

Solutions, Tutorial 1, 2014

1.

$$\rho = \frac{mv_{\perp}}{qB} \quad f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi} \frac{qB}{m}$$

Since nothing is said about how large a part of the velocity is perpendicular to \mathbf{B} , we assume that the particle kinetic energy perpendicular to \mathbf{B} is $W_{\perp} = \frac{2}{3}W$ and get

$$\frac{2}{3}W = \frac{mv_{\perp}^2}{2} \Rightarrow v_{\perp} = \sqrt{\frac{\frac{4}{3}W}{m}}$$

$$\rho = \frac{m\sqrt{\frac{\frac{4}{3}W}{m}}}{qB} = \frac{\sqrt{\frac{4}{3}mW}}{qB}$$

a)

With $W = 5 \text{ keV} = 5 \cdot 1000 \cdot 1.6 \cdot 10^{-19}$, $B = 50 \cdot 10^{-6} \text{ T}$, $m_e = 0.91 \cdot 10^{-30} \text{ kg}$ we get

$$\rho = \frac{\sqrt{\frac{4}{3} \cdot 0.91 \cdot 10^{-30} \cdot 5 \cdot 1.6 \cdot 10^{-16}}}{1.6 \cdot 10^{-19} \cdot 50 \cdot 10^{-6}} = 3.9 \text{ m}$$

$$f_c = \frac{1}{2\pi} \frac{eB}{m_e} = \frac{1.6 \cdot 10^{-19} \cdot 50 \cdot 10^{-6}}{2\pi \cdot 0.91 \cdot 10^{-30}} = 1.4 \cdot 10^6 \text{ Hz}$$

The rest of the problems are solved in a similar way. Note that in c), the charge of the alpha particle is $q = 2e$, and its mass is approximately $4m_p$. You can use $B = 0.3 \text{ T}$ (compare exercise 3).

Answers for the remaining subproblems:

	ρ_L	f_c
a)	3.9 m	1.4 MHz
b)	167 m	762 Hz
c)	0.025 m	2.3 MHz
d)	75 km	0,08 Hz
e)	0.017 m	1.4 MHz

In **d)** we have used a typical solar wind magnetic field of 5 nT (See Fälthammar p. 123).

2.

The Parker spiral angle is given by

$$\Psi = \arctan\left(\frac{\omega_{sun} r}{u_{sw}}\right)$$

where

$$\omega_{sun} = \frac{2\pi}{T_{sun}} = \frac{2\pi}{27 \cdot 24 \cdot 2600} = 2.7 \cdot 10^{-6} \text{ s}^{-1}$$

and we can take u_{sw} to be its average value of 320 km s^{-1} .

a) At the closest approach to the sun (perihelion) $r = 0.31 \text{ AU}$ which gives

$$\Psi = \arctan\left(\frac{2.7 \cdot 10^{-6} \cdot 0.31 \cdot 1.5 \cdot 10^{11}}{320 \cdot 10^3}\right) = 21^\circ$$

At aphelion $r = 0.47 \text{ AU}$ which gives

$$\Psi = \arctan\left(\frac{2.7 \cdot 10^{-6} \cdot 0.47 \cdot 1.5 \cdot 10^{11}}{320 \cdot 10^3}\right) = 30^\circ$$

b) $r = 1.0 \text{ AU} \Rightarrow$

$$\Psi = \arctan\left(\frac{2.7 \cdot 10^{-6} \cdot 1.5 \cdot 10^{11}}{320 \cdot 10^3}\right) = 52^\circ$$

c) $r = 4.95 \text{ AU}$ (perihelion) \Rightarrow

$$\Psi = \arctan\left(\frac{2.7 \cdot 10^{-6} \cdot 4.95 \cdot 1.5 \cdot 10^{11}}{320 \cdot 10^3}\right) = 81^\circ$$

3.

The pressure inside and outside a sunspot is, respectively

$$p_{in} = 2n_{in}k_B T_{in} + \frac{B_{in}^2}{2\mu_0}$$

$$p_{out} = 2n_{out}k_B T_{out} + \frac{B_{out}^2}{2\mu_0}$$

The factor 2 comes from the fact that electrons and ions contribute $nk_B T$ each.

$$p_{in} = p_{out} \Rightarrow$$

$$n_{in} = \frac{2n_{out}k_B T_{out} + \frac{B_{out}^2}{2\mu_0} - \frac{B_{in}^2}{2\mu_0}}{2k_B T_{in}}$$

With

$$B_{in} = 0.3 \text{ T}$$

$$B_{out} = 0 \text{ T}$$

$$T_{in} = 4000 \text{ K}$$

$$T_{out} = 6000 \text{ K}$$

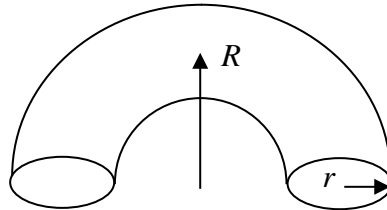
$$n_{out} = 2.1 \cdot 10^{19} \text{ cm}^{-3}$$

we get

$$\frac{n_{in}}{n_{out}} = 1.48$$

4.

We know the size of flares is comparable to sunspots. Model the flare by a half torus with minor axis $r = 1000$ km, and major axis $R = 10\,000$ km.



Let this half-torus be filled with a magnetic field of strength $B \sim 0.3$ T (similar to that in sunspots). If the volume of the half-torus is V and the magnetic energy density is p_B , the total energy is

$$W = V p_B = \pi R \pi r^2 \frac{B^2}{2\mu_0}$$

With the above numbers we get $W \sim 10^{24}$ J, which is similar to the energy released in a solar flare.

5.

a) Wien's displacement law gives

$$\lambda_{max} = \frac{2.9 \cdot 10^{-3}}{T} = \frac{2.9 \cdot 10^{-3}}{310} = 9.4 \cdot 10^{-6} \text{ m} = 9400 \text{ nm} = 9.4 \text{ } \mu\text{m}.$$

This is infra-red radiation.

b)

$$\lambda_{max} = \frac{2.9 \cdot 10^{-3}}{4200} = 6.9 \cdot 10^{-7} \text{ m} = 690 \text{ nm}$$

Dark red.

c)

$$P_{sun} = \sigma_{SB} T_{sun}^4 \cdot 4\pi r_{sun}^2$$

$$r_{sun} = \frac{1.39 \cdot 10^9}{2} \text{ m} = 6.95 \cdot 10^8 \text{ m}$$

$$P_{with\ spot} = \sigma_{SB} T_{sun}^4 \cdot (4\pi r_{sun}^2 - \pi r_{spot}^2) + \sigma_{SB} T_{spot}^4 \pi r_{spot}^2$$

Then

$$\begin{aligned} \frac{P_{with\ spot}}{P_{sun}} &= \frac{\sigma_{SB} T_{sun}^4 \cdot (4\pi r_{sun}^2 - \pi r_{spot}^2) + \sigma_{SB} T_{spot}^4 \pi r_{spot}^2}{\sigma_{SB} T_{sun}^4 \cdot 4\pi r_{sun}^2} \\ &= \frac{T_{sun}^4 \cdot (4r_{sun}^2 - r_{spot}^2) + T_{spot}^4 r_{spot}^2}{T_{sun}^4 \cdot 4r_{sun}^2} \\ &= \frac{6000^4 \cdot (4 \cdot (6.95 \cdot 10^8)^2 - (10^8)^2) + 4200^4 \cdot (10^8)^2}{6000^4 \cdot 4 \cdot (6.95 \cdot 10^8)^2} = 0.99607 \end{aligned}$$

or

$$\frac{P_{sun} - P_{with\ spot}}{P_{sun}} = 1 - \frac{P_{with\ spot}}{P_{sun}} = 1 - 0.99607 = 0.4 \%$$