of subjective probability. This is a problem since subjective probability is not universally accepted as a stringent scientific concept.