KTH Informations- och kommunikationsteknik

Written Re-exam with solutions for IE1204/5 Digital Design Friday 10/4 2015 8.00-12.00

General Information

Examiner: Ingo Sander.

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Exam text does not have to be returned when you hand in your writing.

Aids: No aids are allowed!
The exam consists of three parts with a total of 12 tasks, and a total of 30 points:

Part A1 (Analysis) contains eight short questions. Right answer will for six of the questions give you one point and for two of the questions one or two points. Incorrect answer will give you zero points. The total number of points in Part A1 **is 10 points**. To **pass the Part A1 requires at least 6p**, *if fewer points we will not look at the rest of your exam.*

Part A2 (Methods) contains two method problems on a total of 10 points.

To **pass the exam** requires at least **11 points** from A1 + A2 , *if fewer points we will not look at the rest of your exam.*

Part B (Design problems) contains two design problems of a total of 10 points. Part B is corrected only if there are at **least 11p** from the exam A- Part.

NOTE ! At the end of the exam text there is a submission sheet for Part A1 , which can be separated to be submitted together with the solutions for A2 and B.

For a passing grade (**E**) requires at **least 11 points on the exam**.

Grades are given as follows:

0 –	11 –	16 –	19 –	22 –	25
F	E	D	C	B	A

The result is expected to be announced before Monday 4/5 2015.

Part A1: Analysis

Only answers are needed in Part A1. Write the answers on the submission sheet for Part A1, which can be found at the end of the exam text.

1. 1p/0p
A function $f(x, y, z)$ is described by the equation:

$$f(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{y}z + x\bar{y}\bar{z} + \bar{x}y\bar{z}$$

Minimize the function.

$$f(x,y,z)_{\min} = ?$$

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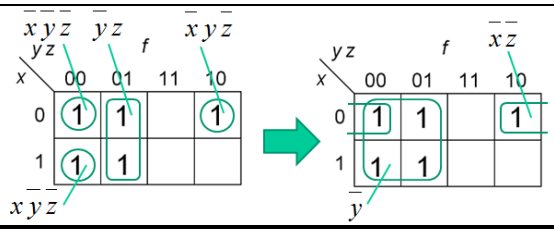
1. Proposed solution

$$f(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{y}z + x\bar{y}\bar{z} + \bar{x}y\bar{z} = \{Kmap\} =$$
$$= \bar{y} + \bar{x}\bar{z}$$



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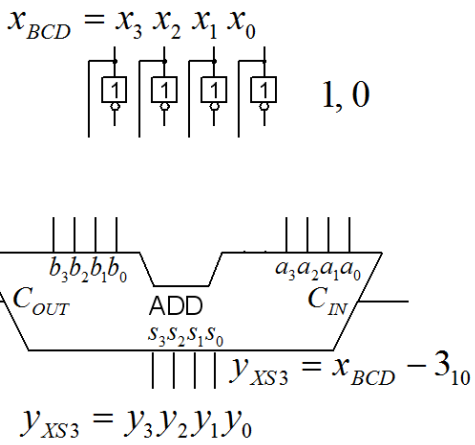
2. 2p/1p/0p
When the numbers 0 ... 9 are coded with the usual 4 bit binary code it is called BCD code (binary code for 10 ... 15 are not included in the BCD code). Sometimes, a 4-bit code in which the BCD code words is reduced by 3, the so-called "exess-3 code", XS3, is used. $(y_3y_2y_1y_0)_{XS3} = (x_3x_2x_1x_0)_{BCD} - 3_{10}$.

a) Use a four-bit adder and, if necessary inverters to make a BCD→XS3 coder. The subtraction should be done with the two complement method. The constants 0 and 1 are available. Draw your solution in the figure on the answer sheet.

b) What **binary** XS3 code has the BCD number 9?



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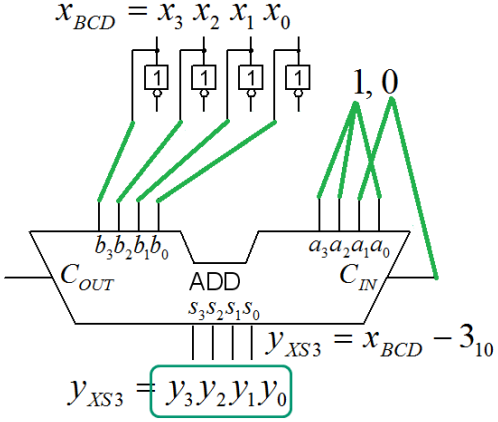
2. Proposed solution **a**) -3=1101.



b) BCD 9 is 1001. XS3 code gets 1001 – 0011 = 0110.

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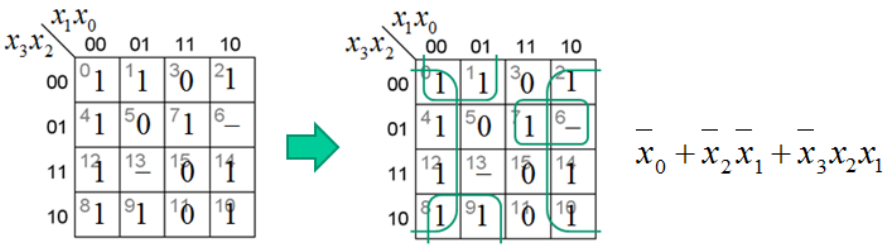
3. 1p/0p
Given is a Karnaugh map for a function of four variables $y = f(x_3, x_2, x_1, x_0)$.
Write the function as a **minimized** sum of products, SP form. ”-” in the map means ”don’t care”.

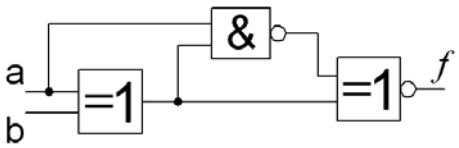
x_3x_2 \ x_1x_0	00	01	11	10
00	1	1	0	1
01	1	0	1	–
11	1	–	0	1
10	1	1	0	1

3. Proposed solution



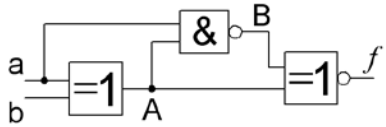
$$\bar{x}_0 + \bar{x}_2\bar{x}_1 + \bar{x}_3x_2x_1$$

4. 2p/1p/0p
The figure shows a circuit consisting of three gates.



a) Specify the logical function f **truth table**.
b) Derive a **simplified expression** for the function $f = f(a,b)$.

4. Proposed solution



a)

a	b	A	B	f
0	0	0	1	0
0	1	1	1	1
1	0	1	0	0
1	1	0	1	0

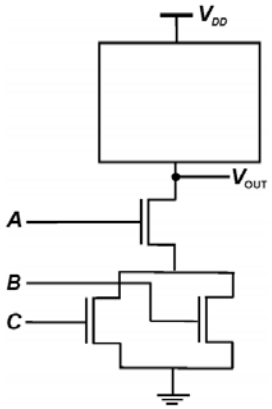
b)

$$f = \overline{\overline{a \cdot (a \oplus b)} \oplus (a \oplus b)}$$

$$f = \bar{a} \cdot b$$

5. 1p/0p

Give an expression for the logical function realized by the CMOS circuit in the figure? Only the "pull-down" circuit appears, the "pull-up" circuit is symbolized by the square at the top of the figure.



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5. Proposed solution

Pull Down circuit gives inverted function.
A is in series (·) with (+) *B* and *C* in parallel.

$$\overline{V}_{OUT} = A\cdot(B+C) \quad V_{OUT} = \overline{A\cdot(B+C)} = \{dM\} = \overline{A} + \overline{(B+C)} = \overline{A} + \overline{B}\,\overline{C}$$

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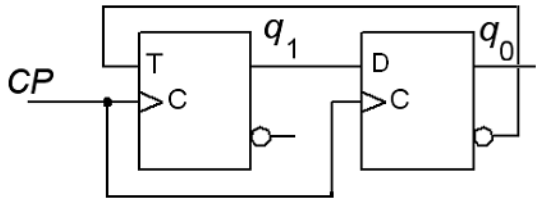
6. 1p/0p



This sequence circuit consists of a T-flip-flop and a D-flip-flop, it starts in the state q_1q_0 00. Analyze the circuit and give the sequence for the following three clock pulses.

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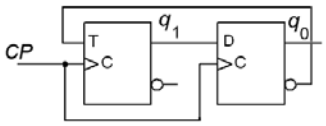
6. Proposed solution

$$q_1^+ = T\cdot\overline{q}_1 + \overline{T}\cdot q_1 \quad T = \overline{q}_0 \quad \Rightarrow \quad q_1^+ = \overline{q}_0\cdot\overline{q}_1 + \overline{\overline{q}_0}\cdot q_1 = \overline{q}_0\,\overline{q}_1 + q_0q_1 = \overline{q_0 \oplus q_1}$$

$$q_0^+ = q_1$$



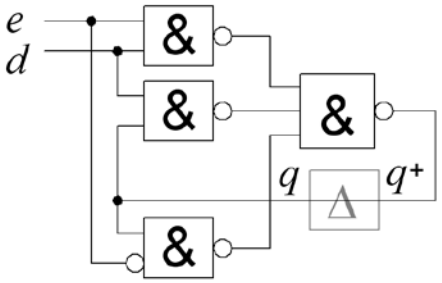
q_1q_0	$\overline{q_1 \oplus q_0}$	$q_1^+q_0^+$	
00	1	10	
10	0	01	$00 \rightarrow 10 \rightarrow 01 \rightarrow 00 \ldots$
01	0	00	
00	1	10	

---

7. 1p/0p

The figure shows a type of *asynchronous* **latch**. It's called the Earle latch (but it is not included in the course materials). Derive the circuit's characteristic function.

$q^+ = f(q,e,d) = ?$



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7. Proposed solution

$$q^+ = f(q,e,d) = \overline{(\overline{e \cdot d}) \cdot (\overline{q \cdot d}) \cdot (\overline{\bar{e} \cdot q})} = \{dM\} = ed + qd + \bar{e}q$$

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8. 1p/0p

The following is the VHDL code for a **2:1 MUX**. Unfortunately, some of the code has fallen away, this is marked by **(????)**
Answer with making the line of code complete!

```
ENTITY MUX_2_1 IS
PORT (   d_in  : IN   STD_LOGIC_VECTOR(1 downto 0) ;
         a     : IN   STD_LOGIC ;
         d_out : OUT  STD_LOGIC ) ;
END MUX_2_1 ;

ARCHITECTURE beh OF MUX_2_1 IS
BEGIN
  d_out <= ( NOT a AND d_in(0) ) OR ( ???? ) ;
END beh ;
```

---

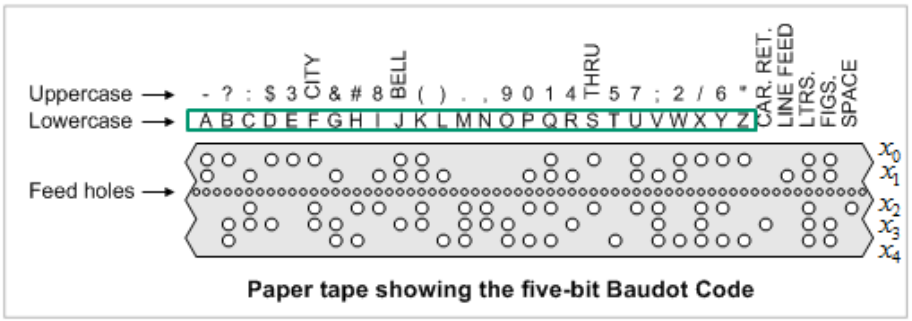
8. Proposed solution

```
d_out <= ( NOT a AND d_in(0) ) OR ( a AND d_in(1) ) ;
```

Part A2: Methods

Note! Part A2 will only be corrected if you have passed part A1 (≥6p).

9. 5p

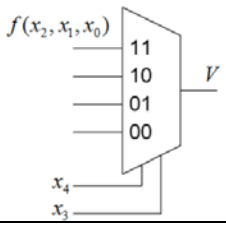


The figure shows an punched tape with the Baudot code. Your task is to make a decoder for vowels (our Swedish vowels Å Ä Ö are *not* included) that provides the output V=1 only when the code is corresponding to a vowel. Assume that the paper tape to read is edited so that only the English alphabet letters **A … Z** are appearing (letters from the line "Lowercase"). No special characters (Car. Return, Line Feed, Ltrs ...) can occur. Consider Y as a vowel. Hole = 1, no hole = 0.

a) (1p) Enter the **truth table** $V = f(x_4,x_3,x_2,x_1,x_0)$, or directly as Karnaugh maps. Specify "do not care". One code word is missing on the character strip - which one? - It could also be used as "do not care".

b) (2p) **Minimize the function** V and express it as a sum of products (SP). Use do not care.

c) (2p) In order to reduce the number of gates a multiplexer is used. Realize the function V with a **4:1 MUX** and a minimized number of optional gates. As select variables x_4 and x_3 should be used.



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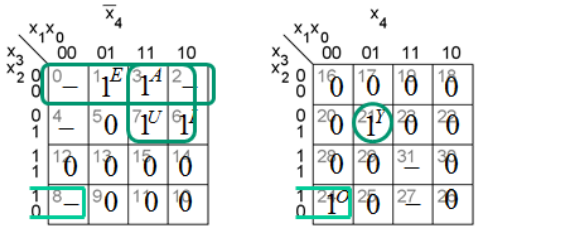
9. Proposed solution

a) $V = 1$ for
A(3), E(1), I(6), O(24), U(7), Y(21).
V is don't care for
BLANK(0), LF(2), CR(8), SP(4),
LTRS(31), FIGS(27).

It is the code for BLANK, "no holes" 00000 that is missing on the character strip. That don't care position is not utilized in this solution.

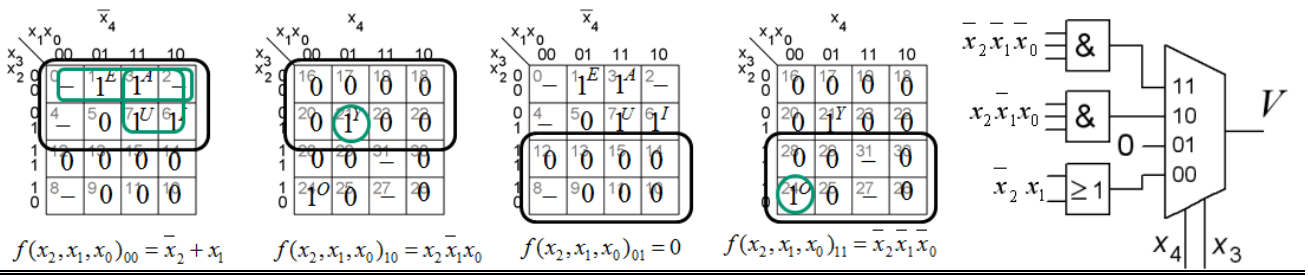
b)

A	00011	3	*O*	11000	24	*BLANK*	00000	0	*SP*	00100	4
E	00001	1	*U*	00111	7	*LF*	00010	2	*LTRS*	11111	31
I	00110	6	*Y*	10101	21	*CR*	01000	8	*FIGS*	11011	27



$$V = \bar{x}_4\bar{x}_3\bar{x}_2 + \bar{x}_4\bar{x}_3x_1 + x_3\bar{x}_2\bar{x}_1\bar{x}_0 + x_4\bar{x}_3\bar{x}_2\bar{x}_1\bar{x}_0$$

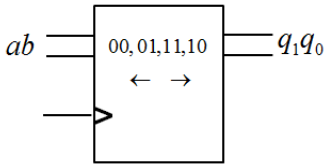
c)

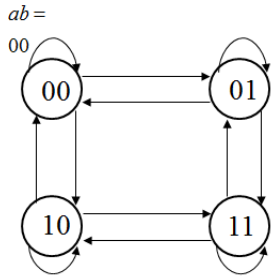


$f(x_2,x_1,x_0)_{00} = \bar{x}_2 + x_1$ $\quad f(x_2,x_1,x_0)_{10} = x_2\bar{x}_1x_0$ $\quad f(x_2,x_1,x_0)_{01} = 0$ $\quad f(x_2,x_1,x_0)_{11} = \bar{x}_2\bar{x}_1\bar{x}_0$

10. 5p

A counter (a Moore-machine) counts Graycode up "→" 00 01 11 10 or down "←" 00 10 11 01. With two mode-signals *a b* (00 01 11 10) you control to which the state counter should count, and then remain in this state until *ab* are changed. Choose to follow up/down sequence so that the desired state *ab* is reached after as few steps as possible – if the choice up/down does not imply any difference in the number of steps, then choose to follow the up "→" sequence.

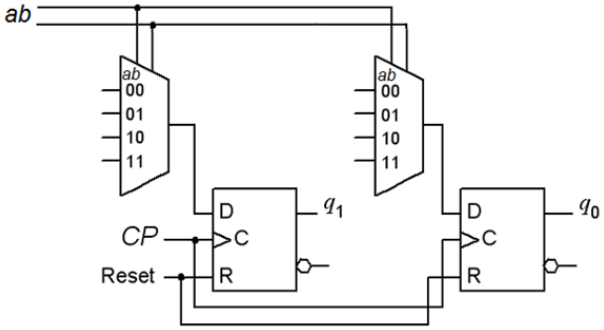




a) (1p) Finish the initiated state diagram. Map the condition for all state transitions.

b) (2p) Write state table based on the state diagram. Derive minimized expressions for the next state. $q_1^+q_0^+ = f(q_1q_0, ab)$ $q_1^+ = f(q_1q_0, ab)$ $q_0^+ = f(q_1q_0, ab)$

c) (2p) Realize the next state decoder with two 4:1 multiplexers. The control signals *a* and *b* are connected to the multiplexers select inputs. Derive the minimized function expressions for the multiplexers data inputs. See the figure.



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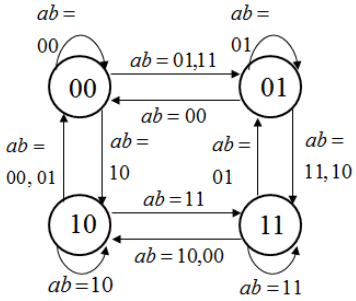
10. Proposed solution

a) b)



$q_1^+q_0^+ = f(q_1q_0, ab)$

q_1q_0 \ ab	00	01	11	10
00	00	01	01	10
01	00	01	11	11
11	10	01	11	10
10	00	00	11	10

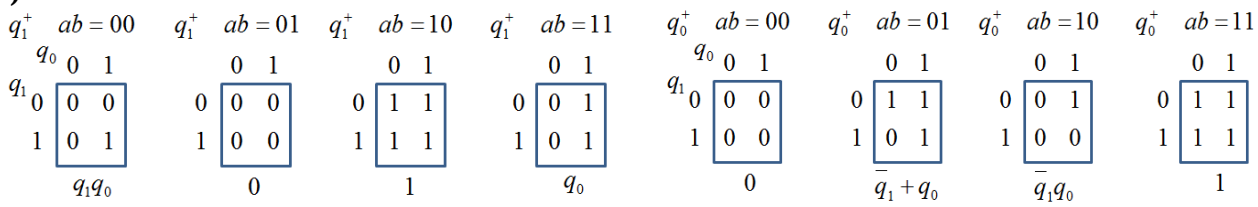
q_1q_0 \ ab	00	01	11	10
00	0	0	0	1
01	0	0	1	1
11	1	0	1	1
10	0	0	1	1

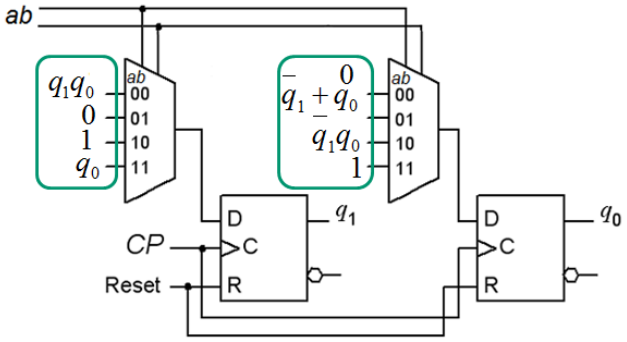
$q_1^+ = aq_0 + aq_1 + a\bar{b} + q_1q_0\bar{b}$

q_1q_0 \ ab	00	01	11	10
00	0	1	1	0
01	0	1	1	1
11	0	1	1	0
10	0	0	1	0

$q_0^+ = b\bar{q}_1 + bq_0 + ab + a\bar{q}_1q_0$

c)

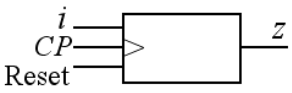



ab
q_1q_0
0
1
q_0
ab
00
01
10
11
$\overline{q}_1 + q_0$
$q_1\overline{q}_0$
1
D
C
R
q_1
q_0
CP
Reset

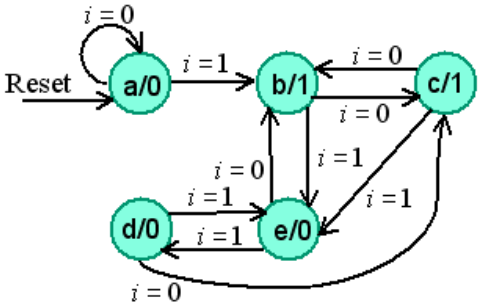
Part B. Design Problems

Note! Part B will only be corrected if you have passed part A1+A2 (≥11p).

11. 5p Synchronous serial two-complementer.



a) (1p) A synchronous sequential circuits, a Moore machine, has the state diagram to the right. Minimize the number of states, set up the minimized state-table and draw the minimized state diagram. (This may very well prove to be time well spent before the subtask b).



b) (1p) The Rapid method for the two complement of a binary number means that, in the direction from the least significant bit to the most significant bit, all the bits including the leading 1 are **copied** and then all of the following bits are **inverted**.
A synchronous sequential circuit, a Moore machine, will at the input i for each clock pulse, get the bits in this serial order. After each clock pulse the sequential circuit output z will present the corresponding bit *copied* or *inverted* according to the rule. After each complete data word the circuit is cleared with the (asynchronous) reset. Derive the circuit's state table and draw the state diagram.
(2p) Derive the **coded state table** (select the code yourself) and derive the minimized functions for **next state** and **output**.
(1p) **Draw schematic** it is free to use any type of gates together with the D-flip-flops wich has asynchronous reset input.

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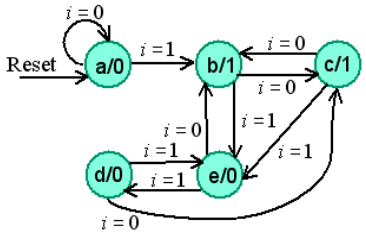
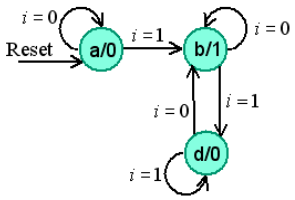
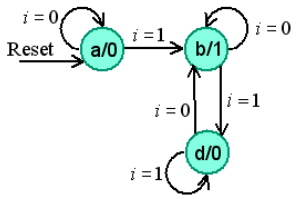
11. Proposed solution



(ade)(bc)
$a_0 \rightarrow$(**b**c**e**) $a_1 \rightarrow$(**bc**)
$d_0 \rightarrow$(**bc**) $d_1 \rightarrow$(**ade**)
$e_0 \rightarrow$(**bc**) $e_1 \rightarrow$(**ade**)
(a)(bc)(de)

(a)(bc)(de)
$d_0 \rightarrow$(**bc**) $d_1 \rightarrow$(**de**)
$e_0 \rightarrow$(**bc**) $e_1 \rightarrow$(**de**)
$b_0 \rightarrow$(**bc**) $b_1 \rightarrow$(**de**)
$c_0 \rightarrow$(**bc**) $c_1 \rightarrow$(**de**)
(a)(bc)(de) a b d





q_1q_0	0	1	z
a 00	a00	b11	0
b 11	b11	d10	1
d 10	b11	d10	0
01	– –	– –	–

$q_1^+q_0^+$

	0	1
00	0	1
01	–	–
11	1	1
10	1	1

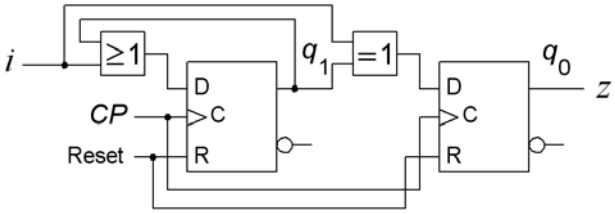
q_1^+

0	1
0	–
1	0
1	0

q_0^+

$$q_1^+ = q_1 + i$$
$$q_0^+ = \bar{q}_1 i + q_1 \bar{i} = q_1 \oplus i$$
$$y = q_0$$

State encoding is free, except that *a* must have the code 00 as we use asynchronous reset.



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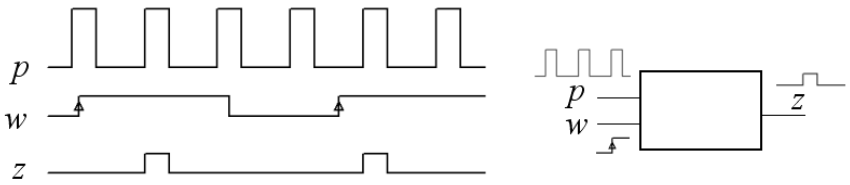
12. (5p) Asynchronous edge triggered pulse gate



An asynchronous sequential circuits has two inputs and one output. On one input there is a pulse train *p*, on the other input there is a slow signal *w* (slow in comparision to *p*). As soon as possible after each rising edge of *w*, the output *z* will ”let through” the next complete pulse (one complete pulse) from *p*. Output *z* is 0 all other times. See the figure's example.

Your answer must include a **state diagram**, if necessary minimized, a **flow table**, and an appropriate **state assignment** with a **excitation table** that gives race-free net. You must also develop the **hazard-free** expressions for the **next state** and an expression for the **output**, but you don’t need to draw the gate circuit.

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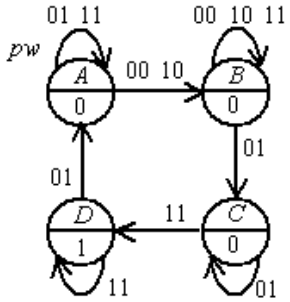
12. Proposed solution



pw	00	01	11	10	z
A	B	(A)	(A)	B	0
B	(B)	C	(B)	(B)	0
C	–	(C)	D	–	0
D	–	A	(D)	–	1

The states can be coded with the gray code. Two of the don’t care entries occur because the signal *w* is slow and then always will be *w* = 1 during the pulses from *p*.

q_1q_0 \ pw	00	01	11	10	z
00	01	00	00	01	0
01	01	11	01	01	0
11	–	11	10	–	0
10	–	00	10	–	1

$z = q_1\bar{q}_0$

q_1^+	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	–	1	1	–
10	–	0	1	–

$q_1^+ = q_1q_0 + q_0\bar{p}w + q_1pw$

q_0^+	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	–	1	0	–
10	–	0	0	–

$q_0^+ = \bar{q}_1q_0 + \bar{w} + q_0\bar{p}$

Good Luck!

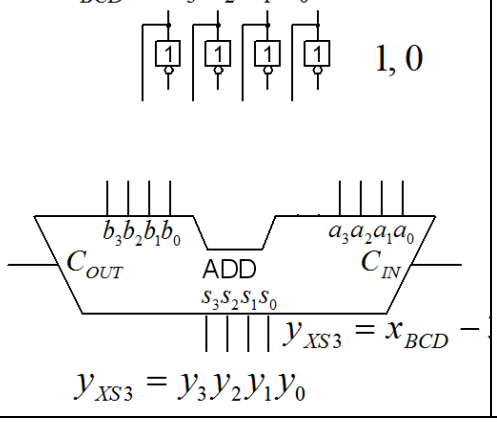
Submission sheet for Part A1 Sheet 1

(remove and hand in together with your answers for part A2 and part B)

Last Name: _____ **Given Name:** _____

Personal code number: _____

Write down your answers for the questions from Part A1 (1 to 8)

Question	Answer	
1 1p	$f(x,y,z) = \{SP\}_{\min} = ?$	
2 1+1p	**a)** $x_{BCD} = x_3\ x_2\ x_1\ x_0$  $y_{XS3} = y_3\ y_2\ y_1\ y_0$	**b)** $x_{BCD} = 9 \rightarrow y_{XS3} = ?_2$ (answer with a binary number)
3 1p	$y = f(x_3, x_2, x_1, x_0) = \{SP\}_{\min} = ?$	
4 1+1p	**a)** Truth table ab | f | ab | f 00 | | 10 | 01 | | 11 |	**b)** Simplified function $f(a,b) = ?$
5 1p	$V_{OUT} = f(A,B,C) = ?$	
6 1p	q_1q_0 00,	
7 1p	$q^+ = f(q,e,d) = ?$	
8 1p	`d_out <= ( NOT a AND d_in(0) ) OR (  ) ;`	

This table is completed by the examiner!!

Part A1	Part A2		Part B		Total	
Points	9	10	11	12	Sum	Grade