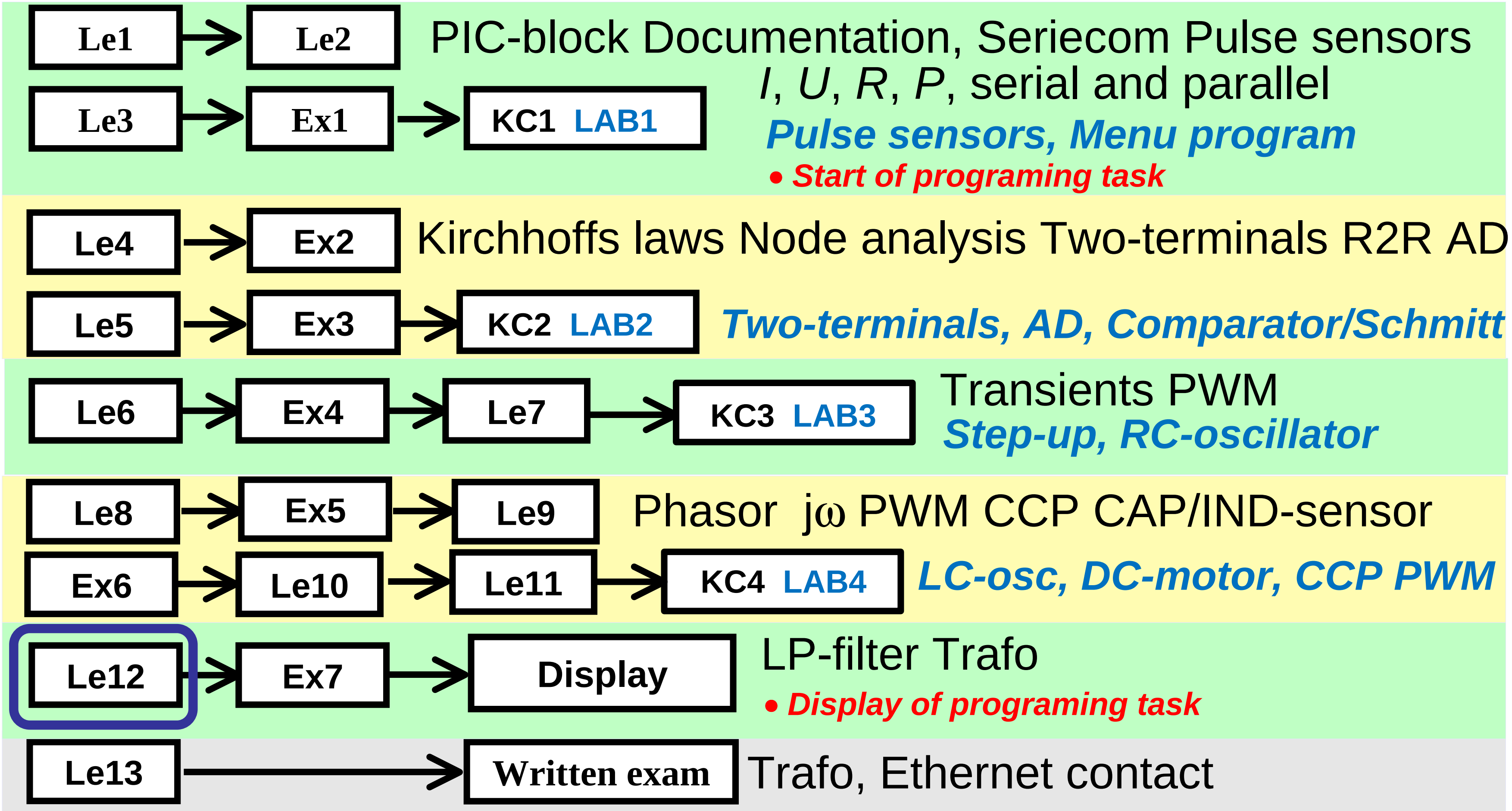
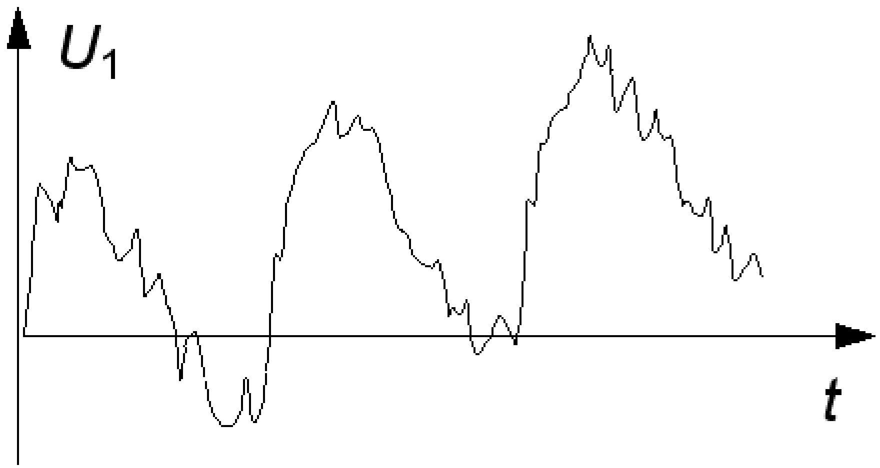
IE1206 Embedded Electronics



A signal in reality …



Actual signals are difficult to interpret. They are often disturbed by noise and hum.

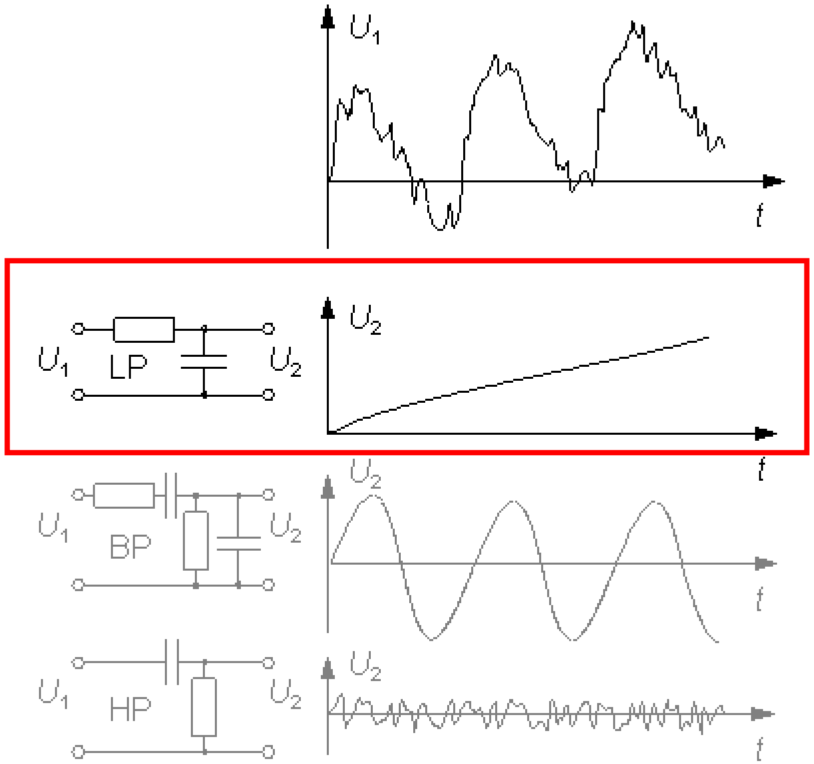
Hum is our 50Hz network induced into the signal lines.

Noise is random disturbances from amplifiers (or even resistors).

Maybe a slow DC …

Perhaps the signal is a slowly increasing direct voltage from eg. a temperature sensor?

In this case, the interference consist of 50 Hz hum and high frequency noise.

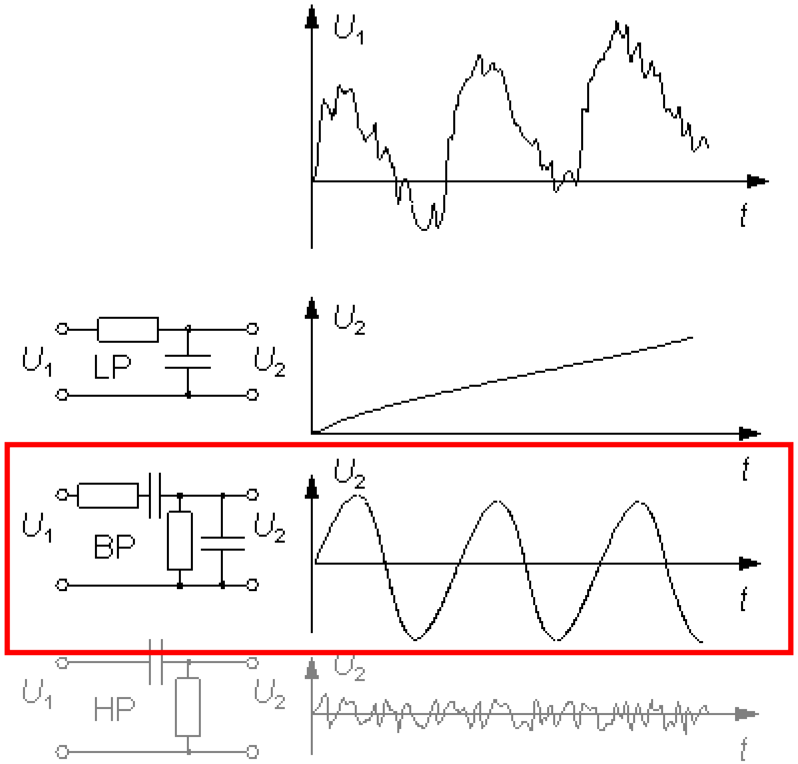


A **LP-filter** (=LowPass) filters away the interference and removes the interferences from the signal.

Maybe a sine wave …

Maybe the signal is a sine wave?

In this case, the interference consist of the DC voltage level slowly changing, offst, and that noise is added.

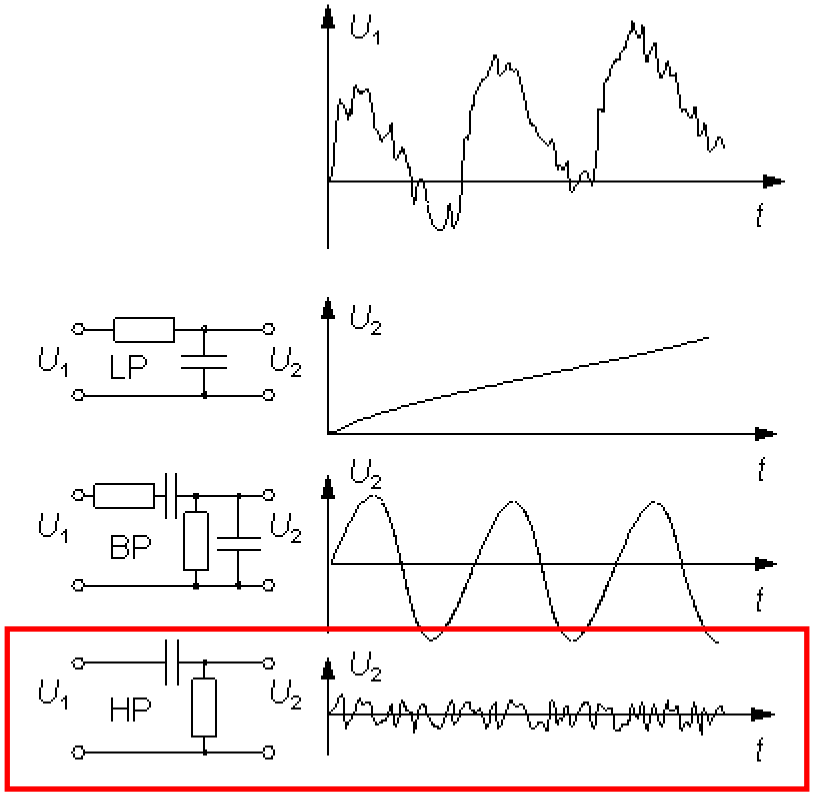


A **BP-filter** (BandPass) will block the offset and filters out the noise.

Maybe rapid variations …

Perhaps the signal is the rapid variations?

In this case, the interference consist of the DC voltage level slowly changing, and that hum has been added.



A **HP-filter** (HighPass) removes the interferences from the signal.

Filter

With *R L* and *C* one can build effective **filters**.

Inductors are more complicated to manufacture than capacitors and resistors, therefore, is typically only combination *R* and *C* used.

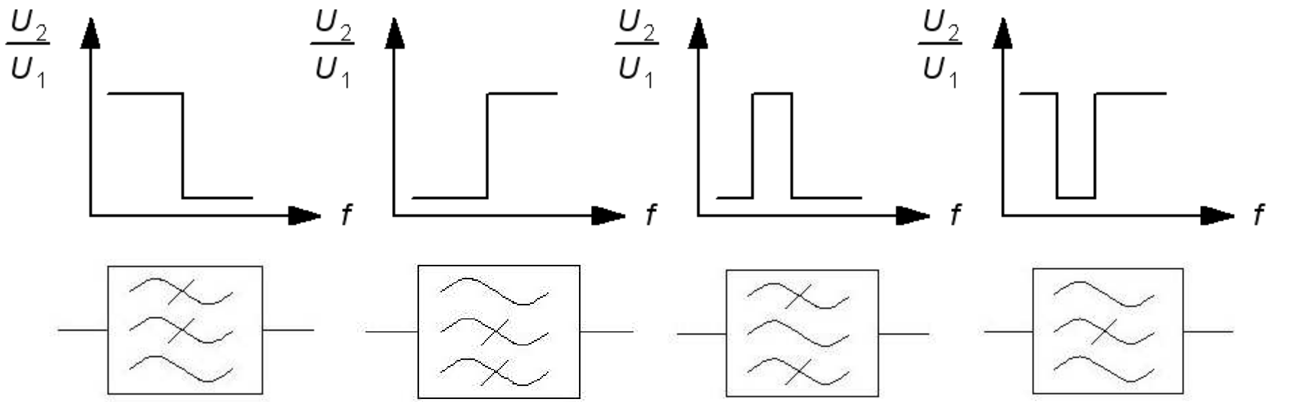
Fast computers can filter signals digitally. Calculating a signal's moving average can for example correspond to the LP filter. Nowadays dominates the digital filter technology over the analog.

Simple RC filter are naturally in most measuring instruments, or even arising from "itself" when linking equipment.

This is the reason that one must know and be able to calculate on simple RC-links, even though they regarded as filters are very incomplete.

LP HP BP BS

LP lowpass HP highpass BP bandpass BS bandstop



BP and BS filters can be seen as different combination of LP and HP filters.

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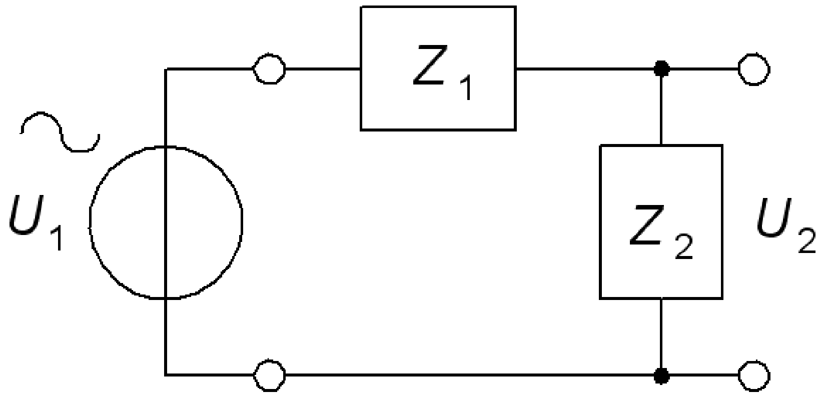
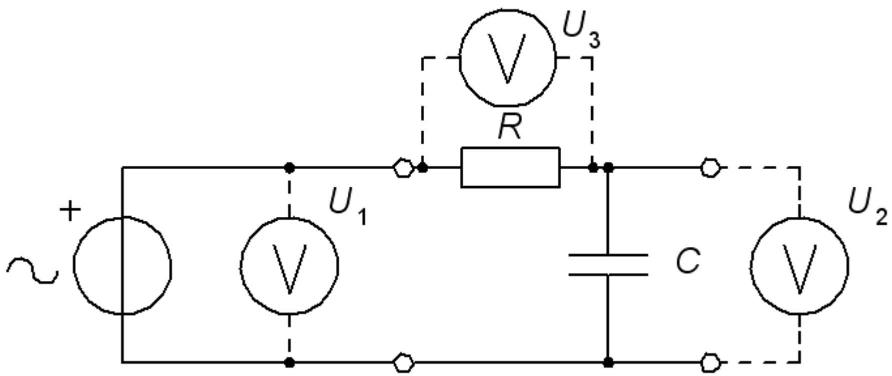
Voltage divider, Transfer function

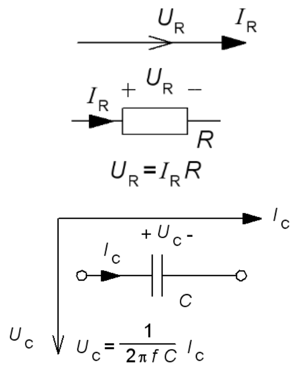
Simple filters are often designed as a voltage dividers.A filter **transfer function**, $H(\omega)$ or $H(f)$, is the ratio between output voltage and input voltage. This ratio we get directly from the voltage divider formula!

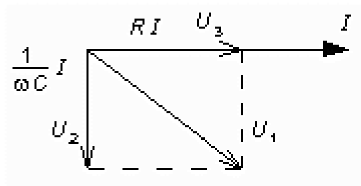


$$\underline{U}_2 = \underline{U}_1 \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \quad \Rightarrow \quad \boxed{\underline{H}(\omega) = \frac{\underline{U}_2}{\underline{U}_1} = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}}$$

RC LP-filter, vectors



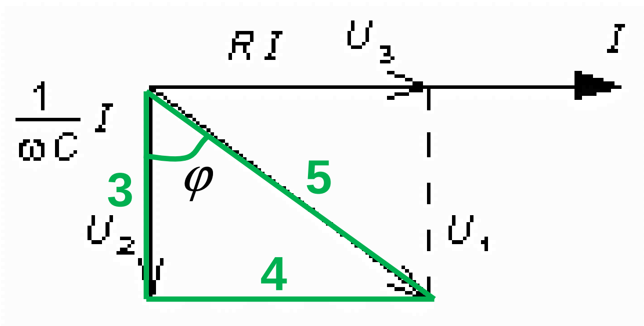




Phasor diagram: *R* and *C* has the current *I* in common. Volyage over resistor and voltage over capacitors kondensatorn therefore becomes perpendicular. Pythagorean theorem can be used:

$$U_1^2 = U_3^2 + U_2^2$$

$$|\varphi| = \arctan \frac{U_2}{U_1}$$



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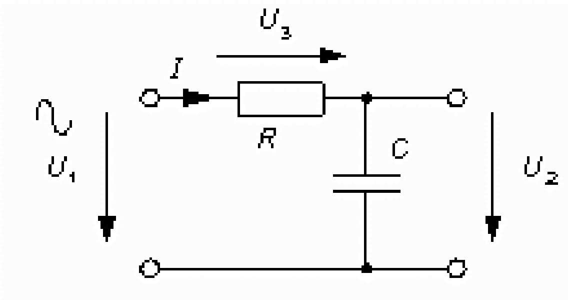
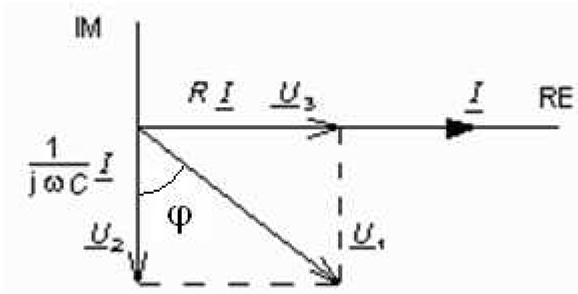
RC LP-filter, jω





$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \cdot \frac{j\omega C}{j\omega C} = \frac{1}{1 + j\omega RC}$$

$$\frac{U_2}{U_1} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\varphi = \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = \arg(1) - \arg(1 + j\omega RC) = 0 - \arctan\left(\frac{\omega RC}{1}\right) = -\arctan(\omega RC)$$

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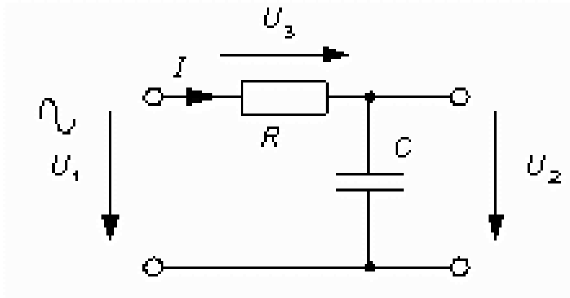
RC LP-filter, $H(\omega)$

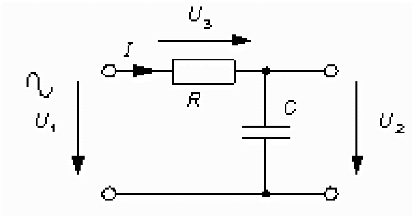


$$\boxed{\underline{H} = \frac{1}{1 + \mathrm{j}\omega RC}} \qquad \mathrm{abs}(\underline{H}) = H = \frac{1}{\sqrt{1+(\omega RC)^2}} \qquad \arg(\underline{H}) = -\arctan(\omega RC)$$

At the angular frequency when $\boxed{\omega RC = 1}$, will the numerator real part and imaginary part be equal. This is the filter cutoff frequency.

$\omega \approx 0$	$\omega \approx \frac{1}{RC} \quad \omega RC = 1$	$\omega >> \frac{1}{RC}$	$\omega \to \infty$
$\frac{U_2}{U_1} \approx \frac{1}{\sqrt{1+0}} \approx 1$	$\frac{U_2}{U_1} \approx \frac{1}{\sqrt{1^2+1^2}} \approx \frac{1}{\sqrt{2}} \approx 0{,}71$	$\frac{U_2}{U_1} \approx \frac{1}{\omega RC}$ avtar med ω 0,1ggr/dekad	$\frac{U_2}{U_1} \to 0$
$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) \approx \arctan 0 \approx 0^\circ$	$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) \approx 0 - \arctan 1 = -45^\circ$	$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) \approx -\arctan(\omega RC)$	$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) \to -90^\circ$

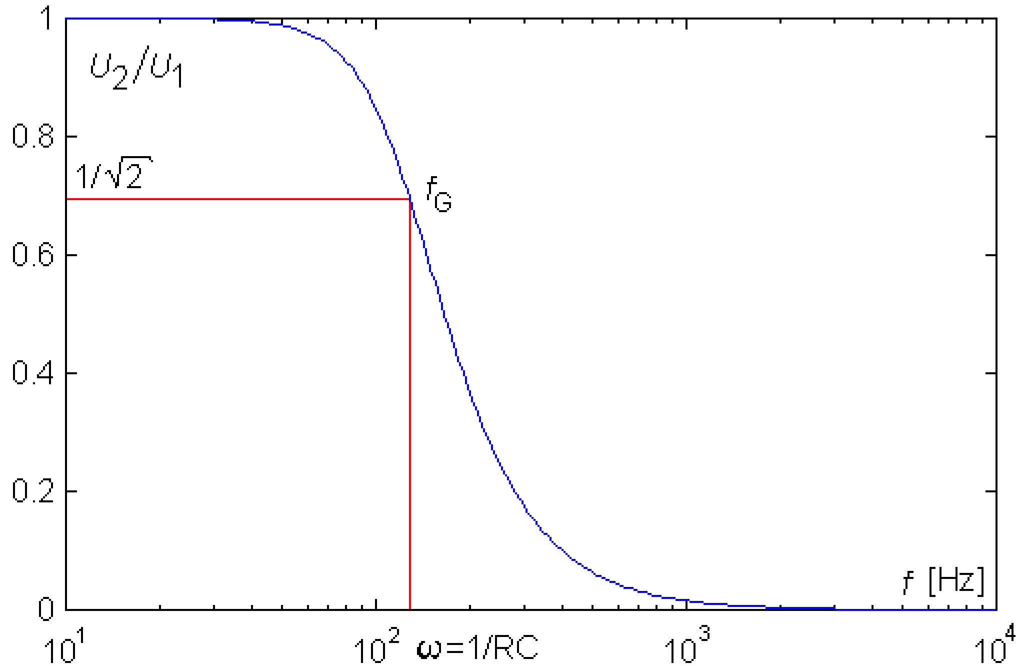
LP-magnitude function



$R = 1\ \text{k}\Omega$

$C = 1\ \mu\text{F}$

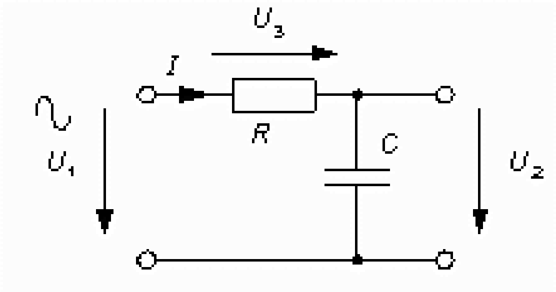
$$f_G = \frac{1}{2\pi \cdot 1 \cdot 10^3 \cdot 1 \cdot 10^{-6}}$$
$$\approx 160\ \text{Hz}$$

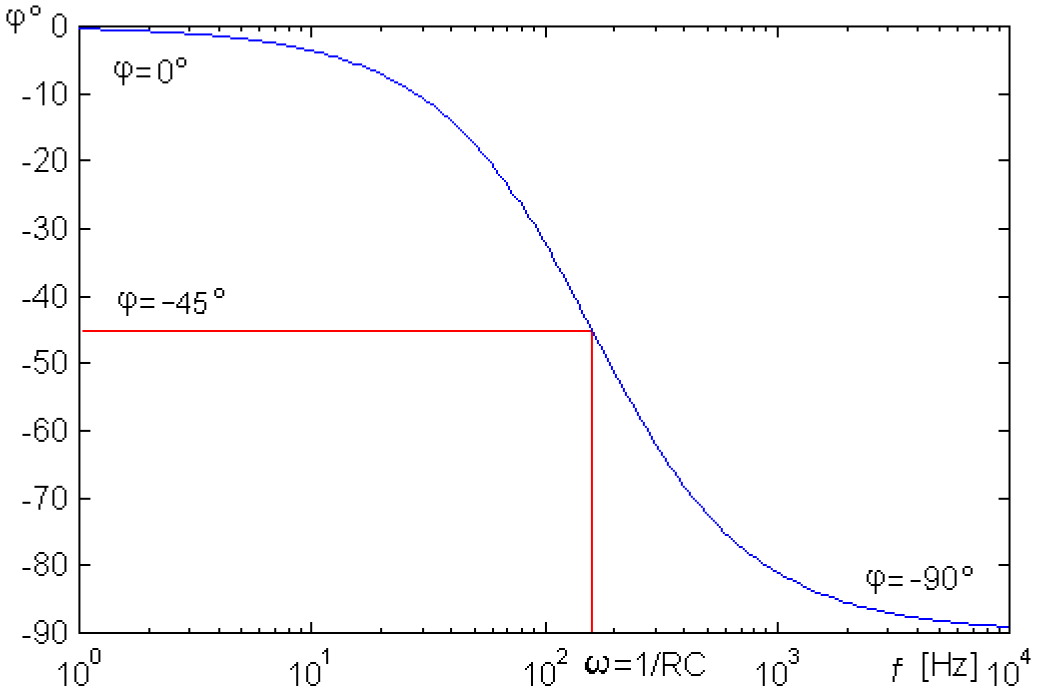


$$H = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\omega_G = \frac{1}{RC} \qquad f_G = \frac{1}{2\pi RC}$$

LP-Phase function

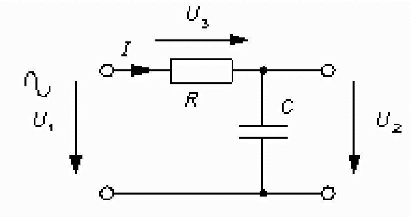




$$\varphi = \arg\left(\underline{H}\right) = -\arctan(\omega RC)$$

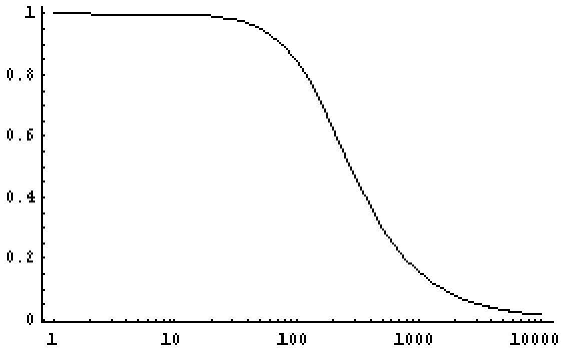
Graphs with Mathematica

Mathematica has commands for complex absolute value (abs []) and argument (arg [], in radians).

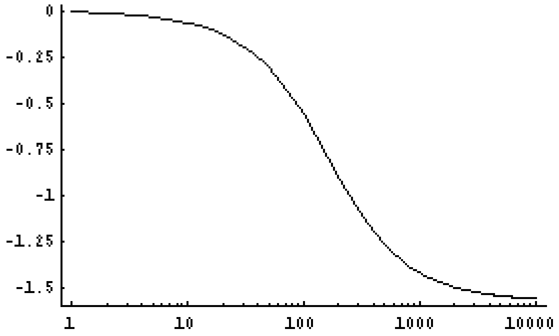



```
<<Graphics
r=1*10^3;
c=1*10^-6;
w=2*Pi*f;
u2u1[f_]=1/(1+I*w*r*c);
LogLinearPlot[Abs[u2u1[f]],{f,1,10000},PlotRange->All,PlotPoints->100];
LogLinearPlot[Arg[u2u1[f]],{f,1,10000},PlotRange->All,PlotPoints->100];
```

Press SHIFT + ENTER to start the calculation and the plot.



Amount plot

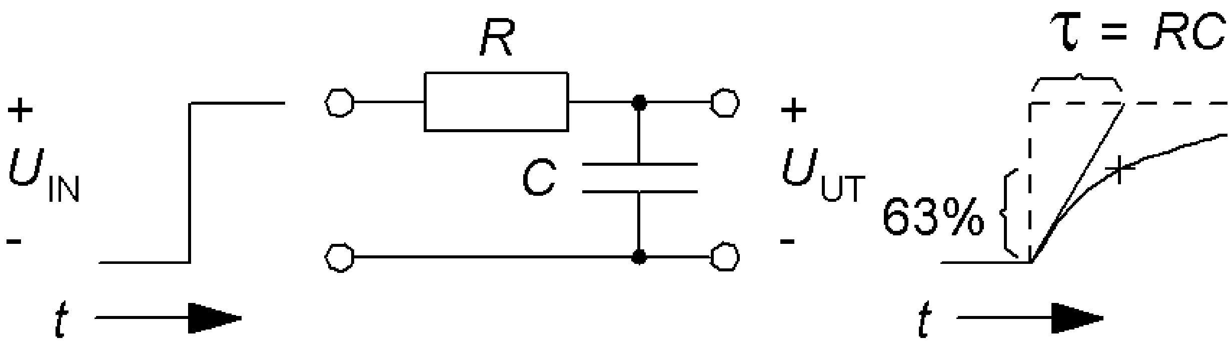


Phase plot [rad]

RC Two sides of the same coin

$$\omega_G = \frac{1}{\boxed{RC}} \qquad \tau = \boxed{RC}$$

Low **cut off frequency** ω_G will supresses interference good, but it will also mean that the time constant τ is long so it takes time until U_{UT} reaches its final value and can be read.



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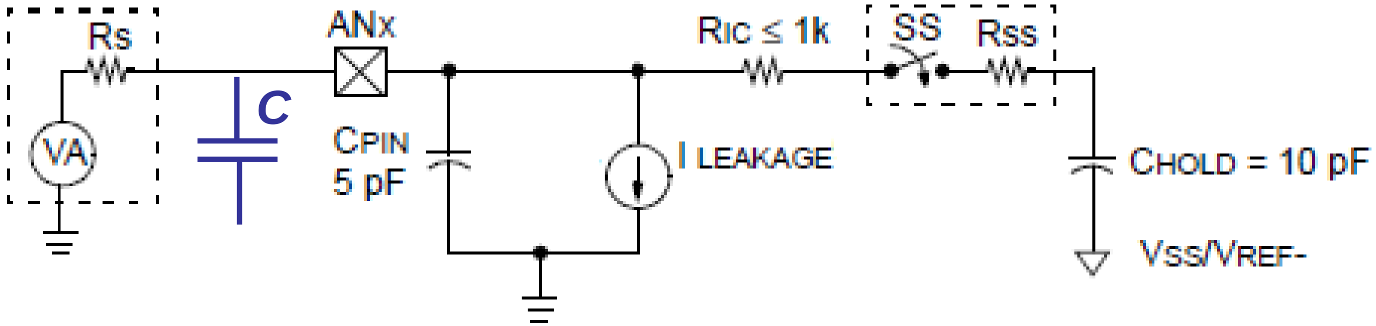
(AD-converter LP-filter)

- *It is necessary to compromise!*



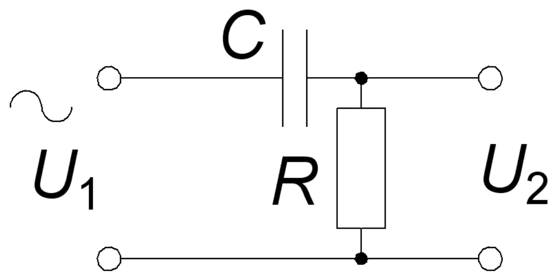
In order to remove noise from the input signal to the AD converter one usually add a capacitor ***C***.

- R_S must have no bigger value than 10kΩ – otherwise you risk losing accuracy because of the leakage current $I_{LEAKAGE}$.

- When the sample charge from ***C*** is taken to sampling capacitor C_{HOLD}. ***C*** should therefore be at least 1024 times greater than C_{HOLD} (10pF) if you do not want to lose accuracy.

- ***C***$\cdot R_S$ gives the cutoff frequency of how fast signals AD converter can follow.

RC HP-filter, jω



$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{R}{R + \frac{1}{j\omega C}} \cdot \frac{j\omega C}{j\omega C} = \frac{j\omega RC}{1 + j\omega RC}$$

$$\frac{U_2}{U_1} = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

$$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = \arg(j\omega RC) - \arg(1 + j\omega RC) = \underbrace{90^\circ - \arctan\left(\frac{\omega RC}{1}\right)}_{= \operatorname{arccot}()} = \arctan\left(\frac{1}{\omega RC}\right)$$

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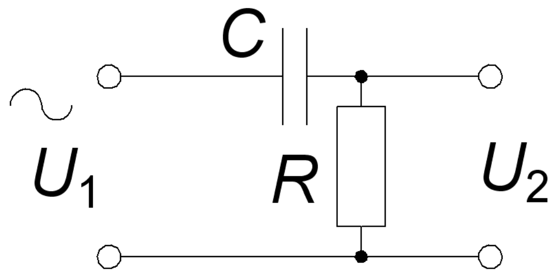
RC HP-filter, $H(\omega)$

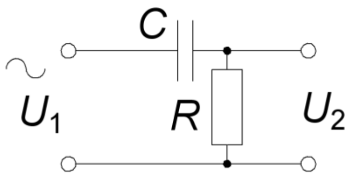


$$\boxed{\underline{H} = \frac{\mathrm{j}\omega RC}{1+\mathrm{j}\omega RC}} \quad \mathrm{abs}(\underline{H}) = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} \quad \arg(\underline{H}) = \arctan\left(\frac{1}{\omega RC}\right)$$

At the angular frequency when $\boxed{\omega RC = 1}$, will the numerator real part and imaginary part be equal. This is the filter cutoff frequency.

$\omega \approx 0$	$\omega << \frac{1}{RC}$	$\omega = \frac{1}{RC}$	$\omega \to \infty$
$\frac{U_2}{U_1} \approx 0$	$\frac{U_2}{U_1} \approx \frac{\omega RC}{1+0} \approx \omega RC$ stiger med ω 10ggr/dekad	$\frac{U_2}{U_1} = \frac{1}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}} \approx 0{,}71$	$\frac{U_2}{U_1} \to 1$
$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) \approx \arg\left(\frac{\approx \mathrm{j}}{1+0\cdot \mathrm{j}}\right) = \arg \mathrm{j} = 90°$	$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) \approx 90° - \arctan(\omega RC)$	$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) \approx 90° - \arctan 1 = 45°$	$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = 90° - 90° = 0°$

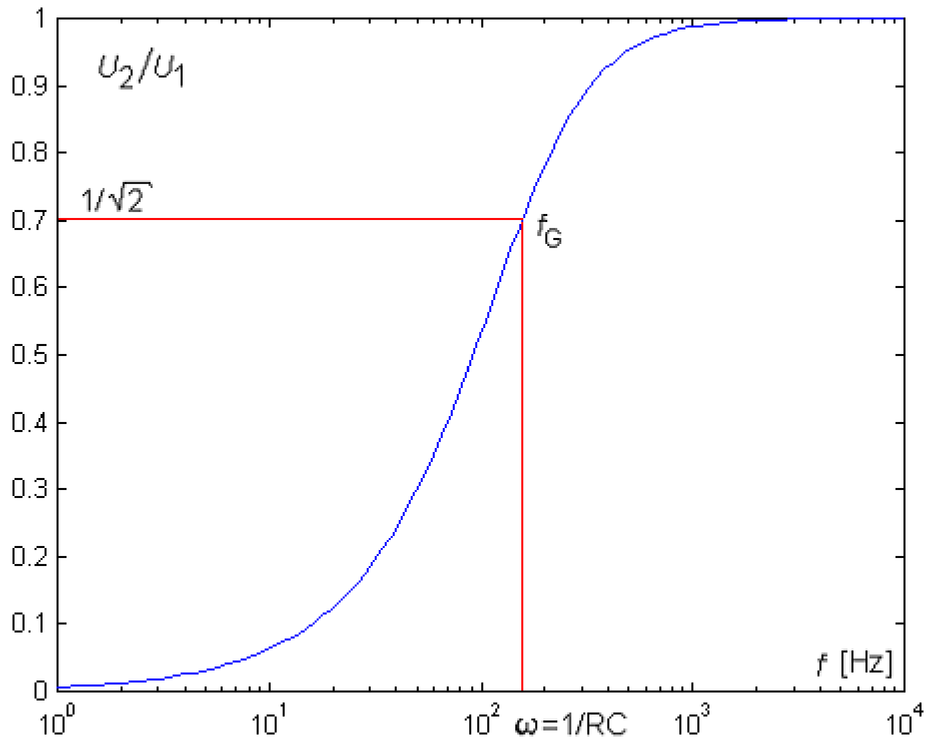
HP-magnitude function



R = 1 kΩ

C = 1 μF

$$f_G = \frac{1}{2\pi \cdot 1 \cdot 10^3 \cdot 1 \cdot 10^{-6}}$$
$$\approx 160 \text{ Hz}$$



$$\text{abs}(\underline{H}) = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$$

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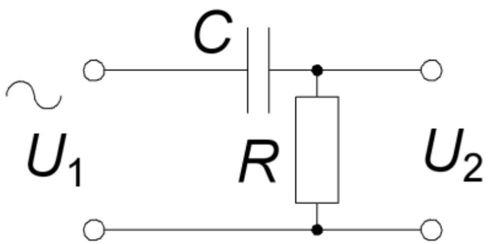
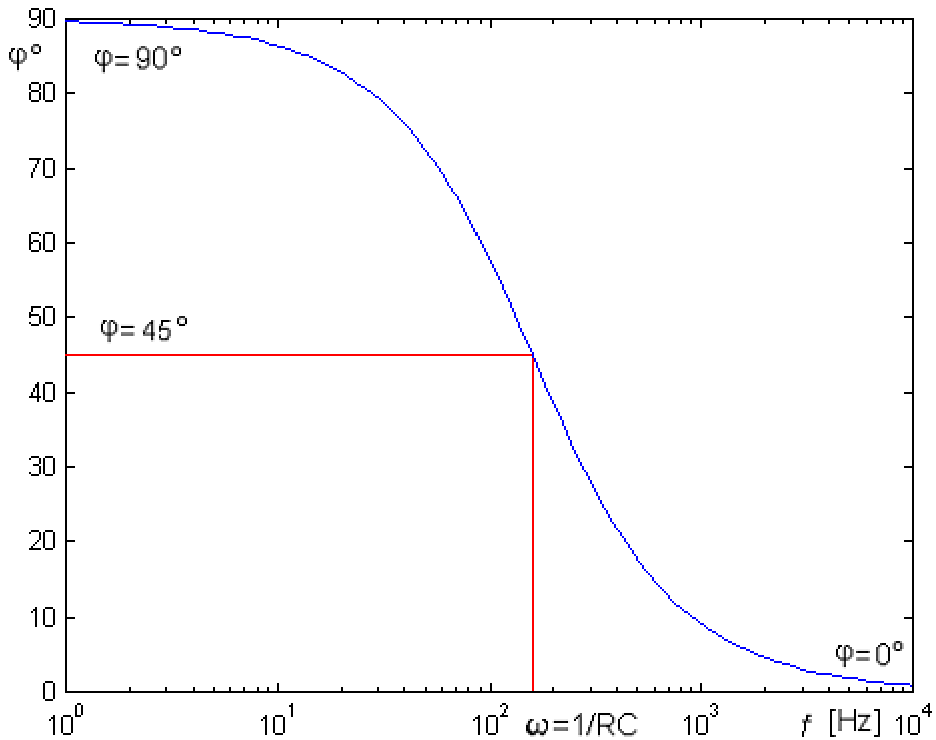
HP-phase function





$$\varphi = \arg(\underline{H}) = \arctan\left(\frac{1}{\omega RC}\right)$$

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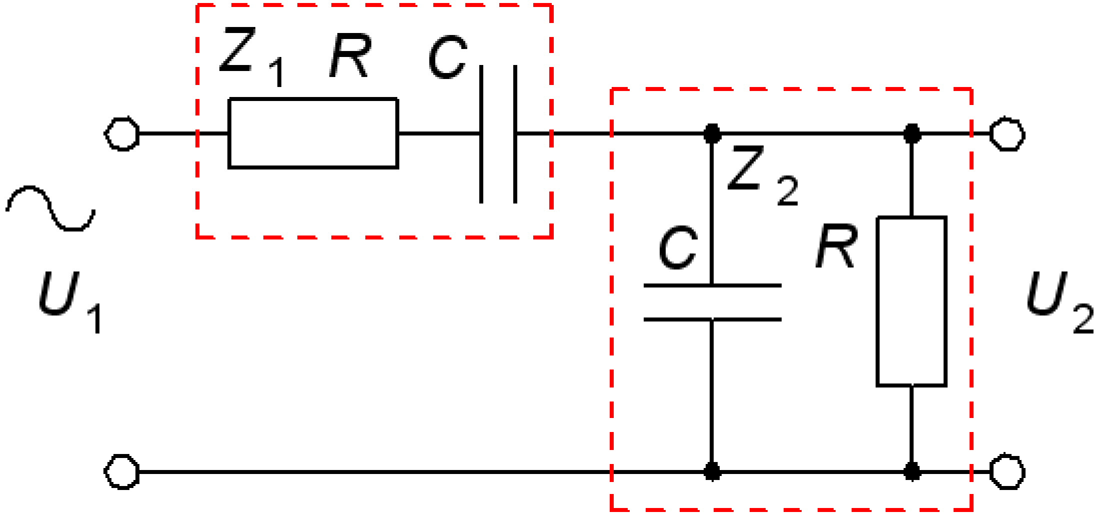
Wienbridge (14.5)

Was investigated by Max Wien 1891

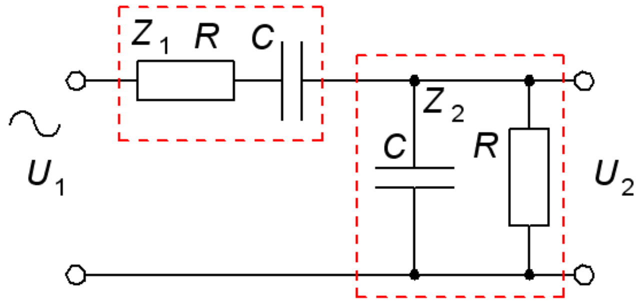


For a certain frequency U_1 and U_2 are in phase. What frequency?

Wienbridge

$$\underline{Z}_1 = R + \frac{1}{j\omega C}$$

$$\underline{Z}_2 = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \cdot \frac{j\omega C}{j\omega C} = \frac{R}{1 + j\omega RC}$$



U_1 and U_2 are in phase if the transferfunction imaginary part is 0!

$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{\frac{R}{1+j\omega RC}}{R + \frac{1}{j\omega C} + \frac{R}{1+j\omega RC}} \cdot \frac{\frac{(1+j\omega RC)}{R}}{\frac{(1+j\omega RC)}{R}} = \frac{1}{3 + j\omega RC + \frac{1}{j\omega RC}} = \frac{1}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)}$$

$= 0$

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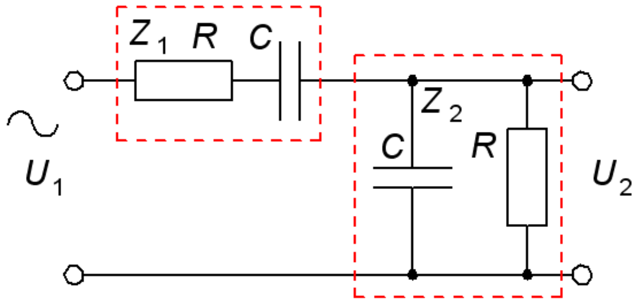
Wienbridge

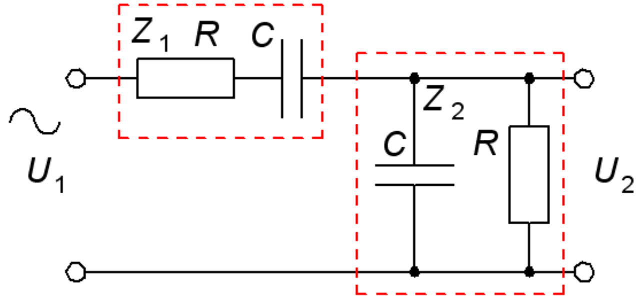
$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{1}{3 + \mathrm{j}(\omega RC - \frac{1}{\omega RC})} \Rightarrow$$

$$\omega RC - \frac{1}{\omega RC} = 0 \Rightarrow \omega_0 = \frac{1}{RC}$$

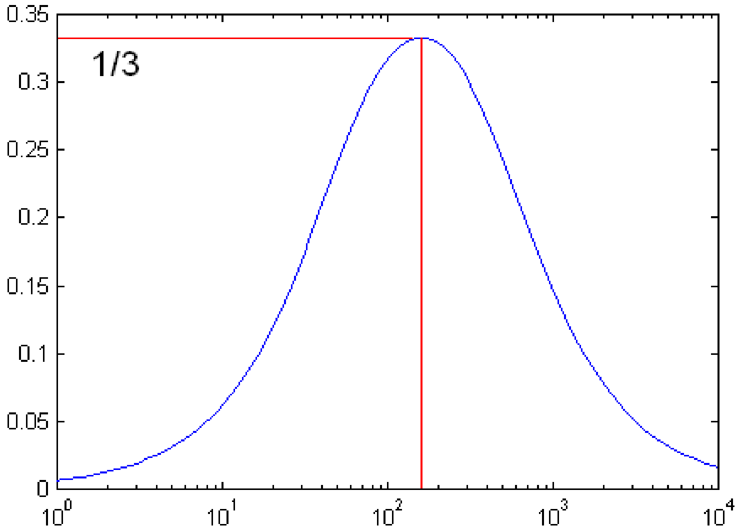


$\omega \approx 0$	$\omega = \frac{1}{RC} \quad (\omega RC - \frac{1}{\omega RC}) = 0$	$\omega \approx \infty$
$\frac{\underline{U}_2}{\underline{U}_1} \approx \frac{1}{\sqrt{\ldots + (-\infty)^2}} \approx 0$	$\frac{\underline{U}_2}{\underline{U}_1} = \frac{1}{\sqrt{3^2 + 0^2}} = \frac{1}{3} \approx 33\%$	$\frac{\underline{U}_2}{\underline{U}_1} \approx \frac{1}{\sqrt{\ldots + (\infty)^2}} \approx 0$
$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) \approx \arg\left(\frac{1}{\ldots + (-\infty \cdot \mathrm{j})}\right) = 90°$	$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = \arg\left(\frac{1}{1 + \mathrm{j} \cdot 0}\right) = 0°$	$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) \approx \arg\left(\frac{1}{\ldots + \infty \cdot \mathrm{j}}\right) \approx -90°$

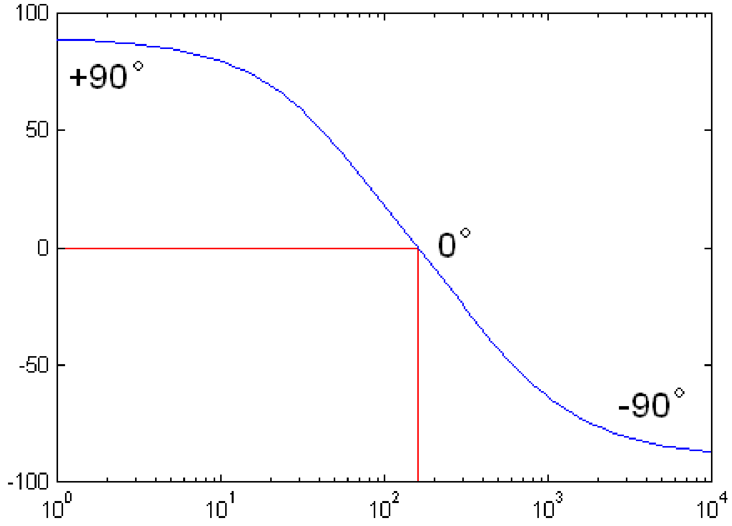
Wienbridge



$$\omega_0 = \frac{1}{RC} \qquad f_0 = \frac{1}{2\pi RC}$$



Magnitude plot



Phase plot

Wienbridge is a band pass filter.

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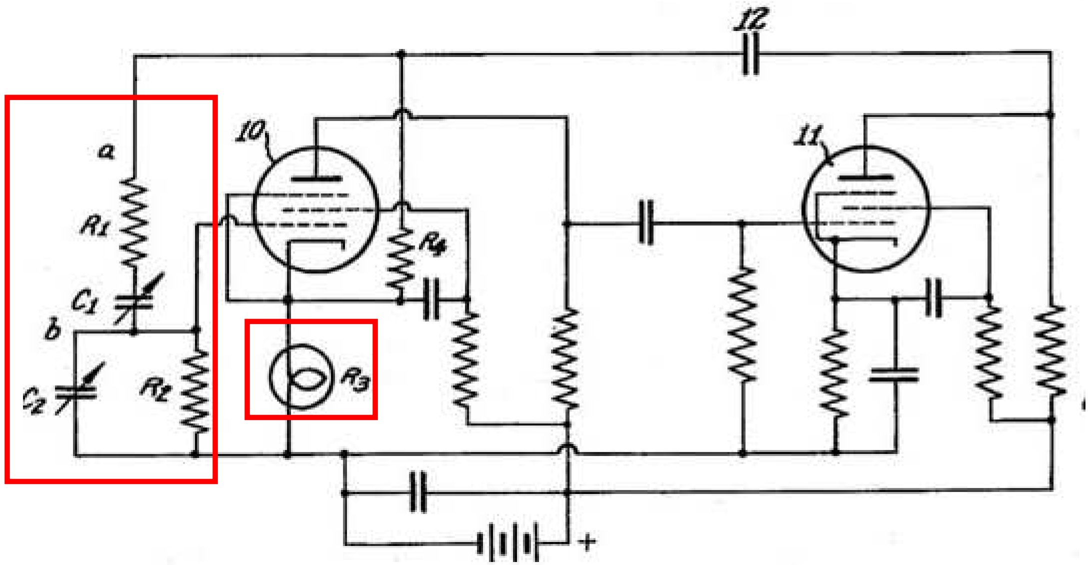
William Hewlett's master thesis

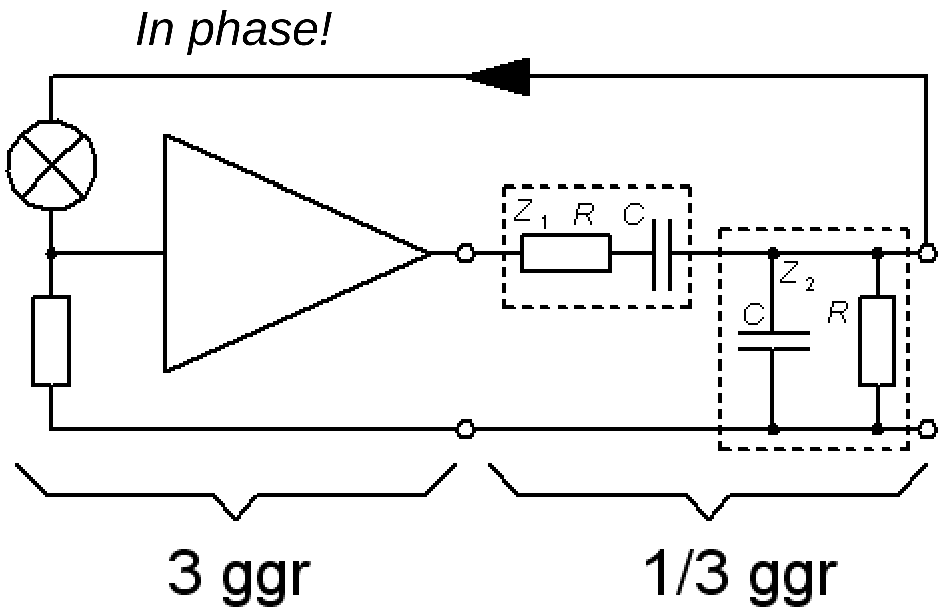
Master thesis 1930. Wienbridge with lamp!



William Hewletts master thesis

Hewlett constructed a tone generator. Wien bridge attenuates the signal to 1/3 so he needed a amplifier with the gain exactly three times.

The bulb stabilizes the signal. If the amplitude becomes too large the lamp will glow and then the signal is attenuated in the voltage divider at the amplifier input.



The Palo Alto garage
the birthplace of **Silicon Valley**

Which global business will you start with your thesis?

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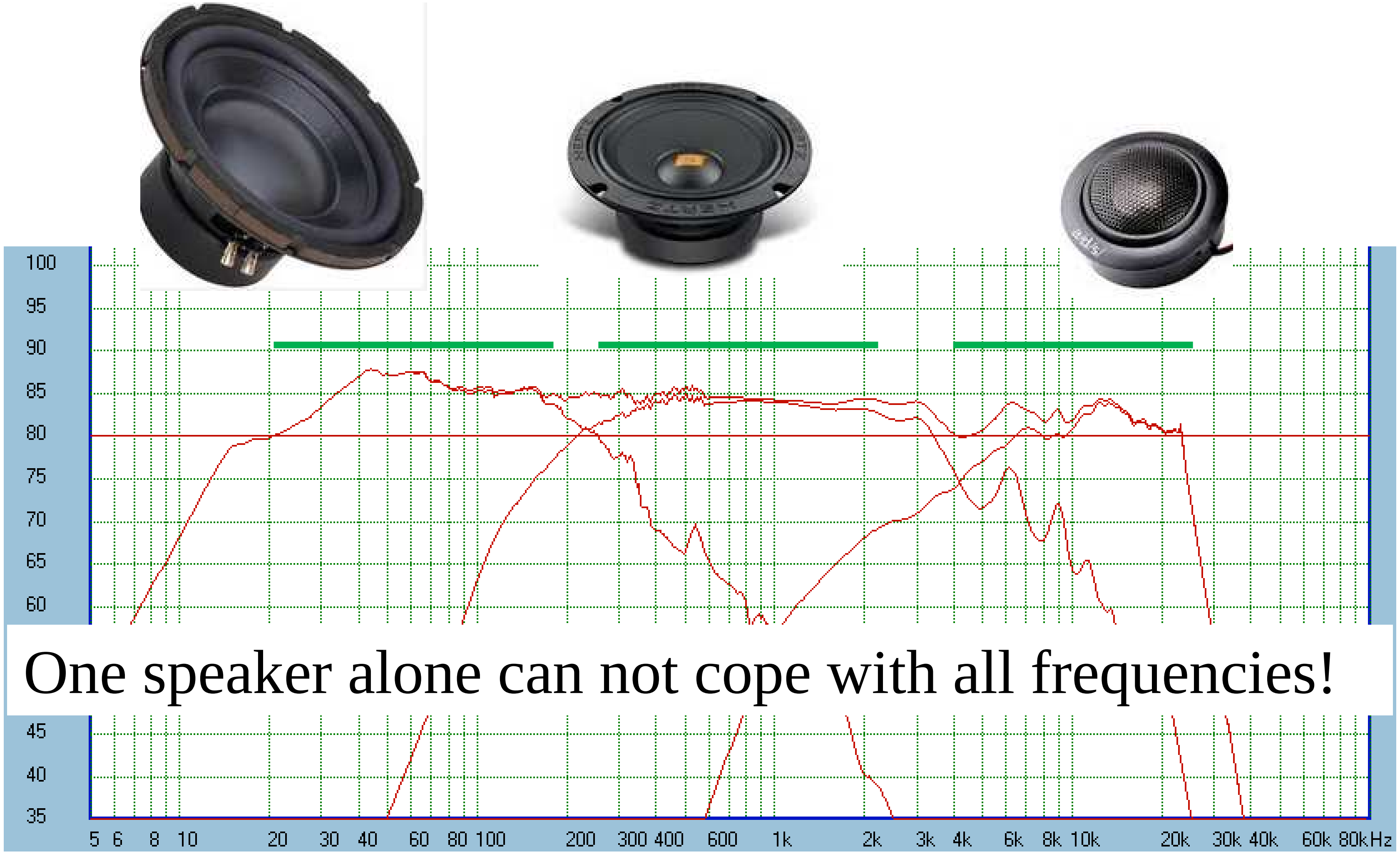
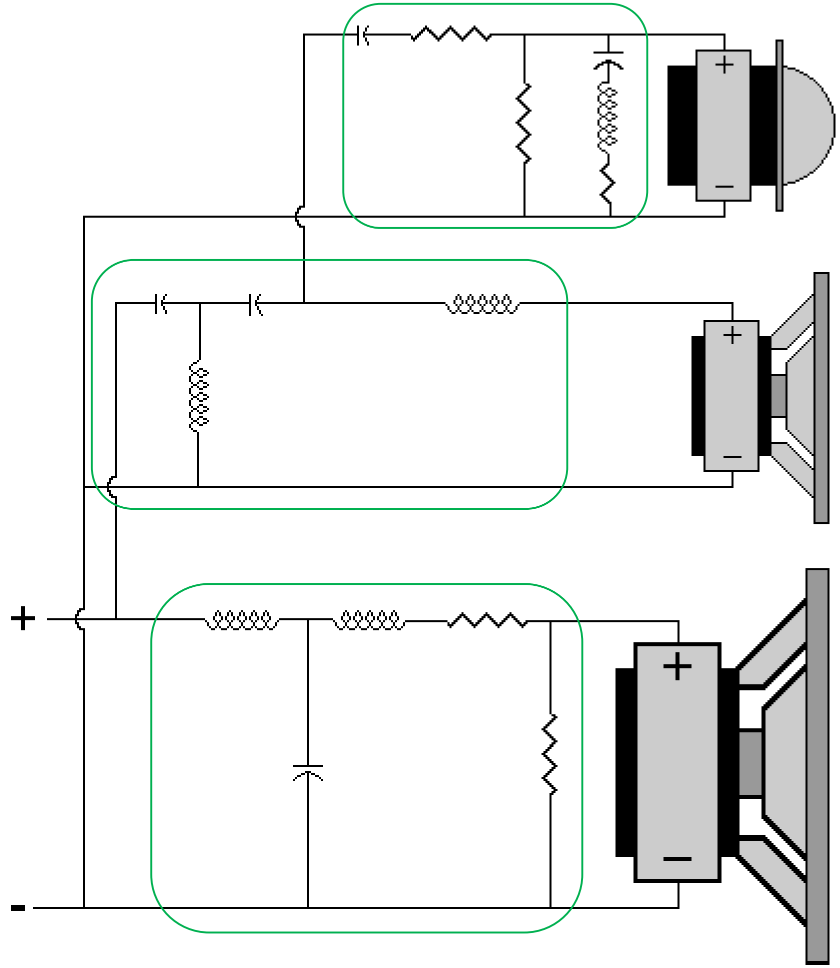
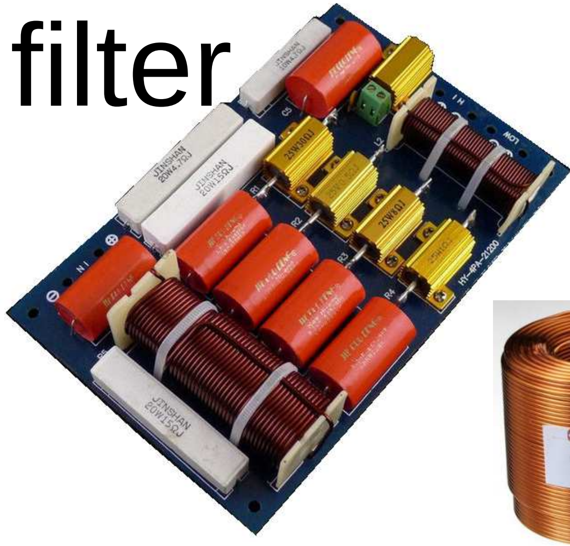
When are filters used?

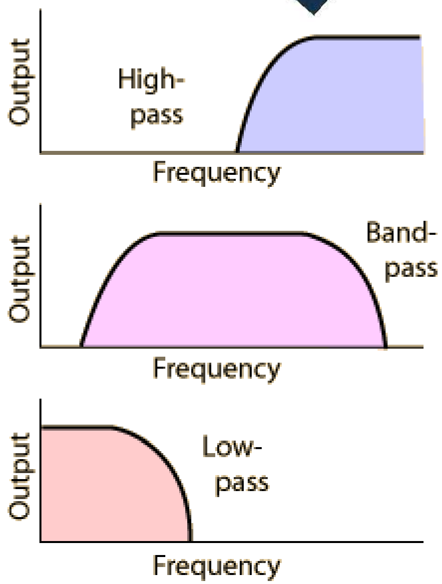


One speaker alone can not cope with all frequencies!

Cross over filter







The crossover filter split the frequencies between the speakers.

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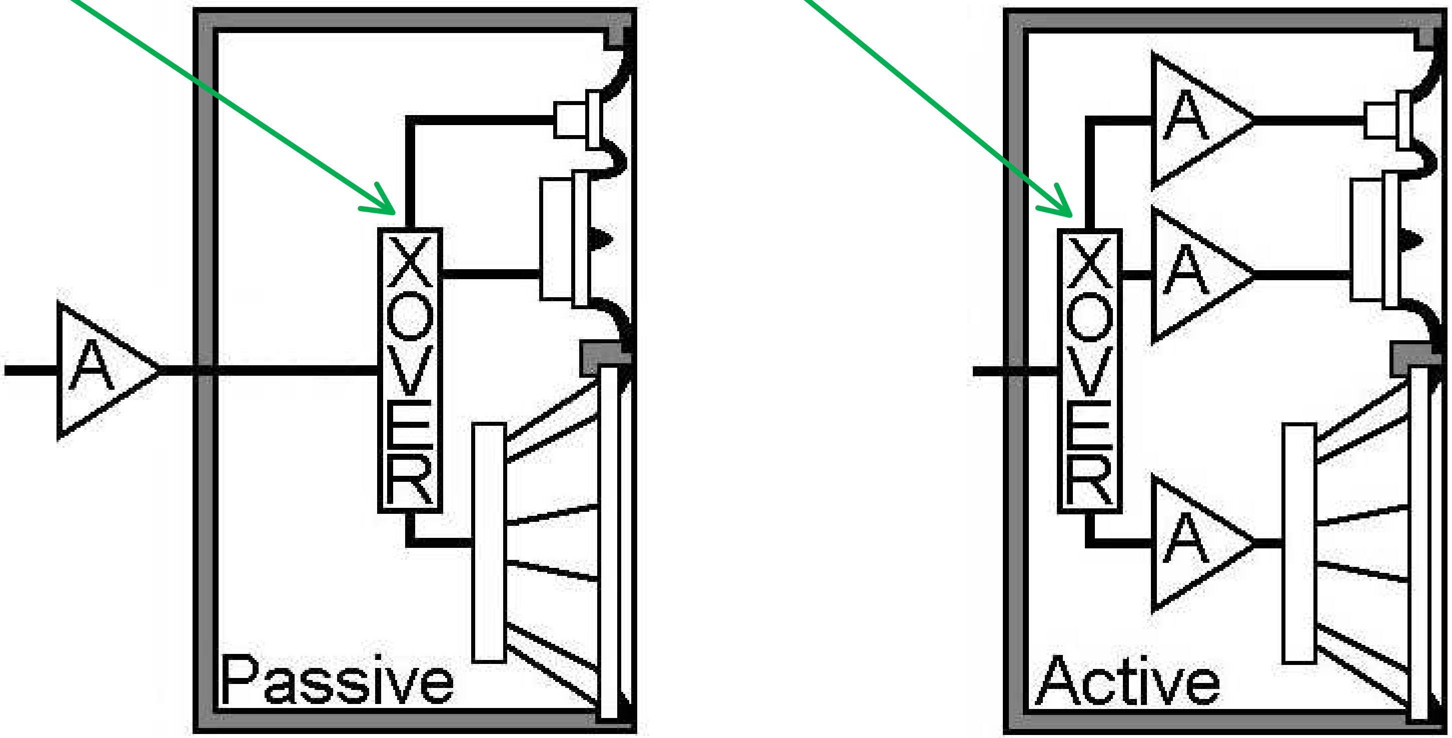
Passive/Active speaker

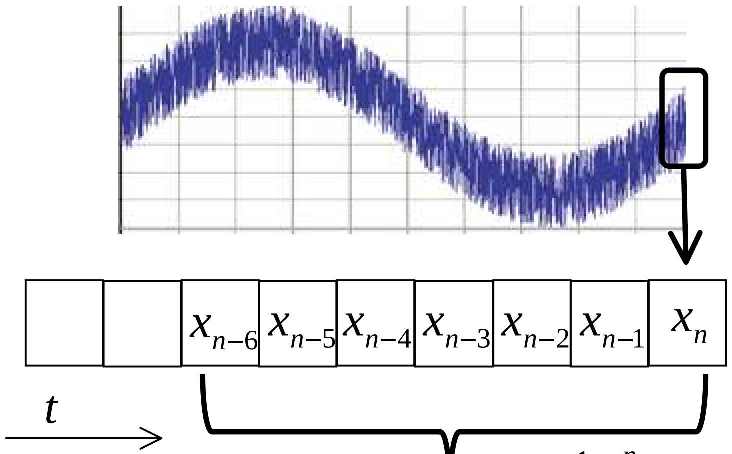
- Analog crossoverfilter *R L C*
- Digital crossoverfilter = computer program



When the amplifier is built in the speaker it becomes possible to use digital crossovers. (XOVER = crossover filter)

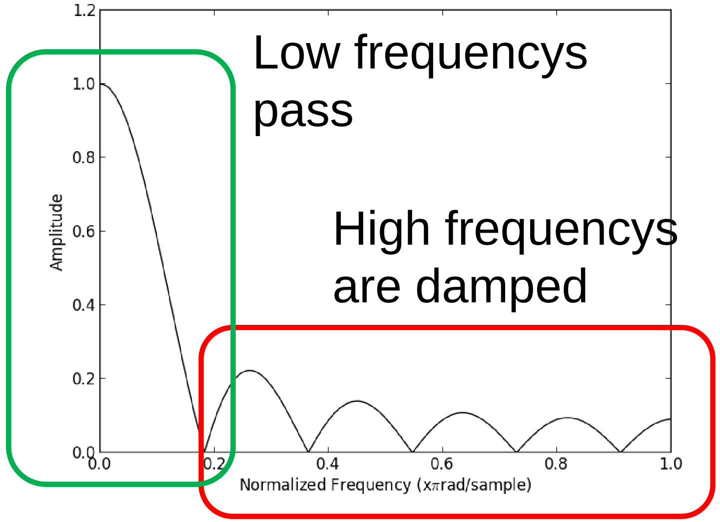
(Digital filter)

Ex. a "rolling" average of the 7 most recent readings.

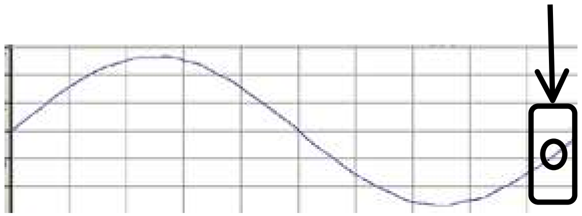


- Brusig signal

LP-filter



$$y_n = \frac{1}{7}\sum_{n-6}^{n} x_n$$



- Filtered signal

There are much better digital filters than this ...