Solutions, Tutorial 2

1.

a) 104 h $\approx$4 days
b) $\approx$20 days *(The distance to Jupiter is approximately 5 A.U.)*

2.

$$\psi = \arctan(\frac{\omega r}{u_{solar\ wind}})$$

The solar rotation is given by $25 \big/ \left(1 - 0.19 \sin^2 \lambda\right)$ where λ is the solar latitude (Fälthammar p. 114). If we use the value for the equatorial plane, we get a solar rotation period of 25 days, and

$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{25 \cdot 24 \cdot 3600} = 2.9 \cdot 10^{-6}\,\text{rad/s}$$

r = 1 A.U. = $1.496 \cdot 10^{11}$ m

$u_{\text{solar wind}} = 150 \cdot 10^3$ m/s

Thus:

$$\psi = \arctan(\frac{2.9 \cdot 10^{-6} \cdot 1.496 \cdot 10^{11}}{150 \cdot 10^3}) = 70.9°$$

3.

Consider a sphere with radius 1 A.U. Now, consider first a small portion (1 m^2) of this sphere. What is the volume of the solar wind which passes this area? By considering how far the solar wind will travel in 1 s after passing this surface, we can work out the volume of solar wind that passes through the surface in every second, per m^2. Since the speed of the solar wind is 320 km/s, this volume is

V = 1 $m^2 \cdot 320 \cdot 10^3$ m = $3.2 \cdot 10^5$ m^3

The mass of material lost per 1 m^2 per second is this volume multiplied by the density ρ of the solar wind at Earth orbit. Approximating that the solar wind contains 100% protons, and that the number density is 8 $cm^{-3} = 8 \cdot 10^6$ m^{-3} (Fälthammar p. 123), we get

$\rho = n_e m_p$ = 8·10^6·1.67·10^{-27} kg/m^3

So the mass loss per second is Vρ/t = 4.3·10^{-15}/1 kgs^{-1}

To get the total mass loss, we multiply this number by the area *S* of the sphere with radius r = 1 A.U.

S = 4πr^2 = 2.8·10^{23}

So the total outflow is $SV\rho/t$ = 1.2·10^9 kg/s. You can easily express this in solar masses per year: 2·10^{-14} M_{sun} yr^{-1}

4.

a)

$$\frac{dn_e}{dt} = q - \alpha n_e^2$$

$$q = 0 \quad \Rightarrow$$

$$\frac{dn_e}{dt} = -\alpha n_e^2 \qquad \Rightarrow$$

$$\int \frac{dn_e}{n_e^2} = -\alpha \int dt \qquad \Rightarrow$$

$$-\frac{1}{n_e} = -\alpha t + C \qquad \Rightarrow$$

$$\alpha t = \frac{1}{n_e} + C$$

Determine *C*:

$$n_e(t=0) \equiv n_{e0} \quad \Rightarrow$$

$$C = -\frac{1}{n_{e0}} \quad \Rightarrow$$

$$\alpha t = \frac{1}{n_e} - \frac{1}{n_{e0}} \quad \Rightarrow$$

$$t = \frac{1}{\alpha n_e} - \frac{1}{\alpha n_{e0}}$$

$n_e = 2 \cdot 10^3$ cm^{-3} and thus $t = \frac{1}{5 \cdot 10^{-7} \cdot 2 \cdot 10^3} - \frac{1}{5 \cdot 10^{-7} \cdot 10^5} = 980\,\text{s} = 16\,\text{min}$

As it happens you didn't need to convert to SI units since $[\alpha]$ = cm^{3}s^{-1}, and $[n_e]$ = cm^{-3}. Thus $[\alpha][n_e]$ = cm^{3}s^{-1}cm^{-3} = s^{-1}.

b)

With $q = 0$, we get

$$\frac{dn_e(t)}{dt} = -\beta n_e(t) \quad \Rightarrow$$

$$\frac{dn_e}{n_e} = -\beta dt \quad \Rightarrow$$

$$\ln(n_e) + C = -\beta t$$

Let us rename the constant C to $-\ln(n_{e0})$. Then

$$\ln(n_e) - \ln(n_{e0}) = -\beta t \quad \Rightarrow$$

$$\ln\left(\frac{n_e}{n_{e0}}\right) = -\beta t$$

To get n_{e0}, I read off a daytime value of the plasma frequency, say for December 6. I get

$$f_p = 12.5 \cdot 10^6 \text{ Hz}$$

Then

$$n_{e0} = \frac{\omega_p^2 \varepsilon_0 m_e}{e^2} = \frac{(2\pi)^2 f_p^2 \varepsilon_0 m_e}{e^2} = 1.94 \cdot 10^{12}\ \mathrm{m}^{-3}$$

Similarly for a value of n_e in the night side, I get

$$f_p = 4 \cdot 10^6\ \mathrm{Hz}$$

$$n_e = \frac{\omega_p^2 \varepsilon_0 m_e}{e^2} = \frac{(2\pi)^2 f_p^2 \varepsilon_0 m_e}{e^2} = 1.99 \cdot 10^{11}\ \mathrm{m}^{-3}.$$

I estimate the time *t* it takes for the density to go from the dayside to the nightside value to be

$$t = 0.2 \cdot 24 \cdot 3600\ \mathrm{s} = 17280\ \mathrm{s}$$

Then

$$\beta = -\frac{1}{t} ln\left(\frac{n_e(t)}{n_{e0}}\right) = -\frac{1}{17280} ln\left(\frac{1.99 \cdot 10^{11}}{1.94 \cdot 10^{12}}\right) = 1.3 \cdot 10^{-4}\ \mathrm{s}^{-1}$$

5.

Differentiate the Chapman distribution expression, and set to zero:

$$n_e = \left(\frac{a_i}{a_r} I_0 n_0 e^{-\left(H a_a n_0 e^{\frac{-z}{H}} + \frac{z}{H} \right)} \right)^{\frac{1}{2}}$$

$$\frac{dn_e}{dz} = \left(\frac{a_i}{a_r} I_0 n_0 \right)^{\frac{1}{2}} \frac{d}{dz} e^{-\frac{1}{2}\left(H a_a n_0 e^{\frac{-z}{H}} + \frac{z}{H} \right)} = 0$$

$$\Rightarrow$$

$$e^{-\frac{1}{2}\left(H a_a n_0 e^{\frac{-z}{H}} + \frac{z}{H} \right)} \left(-\frac{1}{2} \right) \left(H a_a n_0 e^{\frac{-z}{H}} \cdot \left(-\frac{1}{H} \right) + \frac{1}{H} \right) = 0$$

$$\Rightarrow$$

$$e^{\frac{-z_{\max}}{H}} = \left(H a_a n_0 \right)^{-1}$$

$$\Rightarrow$$

$$z_{\max} = H \ln\left(H a_a n_0 \right)$$

6.

a)

The scale height is given by

$$H = \frac{k_B T}{g m_{CO_2}} = \frac{1.38 \cdot 10^{-23} \cdot 400}{8.87 \cdot (12 + 2 \cdot 16) \cdot 1.67 \cdot 10^{-27}} = 8500 \text{ m}$$

b)

The maximum height of a Chapman layer is given by (Equation 3.5.8 in Fälthammar)

$$z_{\max} = H \ln\left(a_a n_0 H \right) \quad \Rightarrow$$

$$n_0 = \frac{e^{\frac{z_{\max}}{H}}}{a_a H} = \frac{e^{\frac{140 \cdot 10^3}{8500}}}{10^{-24} \cdot 8500} = 1.7 \cdot 10^{27} \text{ m}^{-3}$$

where z_{max} was read off of the diagram.