Home Assignment 2

Information

The homework assignments, together with the computer workshops, are compulsory parts of the course. The assignments should be solved in groups of two or three students. The deadline of HW2 is Monday, Oct. 12, 6 pm. A total of 10 points can be obtained on HW2.

Note! If programs like Maple and Matlab are used to solve the homework assignments, print-outs from these programs must only be added as an appendix to the solution. Solutions only consisting of print-outs from Maple/Matlab will be graded with zero points.

Problems

This homework assignment will be about wind turbines and especially the rotating blades. The first part is doing a one dimensional model with truss and beam elements. In the second part, a simple two dimensional model is considered.

Problem 1. [**5 points**] In this problem, the axial and bending deformation of the turbine blade should be analyzed. The turbine blade has a length L=40 m. For simplicity, we assume that the cross-section area is constant and rectangular with dimensions $b = 0.05$ m and $h = 1$ m. The blade is made of fibre-reinforced polymer with Young's modulus $E = 150$ GPa and density $\rho = 1800$ kg/m^3. The angular velocity is $\omega = 1$ rad/s.

(a) Use *two linear and equal* truss elements to perform a FEM analysis of the axial deformation of the blade when it points down to the ground. At this instant, the blade is both loaded with gravity, ρg, and a centrifugal force due to the rotation, $\rho\omega^2 x$. A sketch of the problem is shown to the left in Figure 1. By FEM, compute the displacement at $x = 30$, $u(x = 3L/4)$. Do this by not entering numerical values! Also, on the front page, sketch (as function of x) the stress provided by the FEM solution, with numerical values. Hint: a good equation solving method for this problem is

$$\mathbf{KD} = \mathbf{F_1} + \mathbf{F_2} \implies \mathbf{D} = \mathbf{K}^{-1}(\mathbf{F_1} + \mathbf{F_2})$$

(b) Use *one* beam element to perform a FEM analysis of the bending deformation of the blade when it is horizontal. At this instant of time, the blade will be subjected to bending deformation due to its self-weight, $q = -\rho A g$. A sketch of the problem is shown to the right in Figure 1. By FEM, compute the deflection and rotation at the outermost node, w_2 and θ_2 and also plot the maximum bending stress (as function of x) at the cover page. Do the whole task (b) without using numerical values. Hints :Find the stiffness matrix for a beam element in the lecture notes, be aware of how the length of a beam element is defined and a useful integral might be

$$\int_{-1}^{1} \mathbf{N}^T d\xi = \begin{bmatrix} 1 \\ L_e/3 \\ 1 \\ -L_e/3 \end{bmatrix}$$

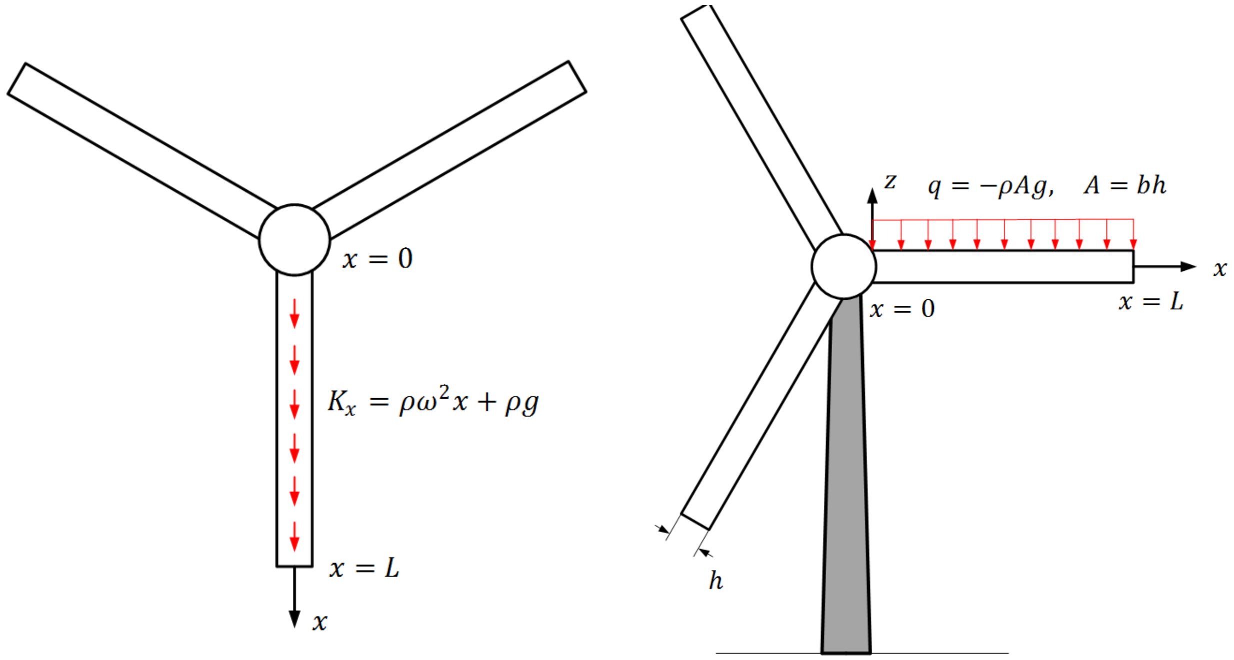

Figure 1: To the left: the problem to be solved in (a): Axial deformation of the blade due to gravity and centrifugal force. To the right: The problem to be solved in (b): Bending of the blade due to gravity.

Problem 2. [5 points] Now, a much more refined model of the rotating blade is studied with particular focus on the mounting points of the blades. The model is made of 2D iso-parametric first order elements and due to symmetry conditions, only one half of the blade needs to be explicitly modeled. The relevant part of the model is sketched to the left in Figure 2. Hint: Think of which matrix elements etc. that you really need. Assume standard numbering of the degrees of freedom throughout the task, i.e that d_1 and d_2 are the displacement of Node 1 in the horizontal and vertical direction respectively.

(a) Compute the components that the element E1, sketched to the right in Figure 2, contributes to the reduced load vector due to the centrifugal force $K_x = \rho\omega^2 x$. The integration should be done using Gauss-integration and choose the number of evaluation points so that you obtain the exact answer.

(b) When the problem is solved, the nodal displacements are calculated. For element E1, the reduced values (non-zero) are

$$\mathbf{d}_e^{(E1)} = \frac{\rho\omega^2 d^2}{E} 10^3 \begin{bmatrix} 2 \\ 4 \\ -3/2 \\ -1/2 \end{bmatrix}$$

(The result is obtained by using $\nu = 1/3$ and the values are *slightly* adjusted to get nicer numbers.) By this result, calculate the normal strain in the x-direction at position $(0, d)$ i. e. at Node 4, $\varepsilon_{xx}(0, d)$.

DON'T FORGET THE UNITS IN YOUR ANSWERS!

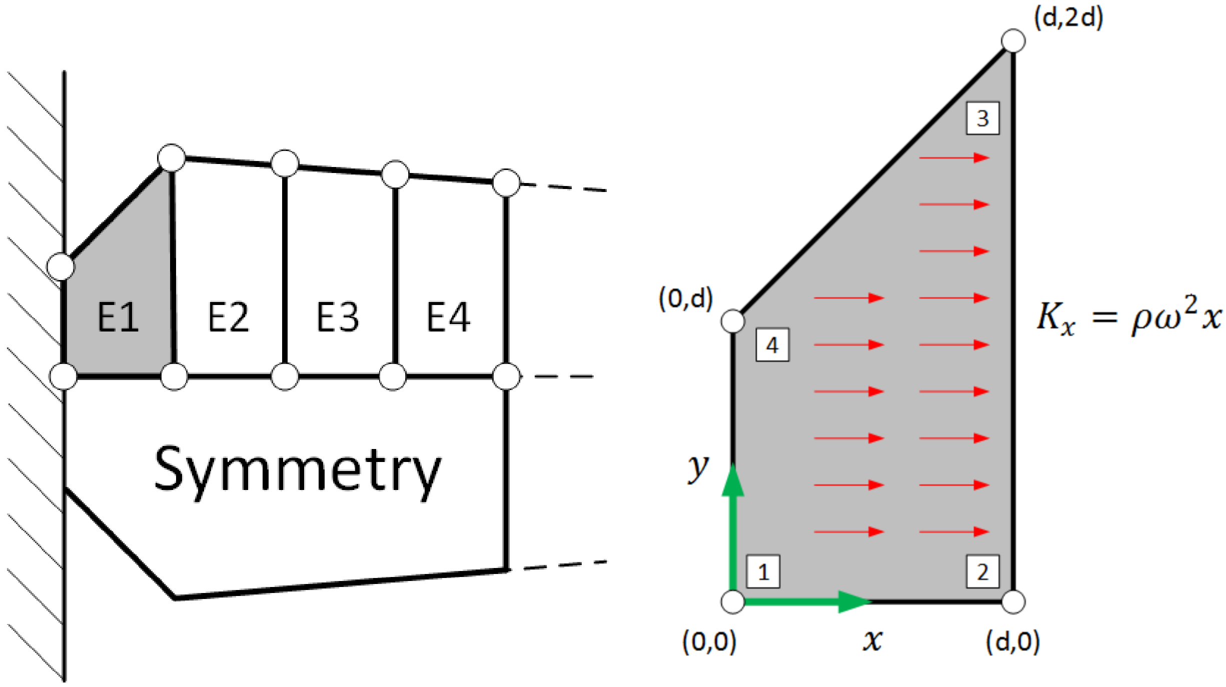

Figure 2: To the left: the FE-model of the turbine blade. To the right: The studied element with nodal positions.

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Name and personal registration number (1):

Name and personal registration number (2):

Name and personal registration number (3):

Summarize the answers below:

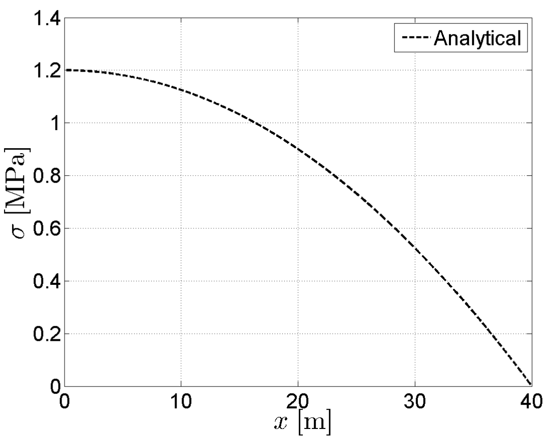
Problem 1

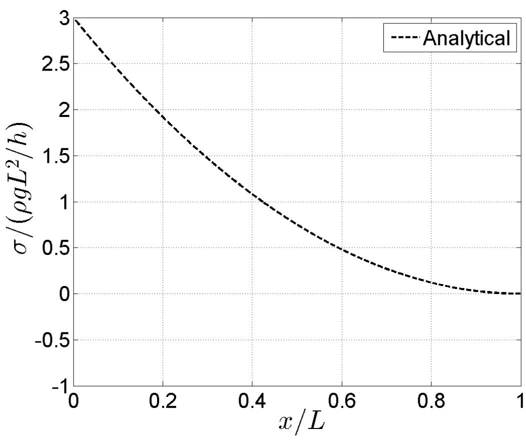
(a) $u(x = 3L/4) =$

(b) $w_2 =$

$\theta_2 =$

Draw the solutions from **(a)** and **(b)**





Problem 2

(a) The reduced force vector is

$\mathbf{F}_{red}^{E1} =$

(b) Normal strain in the x-direction at Node 4 is

$\varepsilon_{xx}(0, d) =$