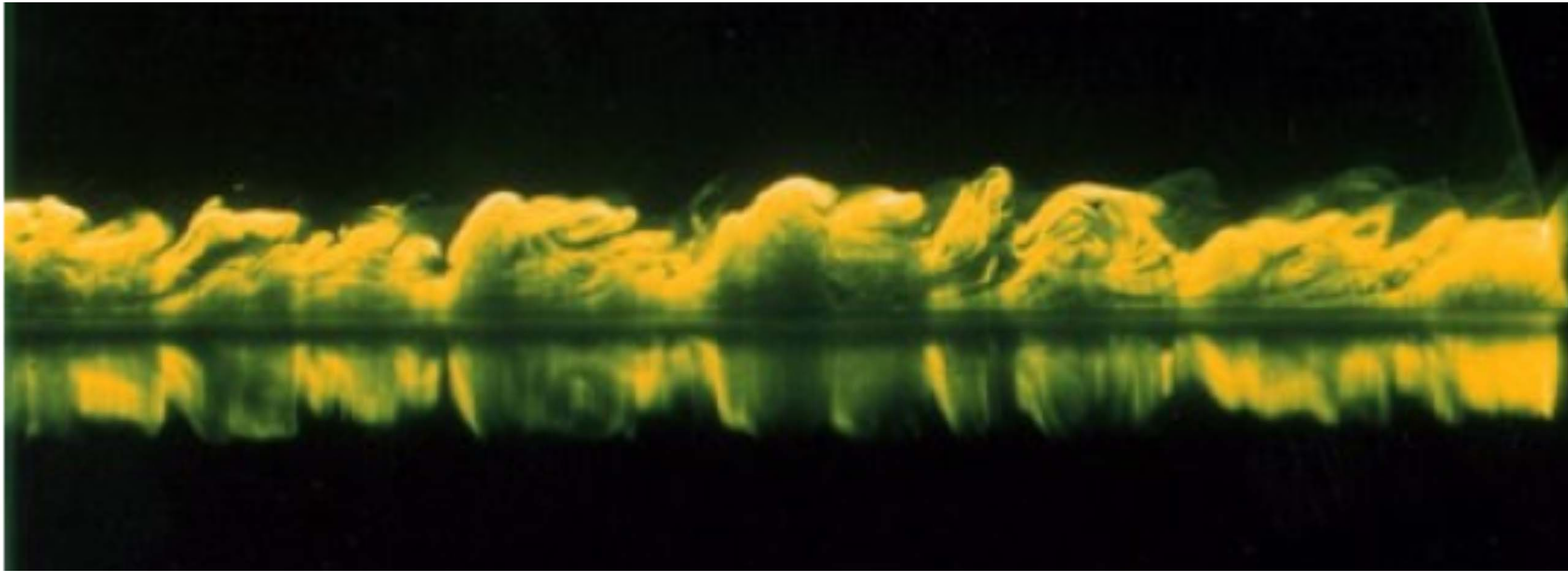
Turbulence

There are no “simple” turbulent flows

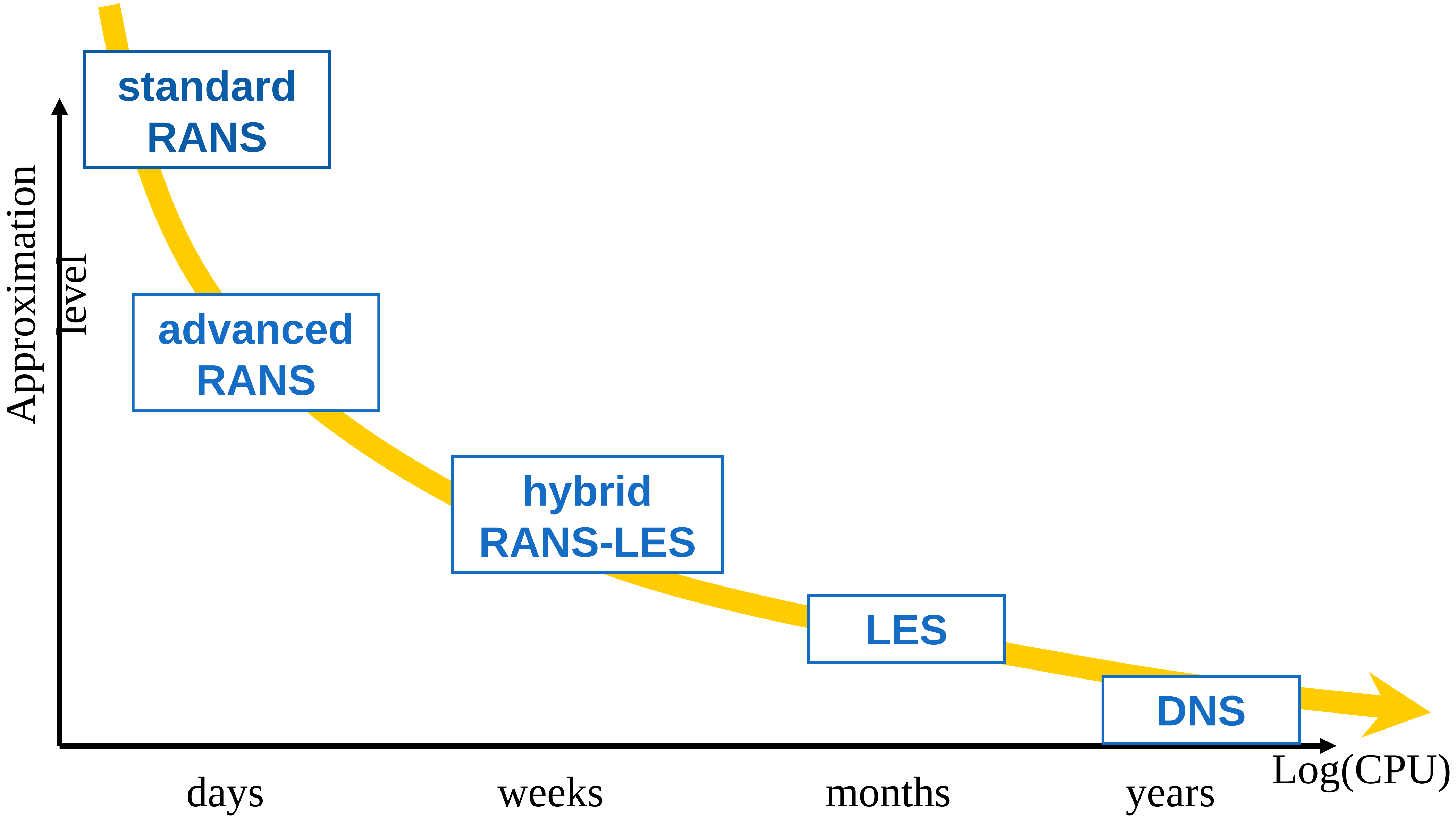
Turbulent boundary layer:

- **Instantaneous velocity field (snapshot)** $u_i(\boldsymbol{x}, t)$

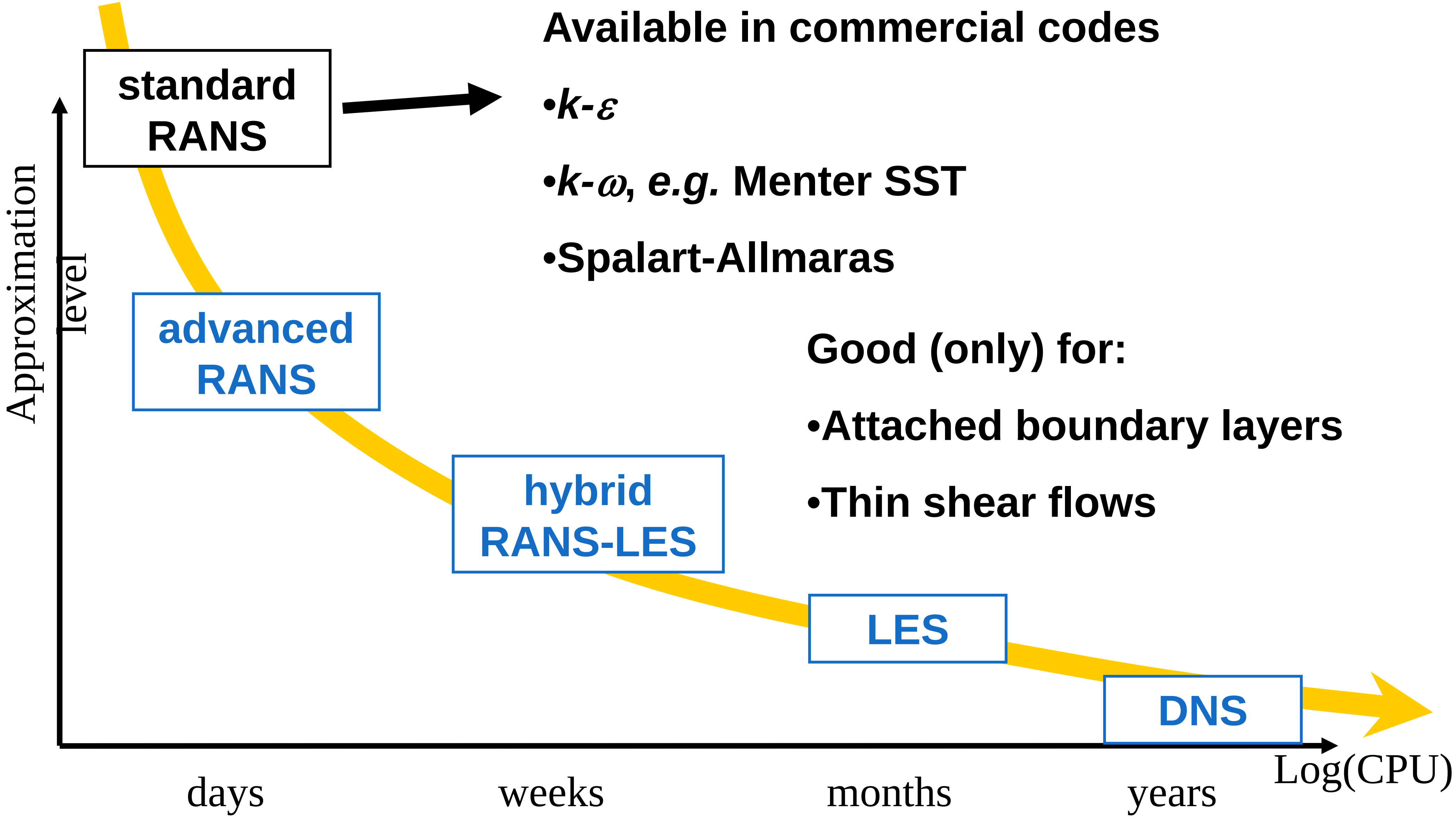


Ref: Prof. M. Gad-el-Hak, University of Notre Dame

Prediction of turbulent flows



Standard RANS models



Available in commercial codes

•*k*-ε

•*k*-ω, *e.g.* Menter SST

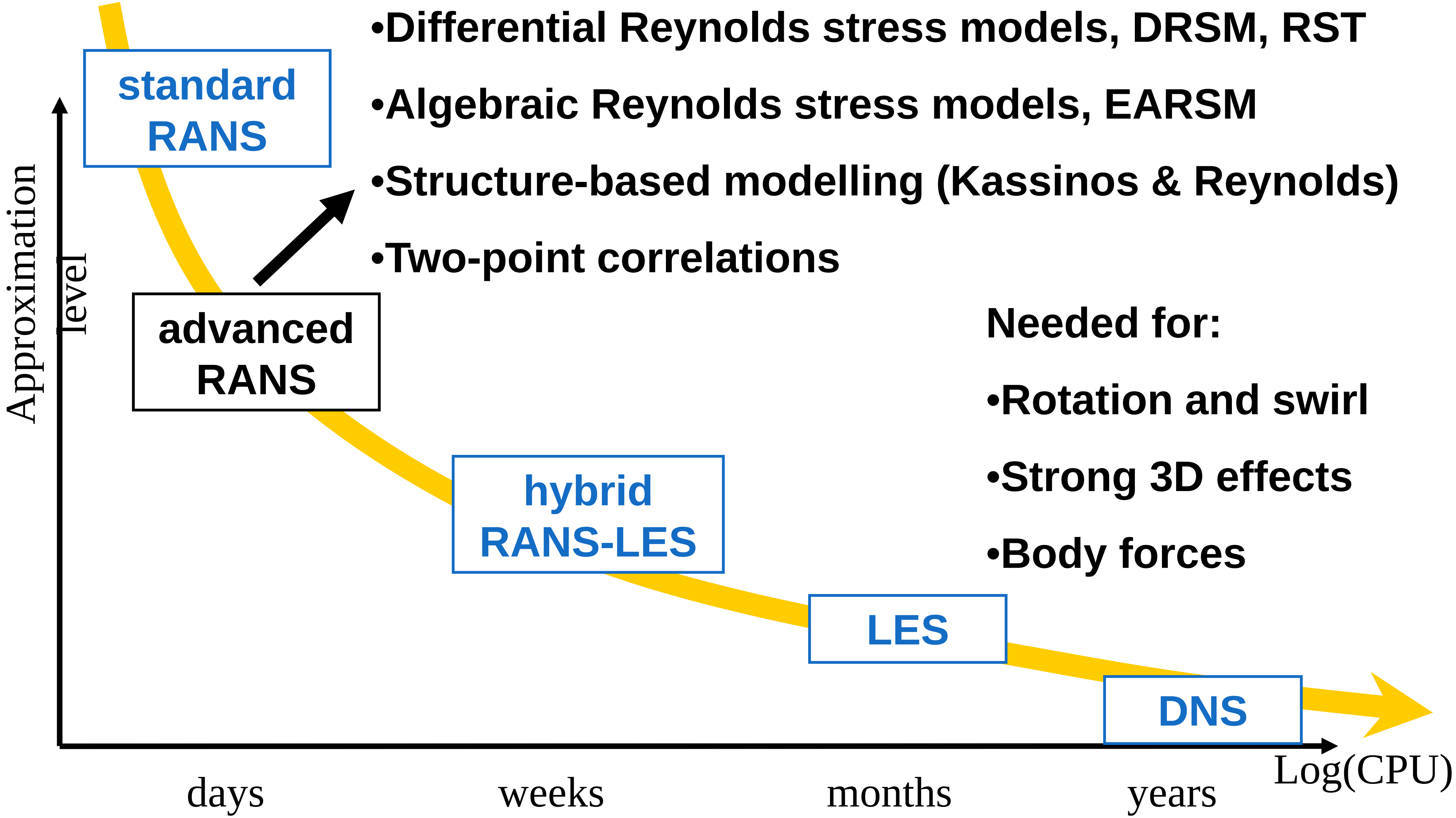
•Spalart-Allmaras

Good (only) for:

•Attached boundary layers

•Thin shear flows

Advanced RANS models



standard RANS

advanced RANS

hybrid RANS-LES

LES

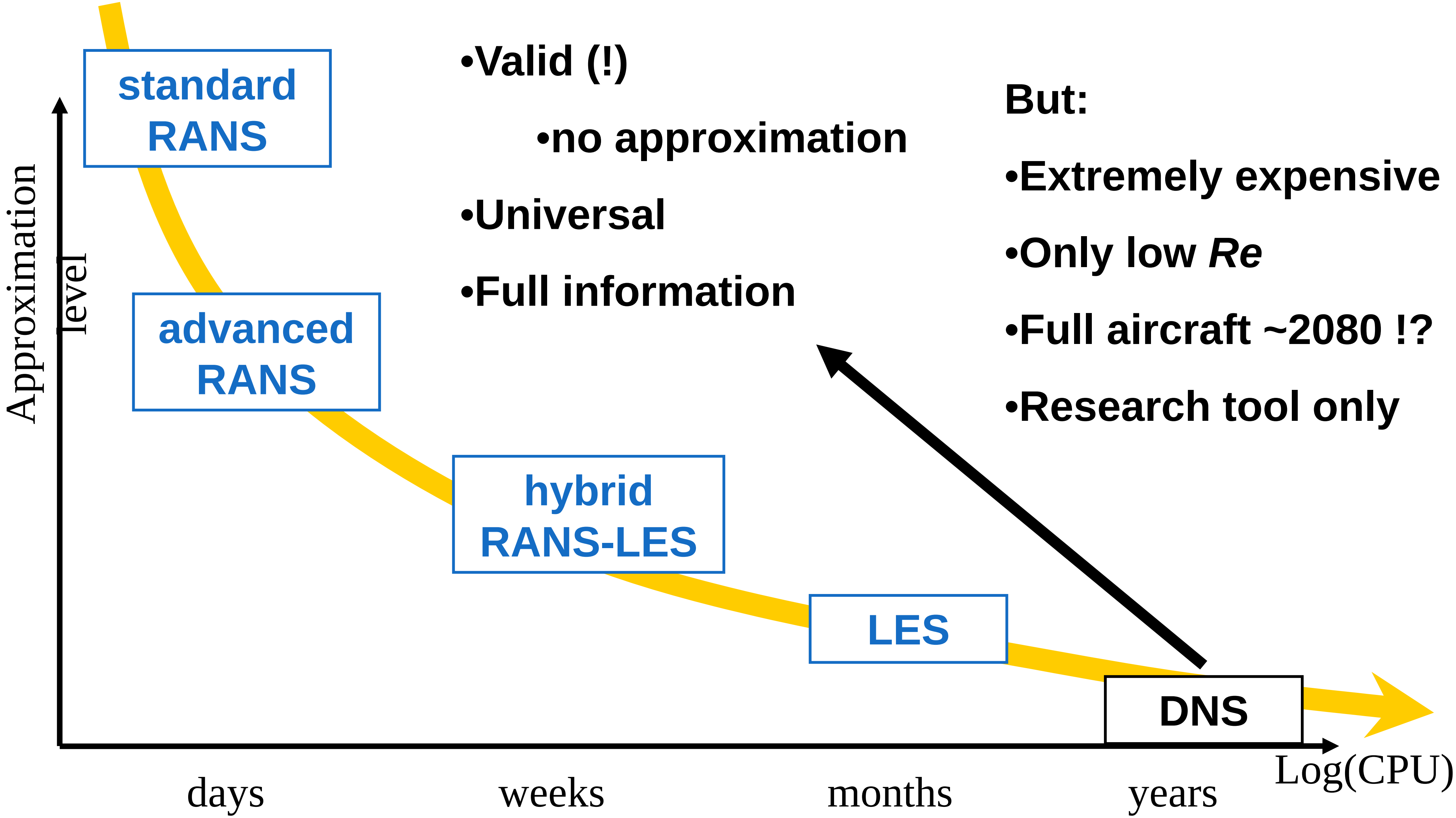
DNS

- **Differential Reynolds stress models, DRSM, RST**
- **Algebraic Reynolds stress models, EARSM**
- **Structure-based modelling (Kassinos & Reynolds)**
- **Two-point correlations**

Needed for:

- **Rotation and swirl**
- **Strong 3D effects**
- **Body forces**

Direct Numerical Simulation – DNS

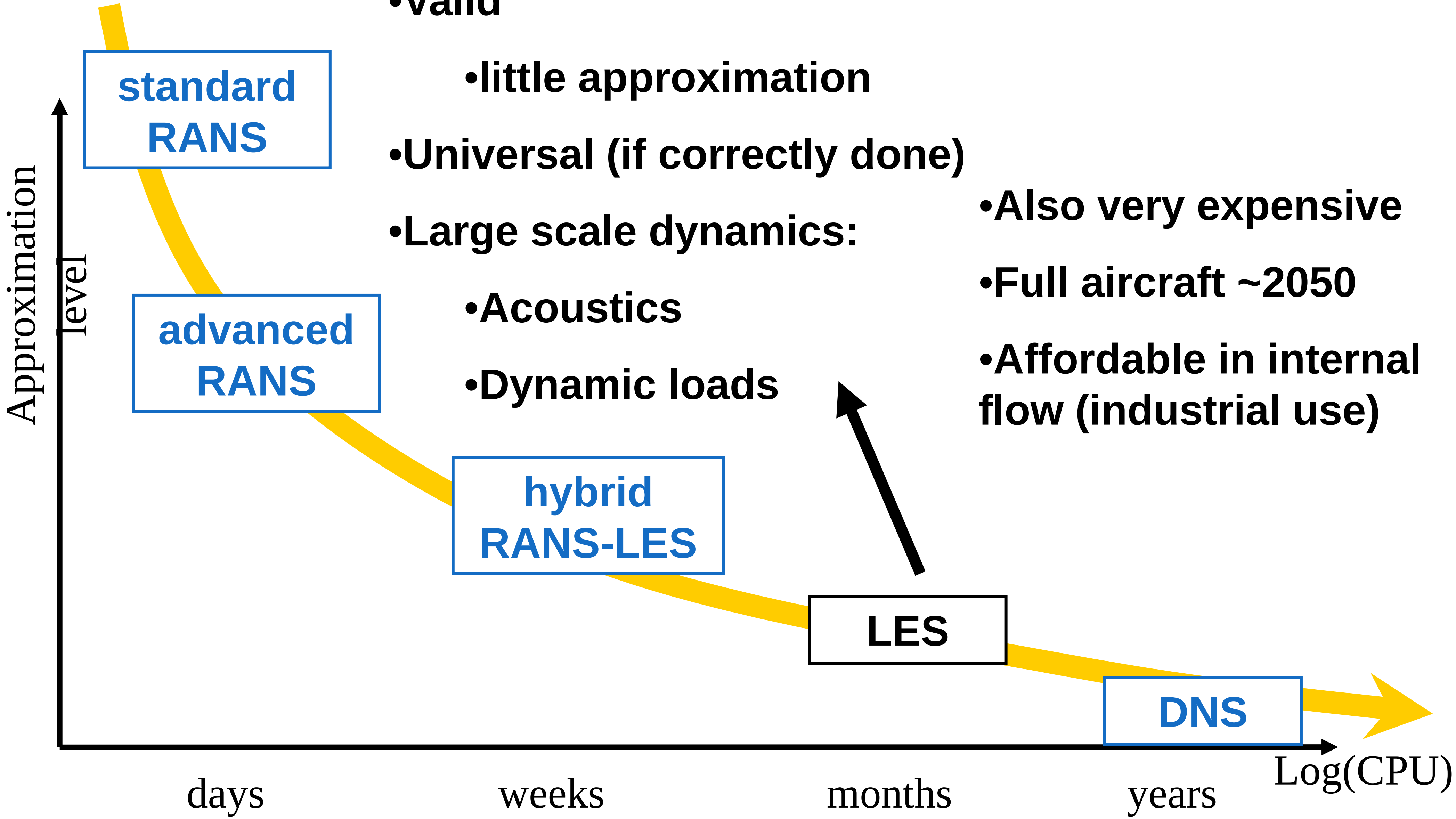


- Valid (!)
  - no approximation
- Universal
- Full information

But:

- Extremely expensive
- Only low *Re*
- Full aircraft ~2080 !?
- Research tool only

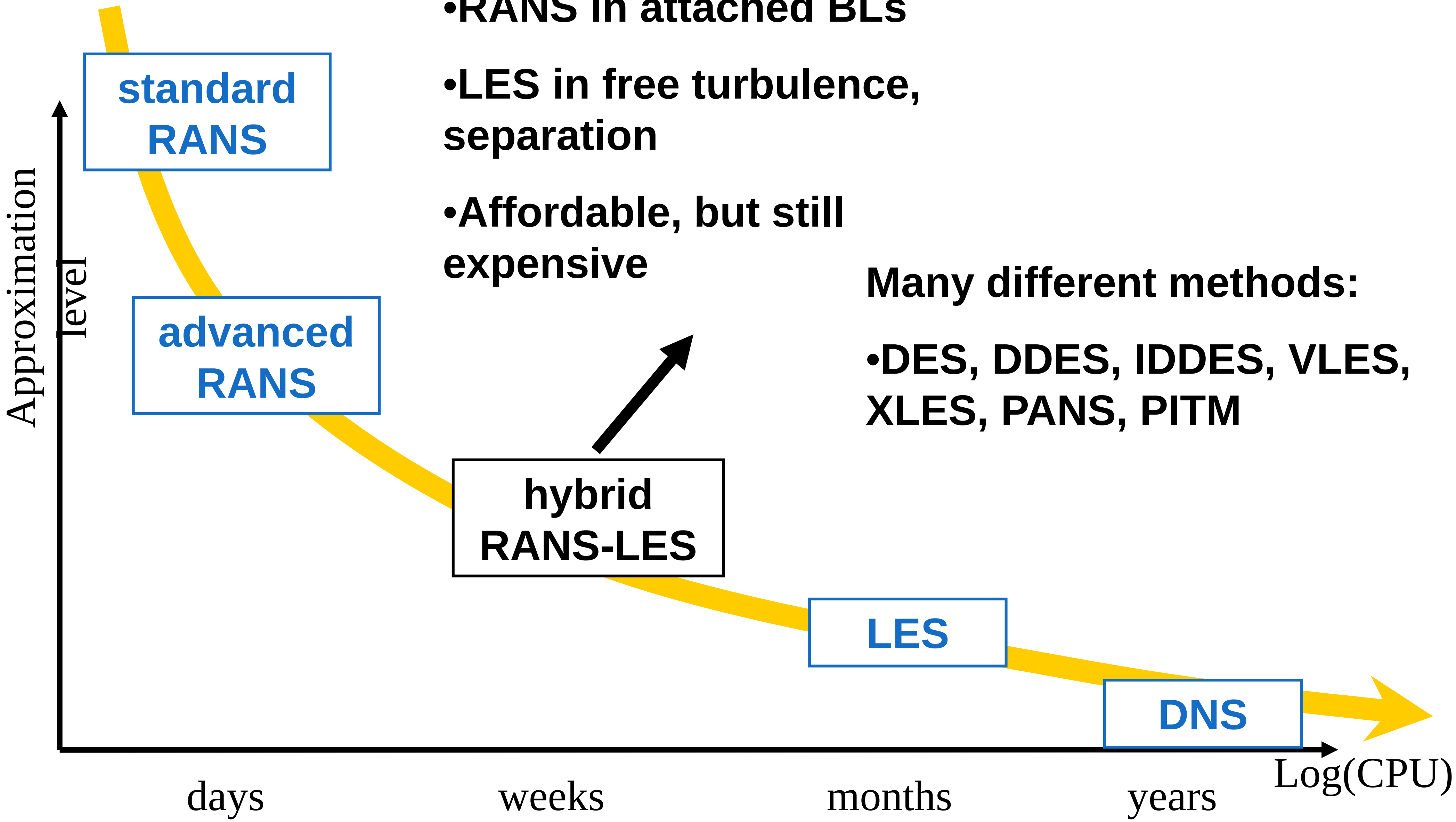
Large Eddy Simulation – LES



- Valid
  - little approximation
- Universal (if correctly done)
- Large scale dynamics:
  - Acoustics
  - Dynamic loads

- Also very expensive
- Full aircraft ~2050
- Affordable in internal flow (industrial use)

Hybrid RANS – LES methods

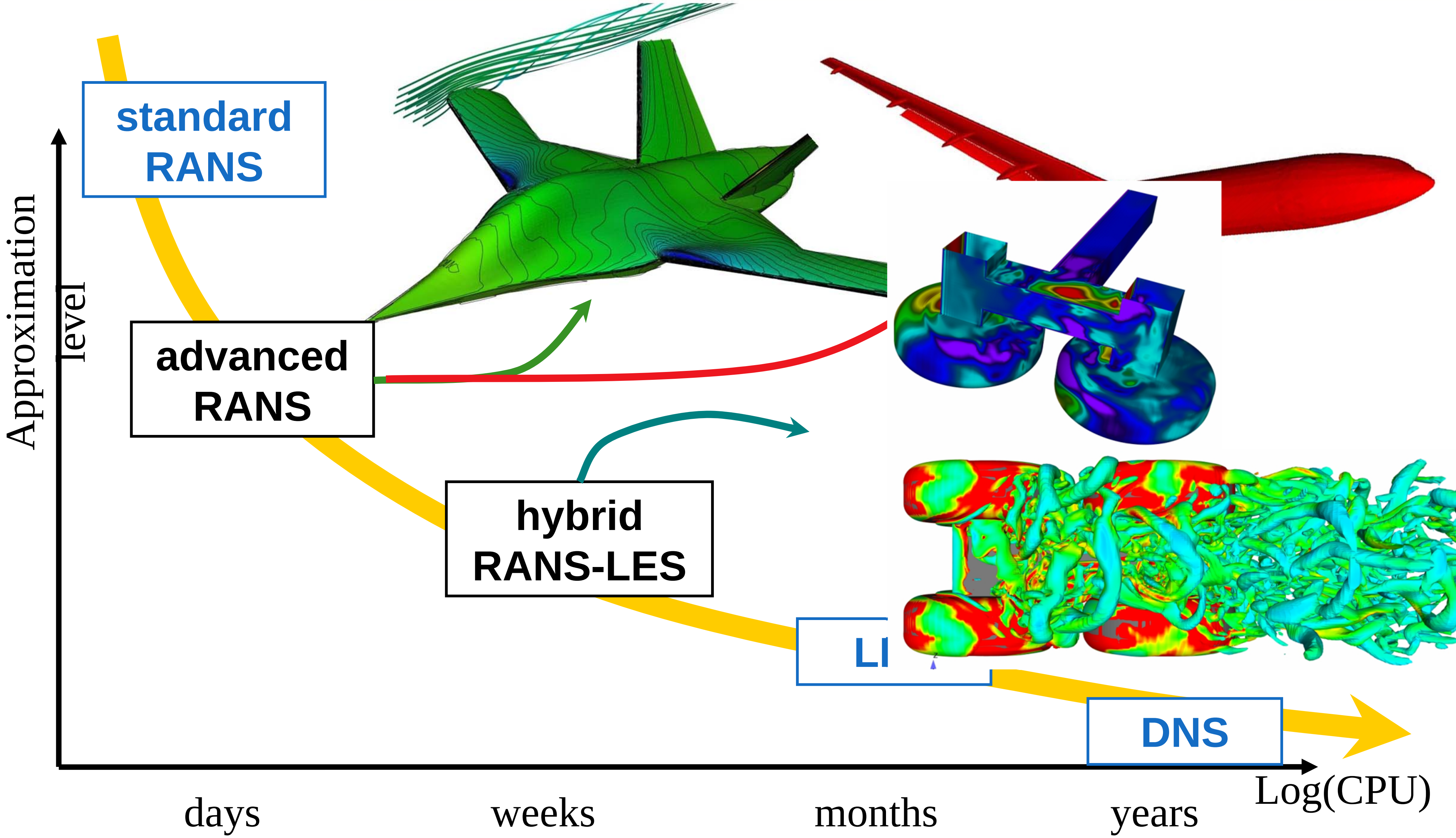


- RANS in attached BLs
- LES in free turbulence, separation
- Affordable, but still expensive

Many different methods:

- DES, DDES, IDDES, VLES, XLES, PANS, PITM

Prediction of turbulent flows



Approximation level

standard RANS

advanced RANS

hybrid RANS-LES

LI

DNS

Log(CPU)

days

weeks

months

years

Basic concepts

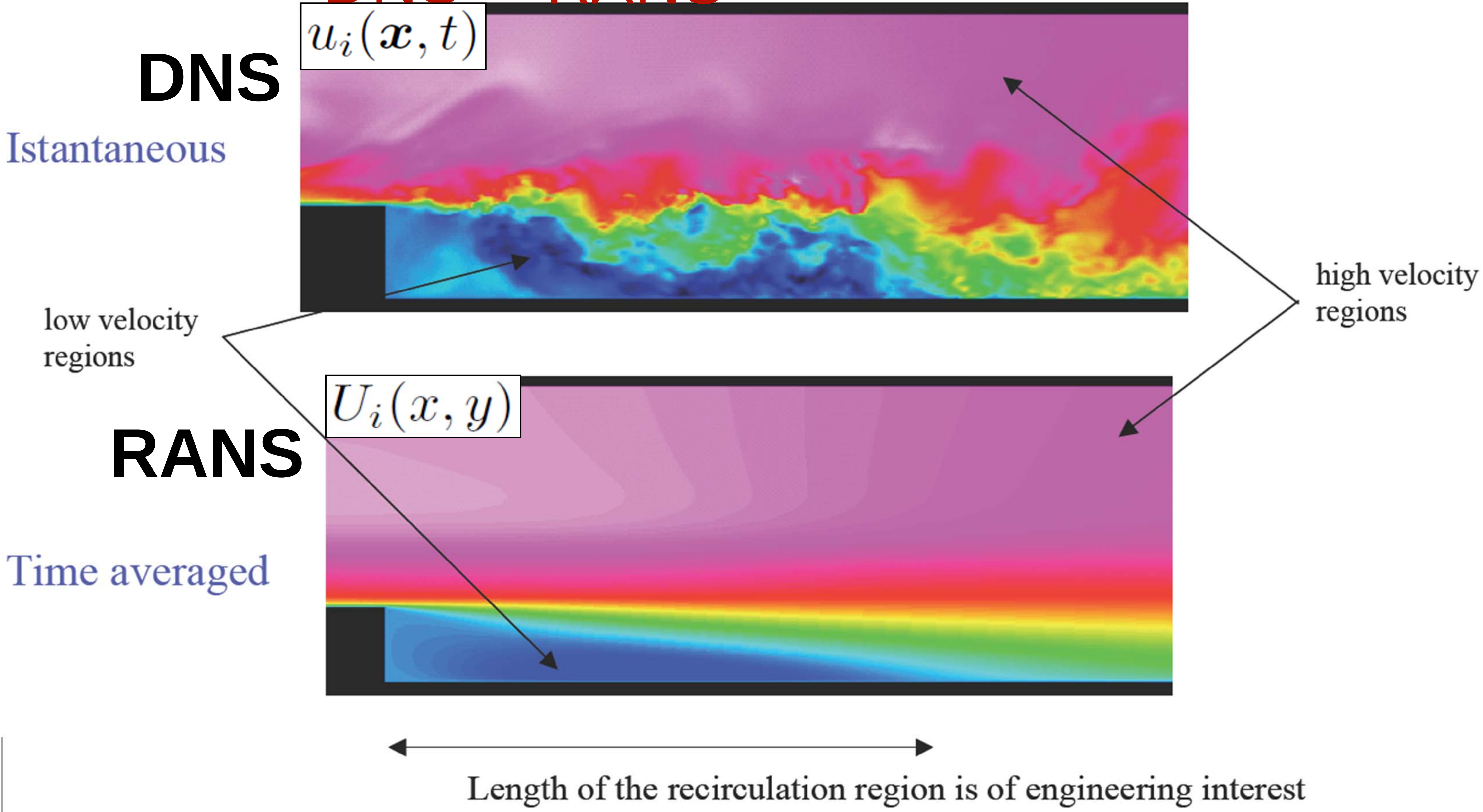
- Turbulence is:
  - random fluctuations
  - 3D
  - time dependent
  - present in most flows of engineering interest
- Energy cascade
  - generated at the largest scales (L and U)
  - large scale vortices breakes down to smaller vortices
  - dissipates to heat at the smallest viscous scales, ε
  - balanced cascade

$$\varepsilon \sim \frac{U^3}{L}$$

  - turbulent kinetic energy

$$K \sim U^2$$

DNS – RANS



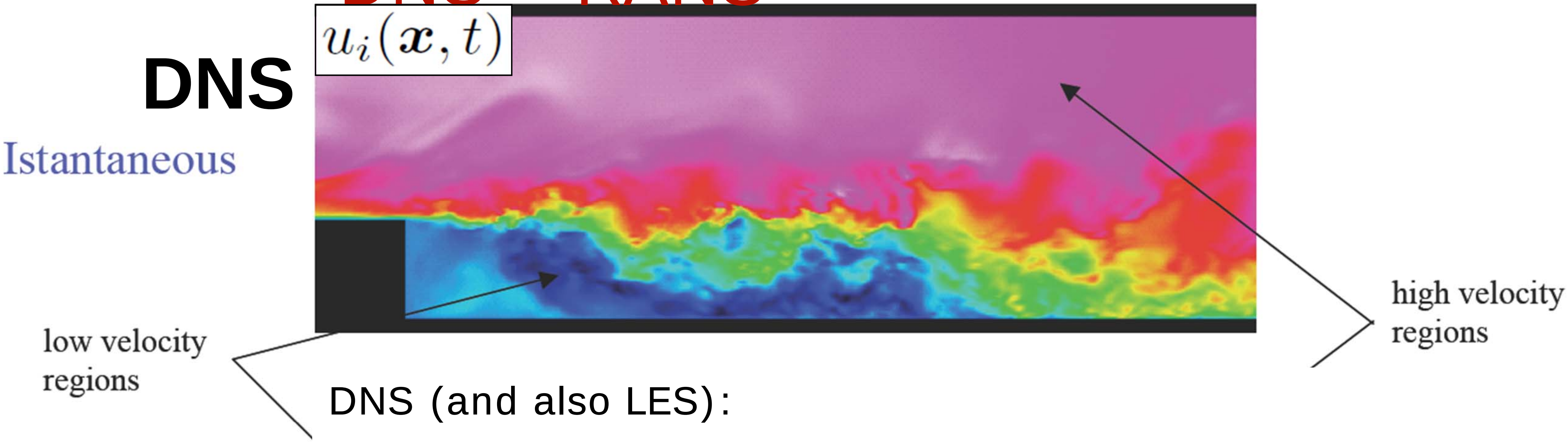
DNS

RANS

Length of the recirculation region is of engineering interest

DNS – RANS

DNS



DNS (and also LES):

- 3D
- Time dependent
  Full information of turbulence scales
  Acoustics and dynamic loads
- Huge Reynolds number dependency

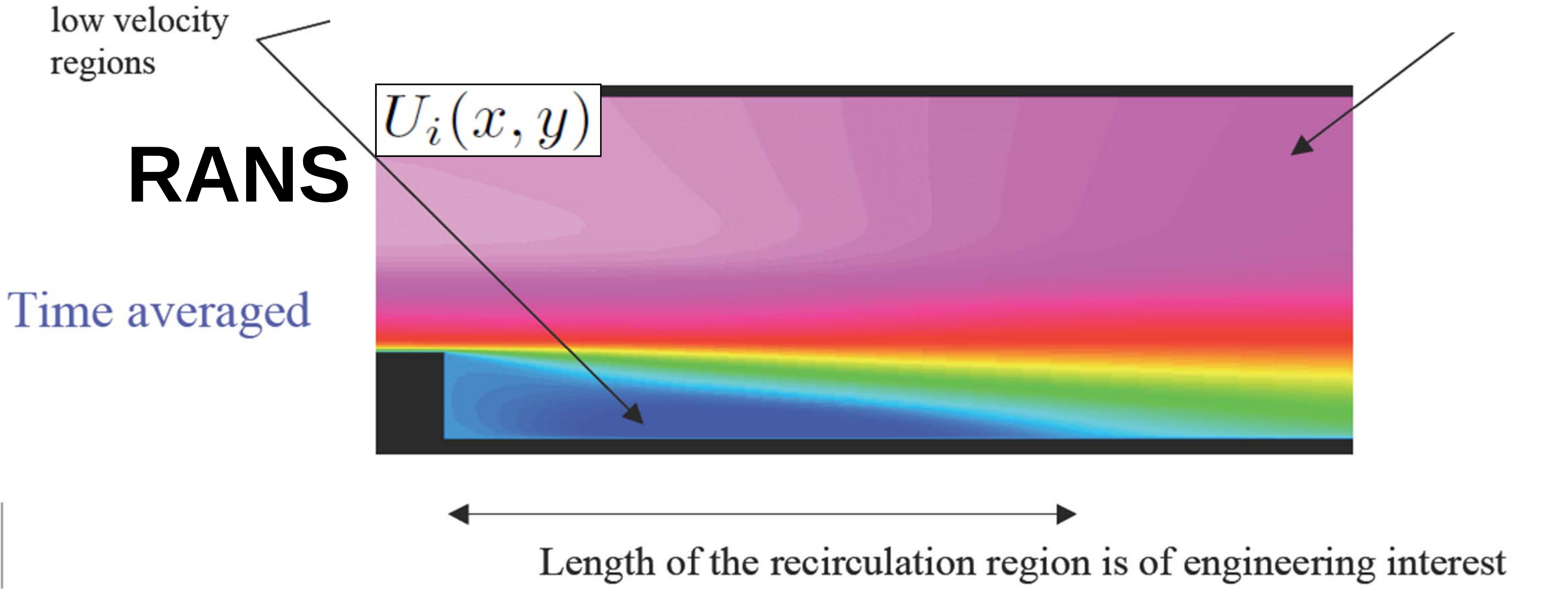
Expensive!

DNS – RANS

RANS:

- Reduction of dimensions –> cheap
  Here: 2D and steady
- Only statistical information of turbulence scales:
  Time and length scales
  rms values

RANS



Length of the recirculation region is of engineering interest

Scale separation

- Large scales L and U related to geometrical scales
- Small viscous scales (related to ν and ε)

$$l_K = \eta \sim \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}, \quad t_K \sim \sqrt{\frac{\nu}{\varepsilon}}$$

- Scale separation

$$\frac{L}{\eta} \sim Re^{3/4}, \quad \frac{t}{t_K} \sim Re^{1/2}$$

- Reynolds number

$$Re = \frac{LU}{\nu}$$

Different Reynolds numbers

- Based on global scales

$$Re_L = \frac{LU}{\nu}$$

- Based on distance x from leading edge (flat plate)

$$Re_x = \frac{xU}{\nu}$$

- Based on boundary layer thickness (δ)

$$Re_\delta = \frac{\delta U}{\nu}$$

- Based on wall skin friction

$$Re_\tau = \frac{\delta u_\tau}{\nu}$$

Viscosity

- Kinematic viscosity, ν
- Dynamic viscosity, μ
- Density, ρ

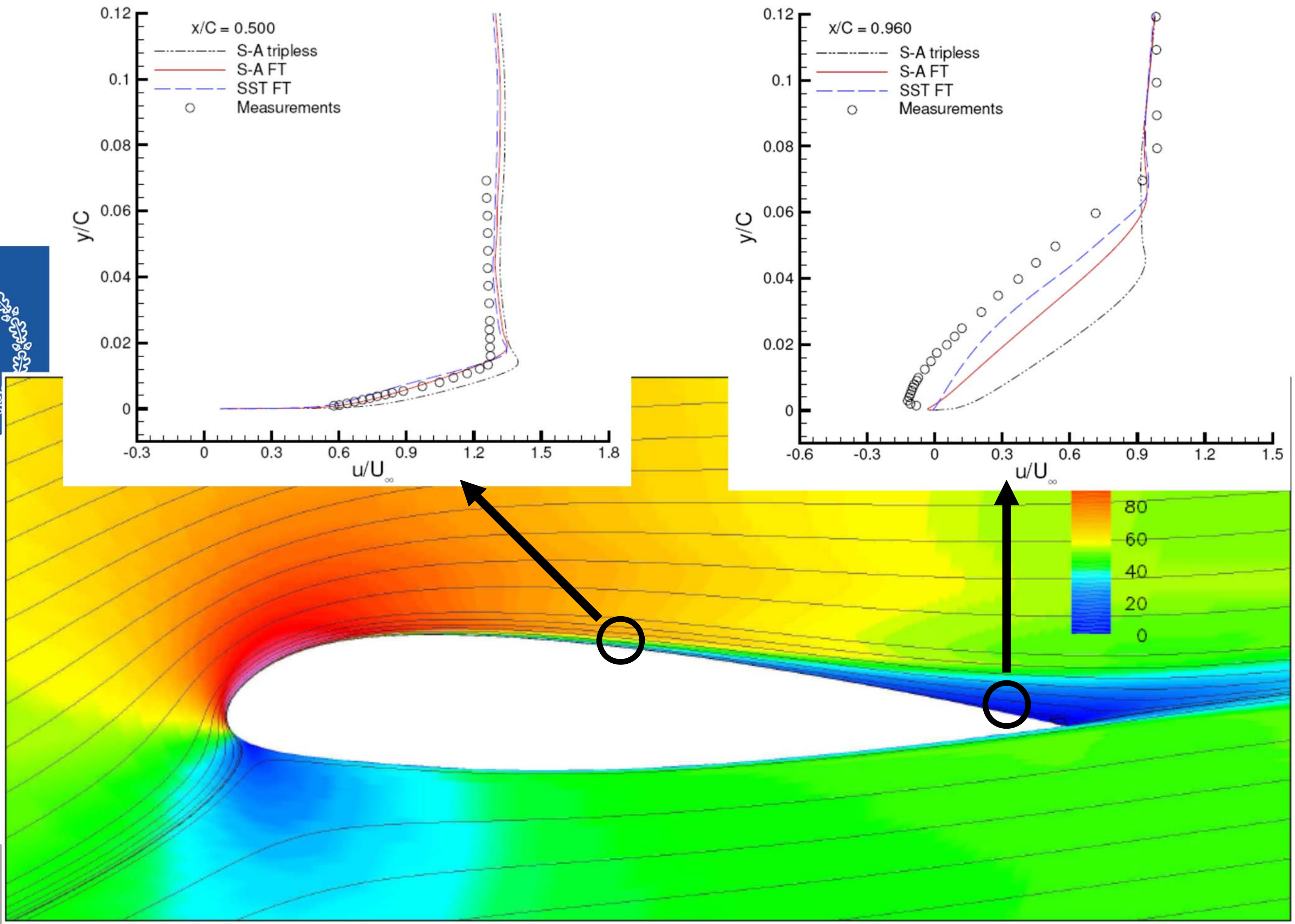
$$\mu = \rho\nu$$

Boundary layers (BL)

- Thin layers
  - Thickness Reynolds number dependent
- Laminar boundary layers
  - Thickness related to wall skin friction
- Turbulent boundary layers
  - Inner and outer scales separated
  - Scale separation Reynolds number dependent

- Figures (Wing+bl – bl – near-wall)

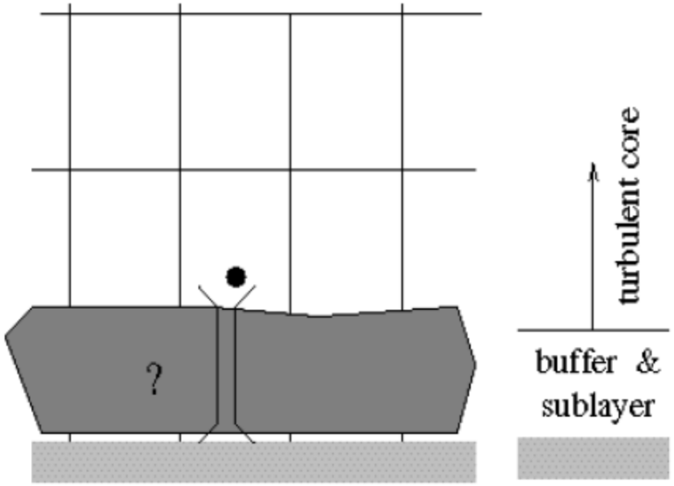
BL on the ONERA A-profile



Approximation of BLs

- Slip wall boundary condition
  - Boundary layer completely neglected
  - Euler (non-viscous) computations possible
  - Slip BC can also be applied to viscous & turbulent CFD
- No slip boundary condition
  - Boundary layer completely resolved ($y^+=1$)
  - Extreme resolution needed ($\Delta y=1\text{-}100\mu\text{m}$)
  - 40-80 grid points within the boundary layer
- Log-law boundary condition (turbulence)
  - First grid point within log layer ($y^+>20$ AND $y<0.1\delta$)
  - 10-20 grid points within the boundary layer
  - Warning: standard log-law BCs inconsistent with too small grid size. READ SOLVER DOCUMENTATION !!!

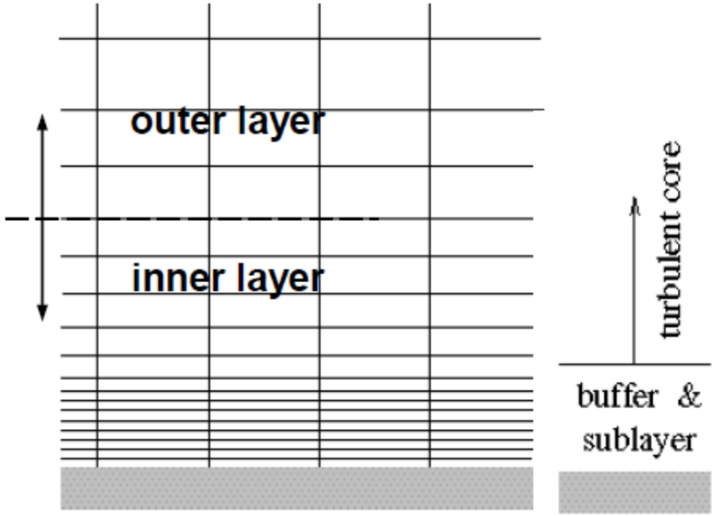
What’s in Fluent?



- Standard and Non-Equilibrium Wall Functions:
  - “Wall adjacent cells should have y+values between 30 and 300–500” – (remember $y < 0.1\delta$!)
  - “The mesh expansion ratio should be small (no larger than around 1.2)”
  - “Non-equilibrium wall function method attempts to improve the results for flows with higher pressure gradients, separations, reattachment and stagnation”
- Scalable Wall Functions:
  - Consistent for all y+values

What’s in Fluent? …

- Enhanced Wall Treatment Option
  - Combines a blended law-of-the wall and a two-layer zonal model.
  - Suitable for low-Re flows or flows with complex near-wall phenomena.
  - Generally requires a fine near-wall mesh capable of resolving the viscous sublayer
  - y+< 5, and a minimum of 10–15 cells across the “inner layer” for best results
  - Valid for all y+
  - Available for all k-e and k-w models
  - Not yet for Spalart-Allmaras (y+<3 OR y+>15)



Recommendations for Fluent

- For K-ε models
  - use Enhanced Wall Treatment: EWT-ε
- If wall functions are favored with K-ε models
  - use scalable wall functions
- For K-ω models
  - use the default: EWT-ω

Wall bounded turbulence

- Viscous wall scales

$$l_* = \frac{\nu}{u_\tau}, \quad t_* = \frac{\nu}{u_\tau^2}$$

- Wall friction velocity

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\nu \frac{\partial U}{\partial y}\bigg|_{\text{wall}}} = U\sqrt{\frac{1}{2}C_f}$$

- Friction coefficient

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = 2\left(\frac{u_\tau}{U}\right)^2$$

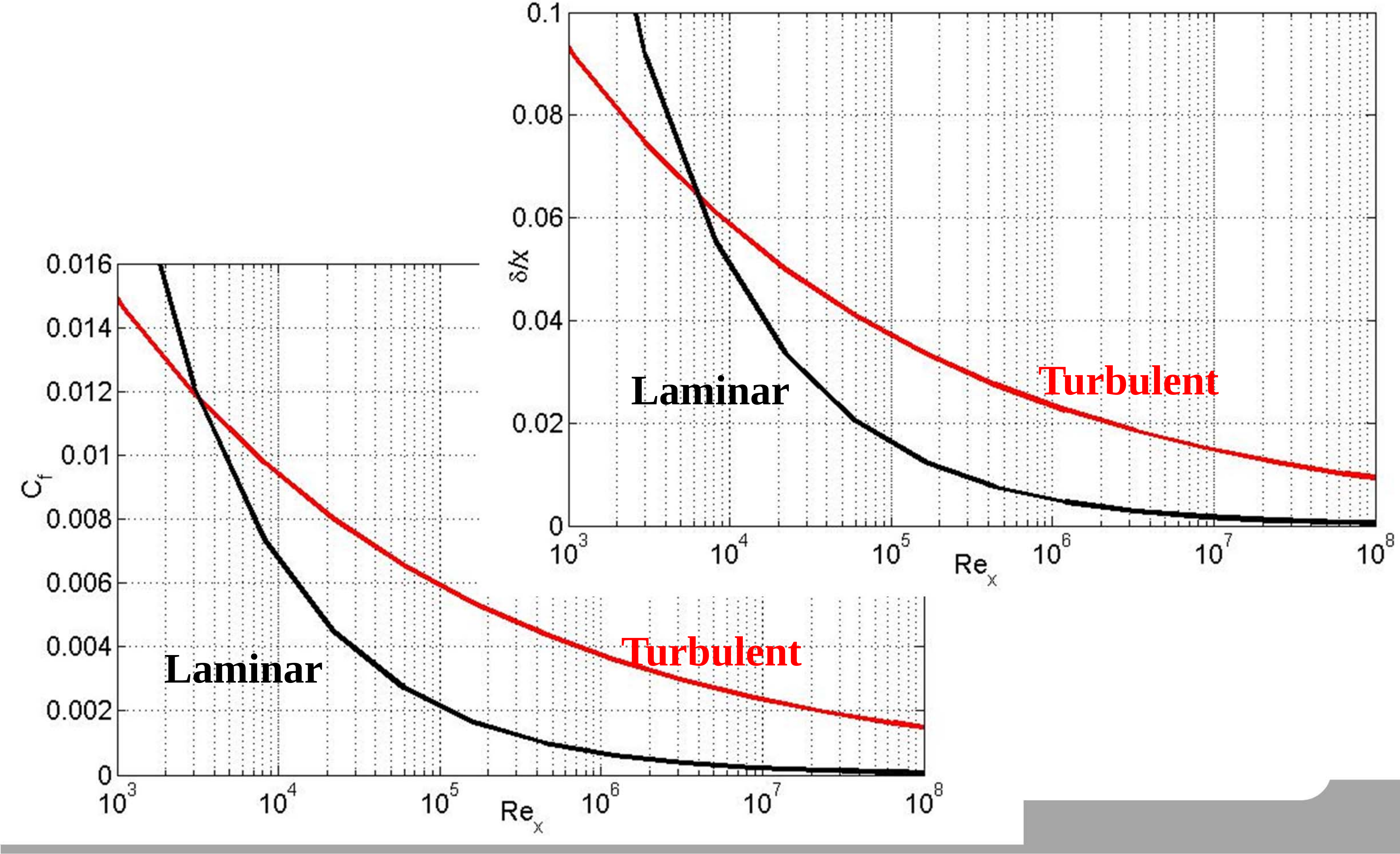
- Viscous wall distance

$$y^+ = \frac{y}{l_*} = \frac{y u_\tau}{\nu}$$

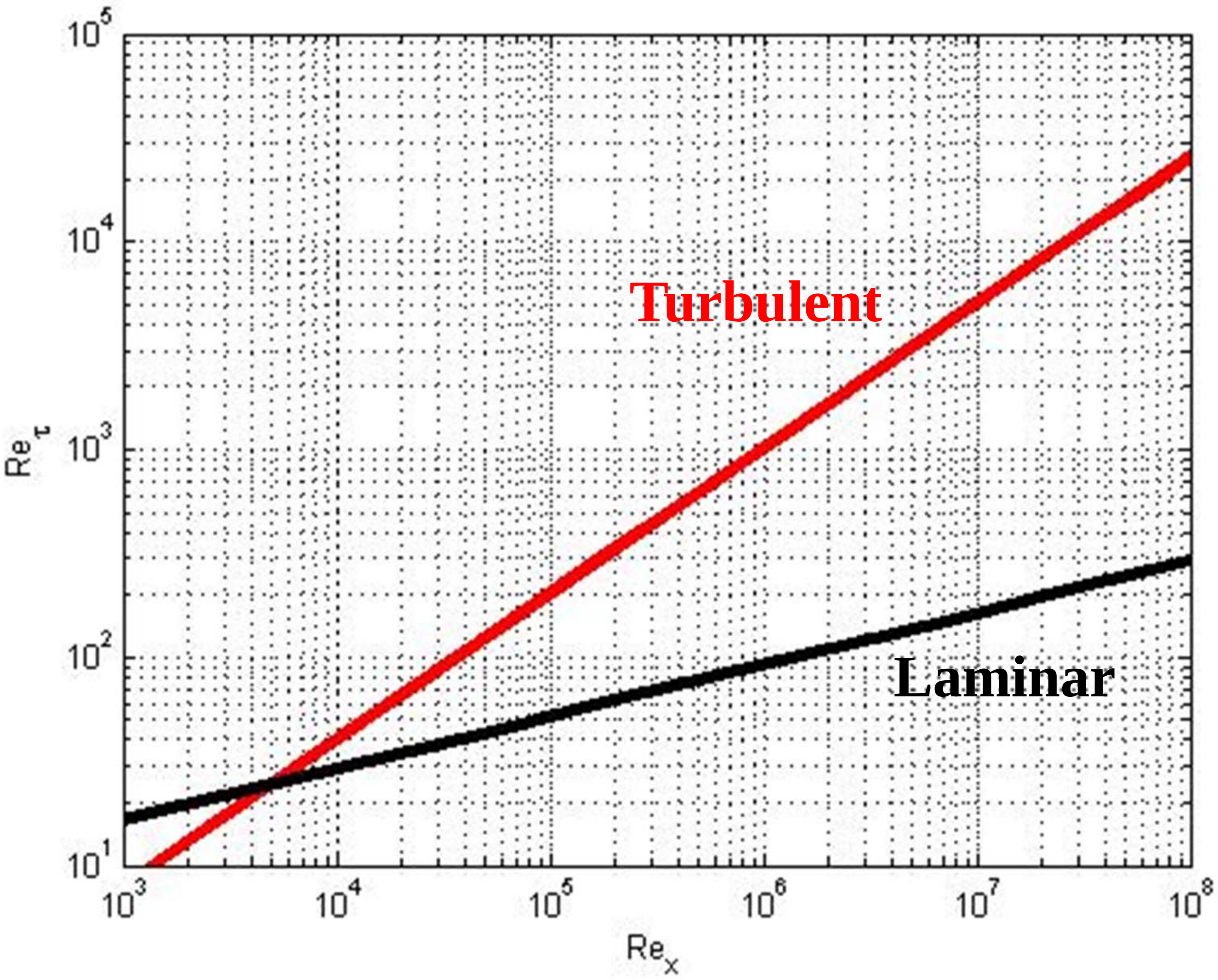
Empirical relations for BLs

- Friction coefficient
  - Turbulent $$\frac{C_f}{2} \approx 0{,}0296 Re_x^{-1/5}$$
  - Laminar $$\frac{C_f}{2} = 0{,}332 Re_x^{-1/2}$$
- Boundary layer thickness
  - Turbulent $$\frac{\delta}{x} \approx 0{,}37 Re_x^{-1/5}$$
  - Laminar $$\frac{\delta}{x} = 5{,}0 Re_x^{-1/2}$$

Empirical relations plotted



Empirical relations plotted



Turbulence modelling

- Reynolds decomposition

$$\tilde{u}_i(\boldsymbol{x}, t) = U_i(\boldsymbol{x}) + u_i(\boldsymbol{x}, t)$$

where $U_i(\boldsymbol{x}) = \overline{\tilde{u}_i(\boldsymbol{x}, t)}$ and $\overline{u_i(\boldsymbol{x}, t)} = 0$

- The “mean” is time average, ensemble average or averaging in homogeneous directions. $U_i(\boldsymbol{x})$ may actually vary in time with a time scale much longer than the turbulent time scale.
- Take the mean of the Navier-Stokes equations -> RANS

$$\frac{\partial U_i}{\partial x_i} = 0$$

$$\frac{\partial U_i}{\partial t} + U_k \frac{\partial U_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_k}\left(\nu \frac{\partial U_i}{\partial x_k} - \overline{u_i u_k}\right)$$

Reynolds stresses

- Not “small”
- Significant effects on the flow
- Needs to be modelled in terms of mean flow quantities
- Reduces the problem to steady (or slowly varying)
- 2D assumptions possible

- Equation can be derived from Navier-Stokes equations
- Need modelling

Eddy-viscosity models (EVM)

- Assume: Reynolds stresses related to an “eddy viscosity”, ν_T

$$\overline{u_i u_j} = -2\nu_T S_{ij} \qquad \left(+ \frac{1}{3}\overline{u_k u_k}\delta_{ij} \right)$$

where $S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$

Based on Boussinesq (1877)

- Eddy viscosity ~ turbulence velocity V and length L scales: $\nu_T \sim VL$

One-equation models

- One transport equation for K (turbulent kinetic energy) or ν_T.
- Additional information from global conditions (typically wall distance)
- Works well for attached boundary layers
- Not very general, but more than algebraic models

- Example: Spalart-Allmaras (1992)
  - reasonable and robust model for external aerodynamics
  - Boeing’s ”standard model”

Two-equation models

- Two transport equations for the turbulence scales (K–ε or K–ω)
- Completely determined in terms of local quantities (except near-wall corrections which may be dependent on wall distance)
- Works well for attached boundary layers
- Somewhat more general than zero-, one-equation models
- Model transport equations loosely connected to the exact equations.

- Examples:
  - Standard K–ε model (Launder & Spalding 1974)
  - Wilcox K–ω (1988, ...) models
  - Menter (1994) SST K–ω model (performing reasonable well also in separated flows)

    Airbus' ”standard model”

Eddy-viscosity models ...

- Problems:
  - No dependency on rotation or curvature. Real turbulence strongly dependent.
  - Modelled production proportional to strain rate squared ~ S^2. Exact production ~ S. Results in a overestimated production of K in highly sheared flows (around stagnation points, impinging jets, pressure gradient BLs, separated flows).
- Fixes
  - Rotation & curvature corrections
  - Yap correction (limit excessive turbulent lengthscale)
  - Menter SST correction (limit excessive ν_T).

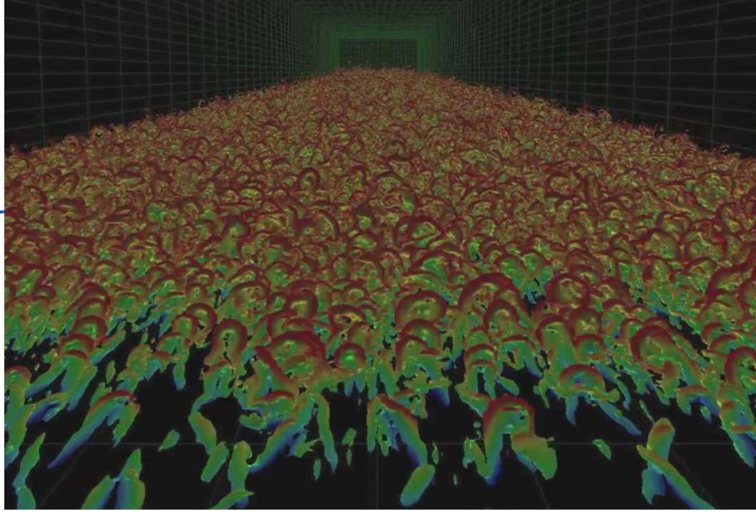
LES and LES/RANS hybrids

- Simulation of only the large scale turbulence (compare with DNS, simulation of all scales)
  - Always time dependent and 3D -> expensive
- Wall free turbulence simulations almost *Re* independent
- Wall bounded turbulence largely *Re* dependent
  - fully resolved near-wall region very expensive (almost as DNS)
  - wall-function or near-wall RANS coupling saves computational cost
  - hybrid RANS-LES (RANS in attached BLs and LES in wall-free separated regions) a very active research field, eg DES

LES and LES/RANS hybrids ...

- LES in academic research for:
  - low *Re* generic flows
  - complement to DNS for higher *Re*
  - gives detailed knowledge about turbulence
- LES in industrial use in:
  - internal flow with complex geometries
  - flows around blunt bodies (with large separated regions)
  - atmospheric boundary layers (e.g. weather forecasts)
  - combustion simulation
  - other complex flow physics at moderate *Re*
- Warning: LES is extremely expensive in high attached and slightly separated wall-bounded flows, if properly resolved.

How expensive is DNS?



- **DNS of flat plate turbulent boundary layer**
    - Schlatter, et al., KTH, Dept. of Mechanics
    - APS meeting 2010: http://arxiv.org/abs/1010.4000
    - http://www.youtube.com/watch?v=4KeaAhVoPIw
    - http://www.youtube.com/watch?v=zm9-hSP4s3w
  - $Re_\theta = 4300$
  - $8192 \times 513 \times 768 = 3.2 \times 10^9$ spectral modes (7.5×10^9 nodes)
  - $\Delta x^+ = 9$, $\Delta z^+ = 4$ –> box: $L^+ = 70\ 000$, $H^+ = W^+ = 3\ 000$
  - BL relations: $Re_x = 1.4 \times 10^6$
  - CPU time: 3 months @ 4000 CPU cores = 1 unit
- **DNS of model airplane, same Reynolds number ($Re_x = 1.4 \times 10^6$)**
  - Only a narrow stripe – wing requires about 1 000 stripes
  - $N_{nodes} = 10^{13}$
  - CPU = 10^3 units

Empirical turbulent BL relations

- **Skin friction coefficient:** $\frac{C_f}{2} = \frac{\tau_w}{\rho U_\infty^2} = \left(\frac{Re_\tau}{Re_\delta}\right)^2 \approx 0.0296 Re_x^{-1/5}$
- **Boundary layer thickness:** $\frac{\delta}{x} = \frac{Re_\delta}{Re_x} \approx 0.37 Re_x^{-1/5}$
- **Boundary layer momentum thickness:** $Re_\tau \approx 1.13 Re_\theta^{0.843}$
- **Reynolds numbers:**

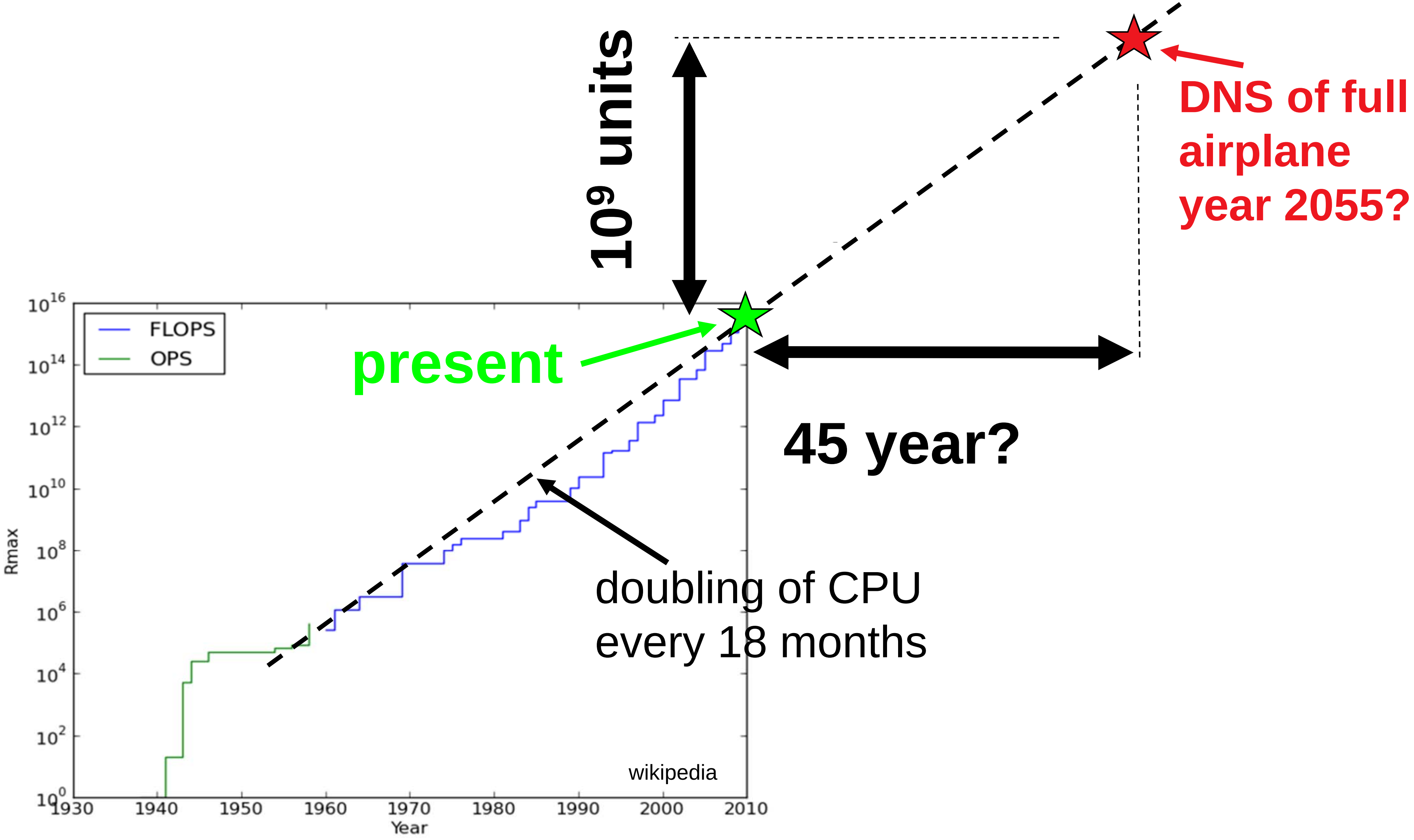
$$Re_\tau \equiv \frac{\delta u_\tau}{\nu} \qquad Re_\theta \equiv \frac{\theta U_\infty}{\nu} \qquad Re_\delta \equiv \frac{\delta U_\infty}{\nu} \qquad Re_x \equiv \frac{x U_\infty}{\nu}$$

DNS – full scale airplane

- **Re scaling – wall bounded flow**
  - **Nodes:** $N_{nodes} \sim \frac{L \times B \times H}{\Delta x \Delta z \Delta y} \sim L^{+2} H^{+} \sim Re_x^{5/2}$
  - **Time steps:** $N_{\Delta T} \sim \frac{T}{\Delta T} \sim T^{+} \sim Re_x^{4/5}$
  - **CPU time:** $N_{CPU} \sim N_{nodes} \times N_{\Delta T} \sim Re_x^{33/10}$

- **DNS of Airplane (Re_x = 70×10^6) (factor of 50)**
  - **N_{nodes} = 10^{17}**
  - **CPU = 10^9 units**

Supercomputer development



10^9 units

DNS of full airplane year 2055?

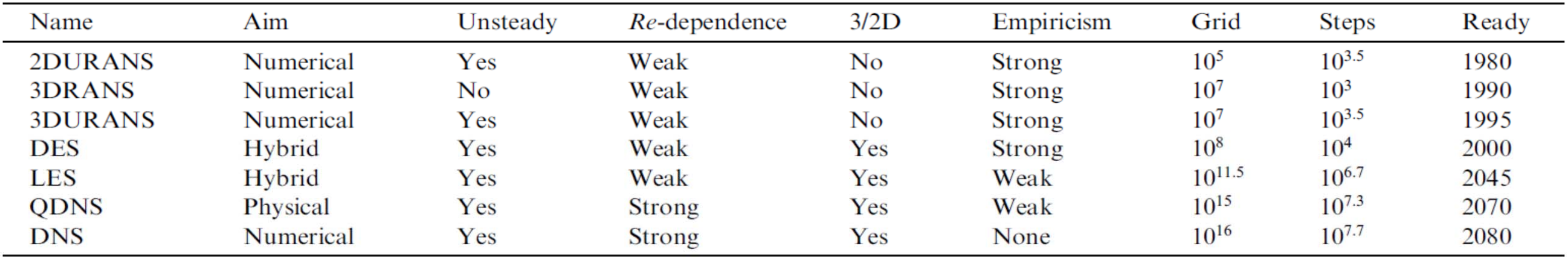
present

45 year?

doubling of CPU every 18 months

wikipedia

Computational effort – different approaches



Name	Aim	Unsteady	*Re*-dependence	3/2D	Empiricism	Grid	Steps	Ready
2DURANS	Numerical	Yes	Weak	No	Strong	10^5	$10^{3.5}$	1980
3DRANS	Numerical	No	Weak	No	Strong	10^7	10^3	1990
3DURANS	Numerical	Yes	Weak	No	Strong	10^7	$10^{3.5}$	1995
DES	Hybrid	Yes	Weak	Yes	Strong	10^8	10^4	2000
LES	Hybrid	Yes	Weak	Yes	Weak	$10^{11.5}$	$10^{6.7}$	2045
QDNS	Physical	Yes	Strong	Yes	Weak	10^{15}	$10^{7.3}$	2070
DNS	Numerical	Yes	Strong	Yes	None	10^{16}	$10^{7.7}$	2080

From Spalart, Int. J. Heat and Fluid Flow, 2000

- **RANS: Reynolds Averaged Navier-Stokes**
- **URANS: Unsteady RANS – slowly in time**
- **DES: Detached Eddy Simulation**
- **LES: Large eddy simulation**
- **QDNS: Quasi DNS, or wall resolved LES**
- **DNS: Direct Numerical Simulation (of the Navier-Stokes eq’s)**
- **“Ready”: When first results can be expected**