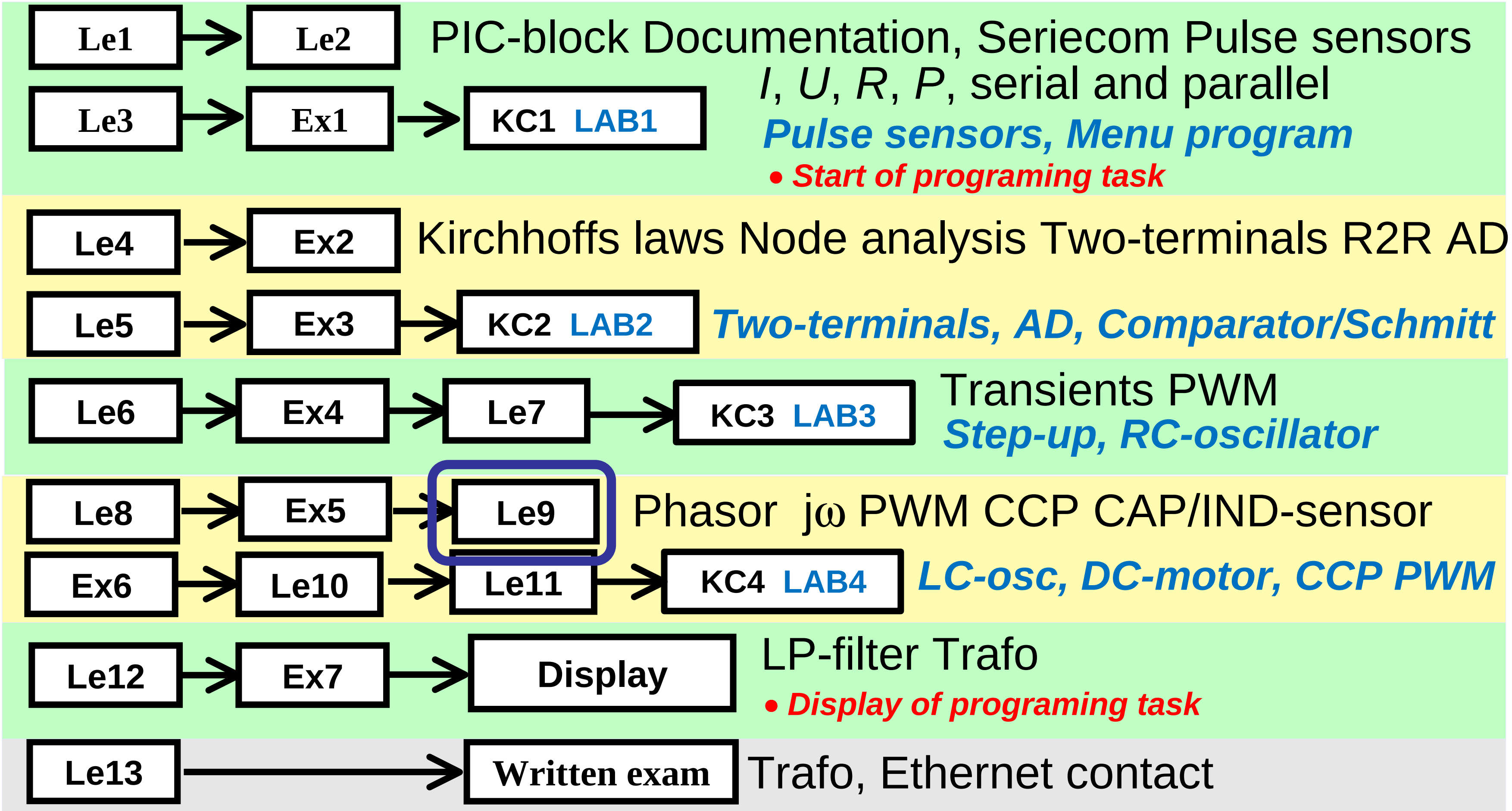
IE1206 Embedded Electronics



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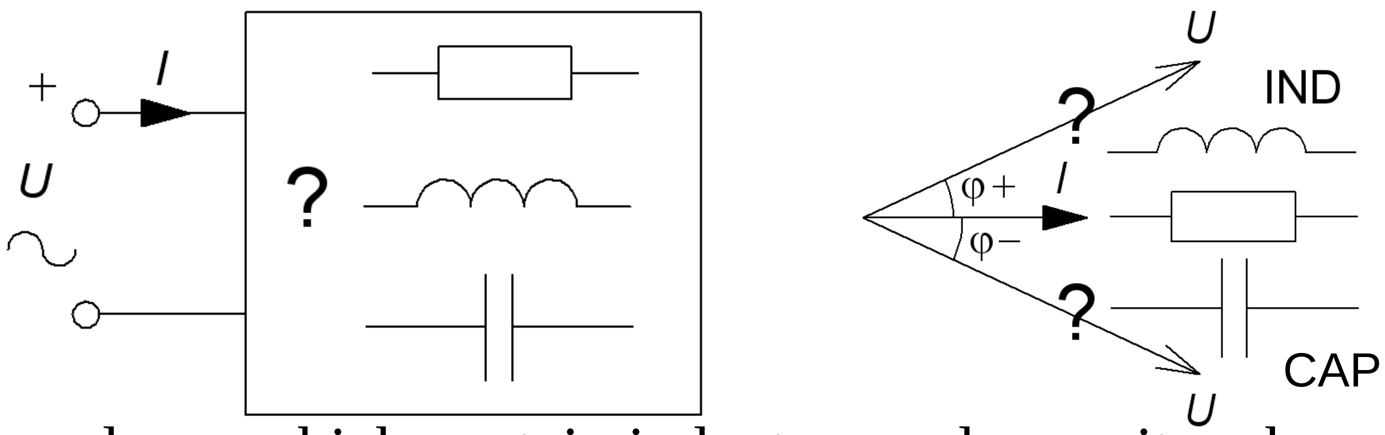
R L C



An impedance which contain inductors and capacitors have, depending on the frequency, either inductive character **IND**, or capacitive character **CAP**.

An important special case occurs at the frequency where capacitances and inductances are equally strong, and their effects cancel each other out. The impedance becomes purely resisistiv. The phenomenon is called the **resonance** and the frequency on which this occurs is the **resonant frequency**.

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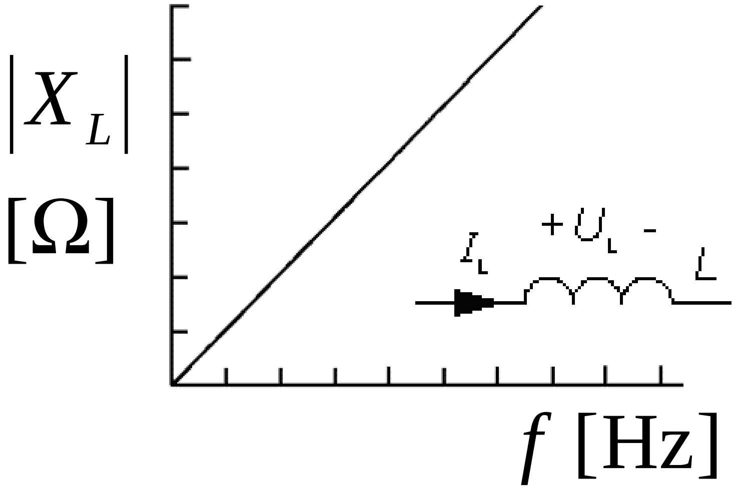
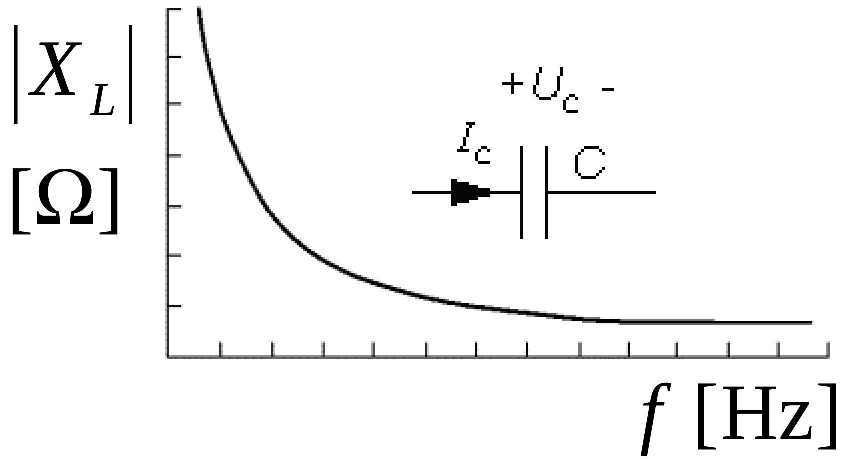
Reactance frequency dependency

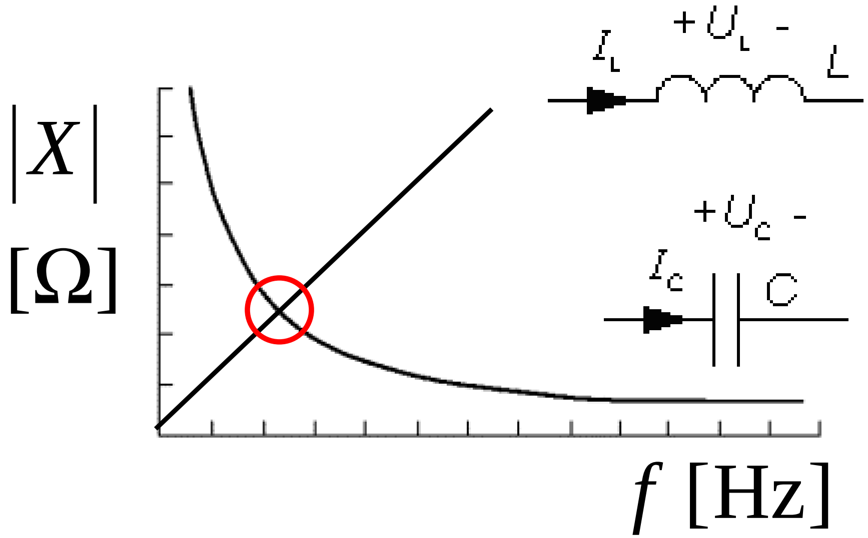




$$\left|X_L\right| = \omega \cdot L \qquad \left|X_C\right| = \frac{1}{\omega \cdot C}$$

$$\omega = 2\pi f$$

R L C impedances



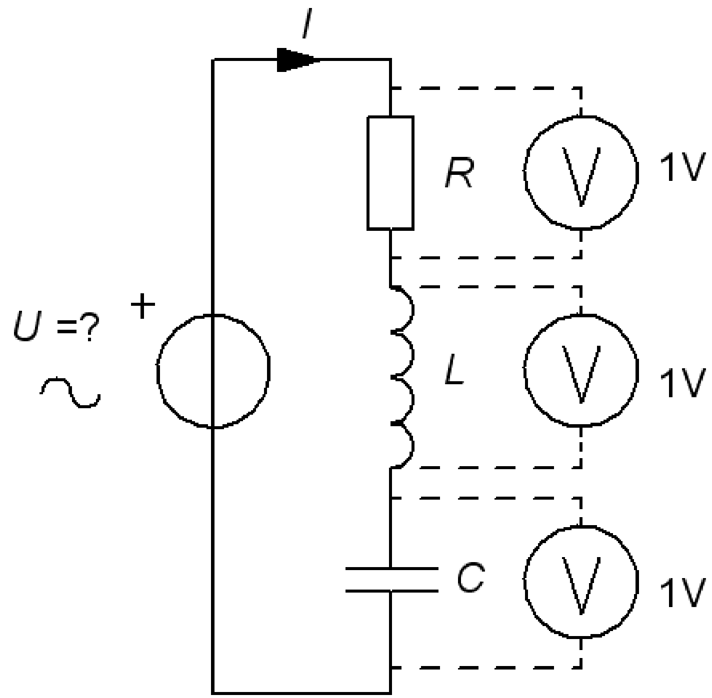
- At a certain frequence X_L and X_C has the same amount.

$$|X_L| = \omega \cdot L \qquad |X_C| = \frac{1}{\omega \cdot C}$$

$$\omega = 2\pi f$$

How big is *U* ? (13.1)

The three volt meters show the same, 1V, how much is the alternating supply voltage *U* ? (*Warning*, *teaser*)



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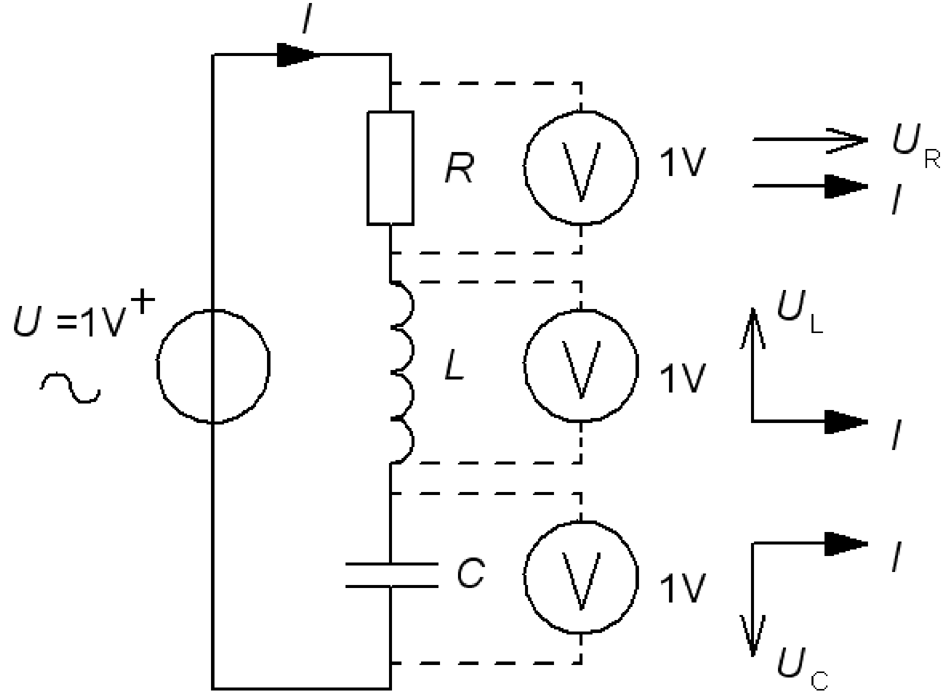

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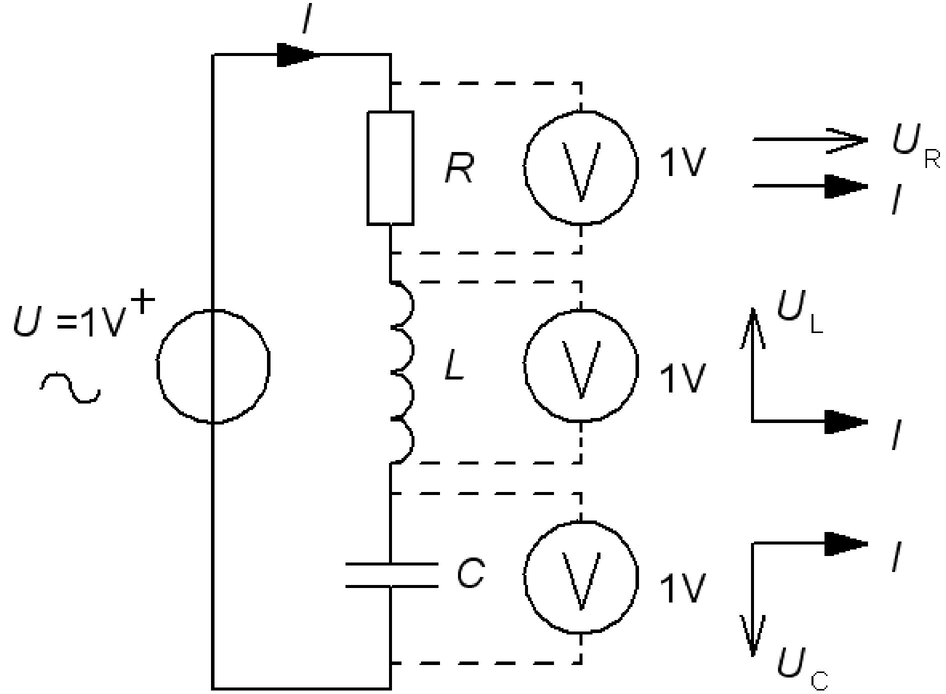


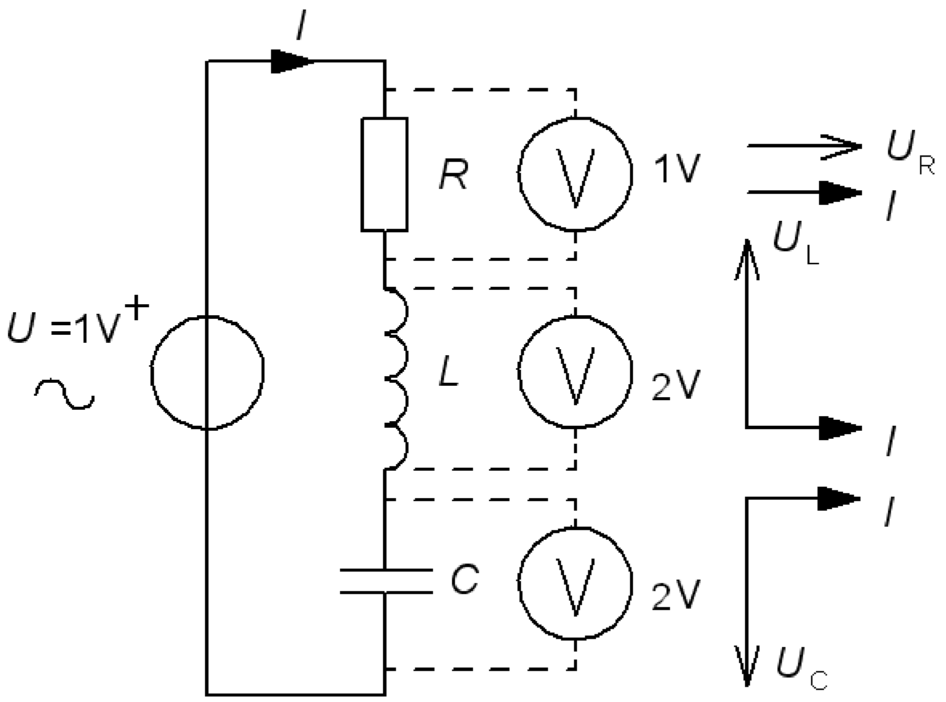
Since volt meters show the "same" and the current *I* is common:

$$R = |X_L| = |X_C| \qquad R = \omega L = \frac{1}{\omega C}$$

If $|X_L|=|X_C|=2R$?

Suppose the AC voltage *U* still 1 V, but the reactances are *twice* as big. What will the voltmeters show?

$$\omega L = \frac{1}{\omega C} = 2 \cdot R$$



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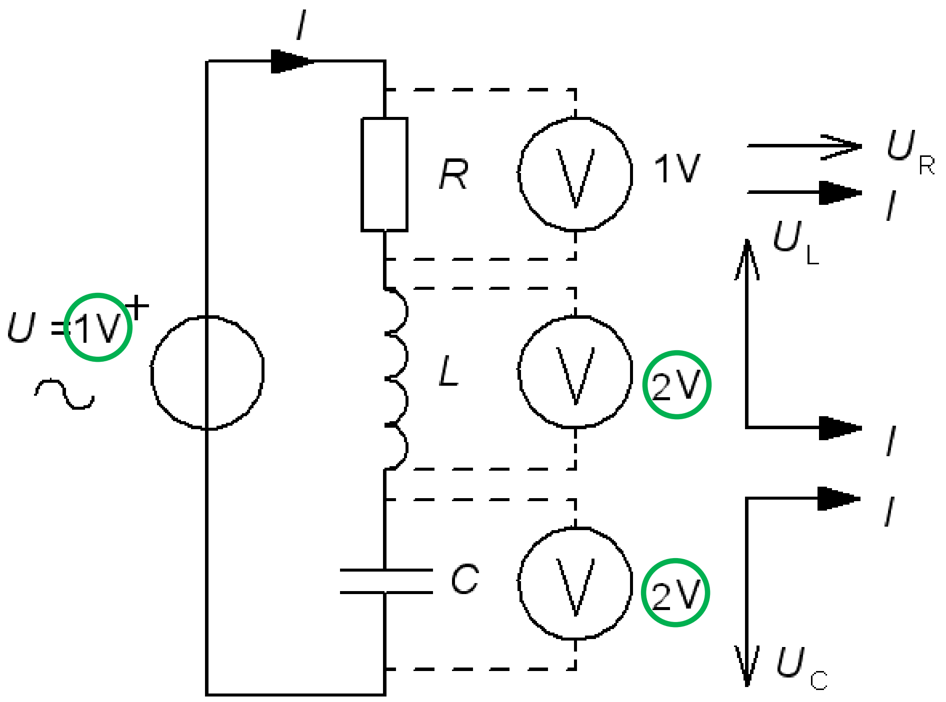

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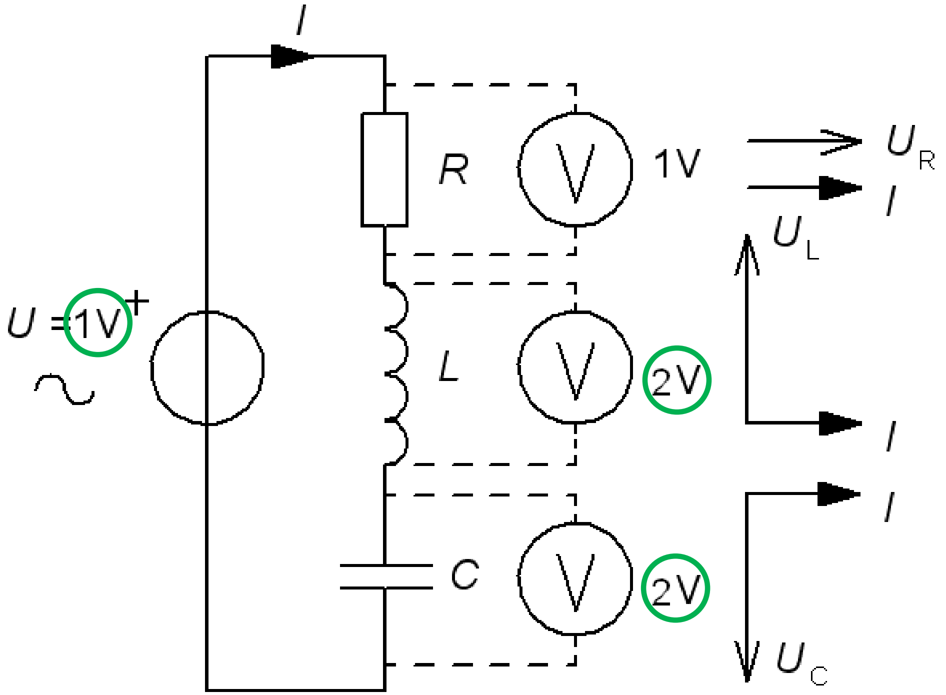

At resonance, the voltage over the reactances can be many times higher than the AC supply voltage.

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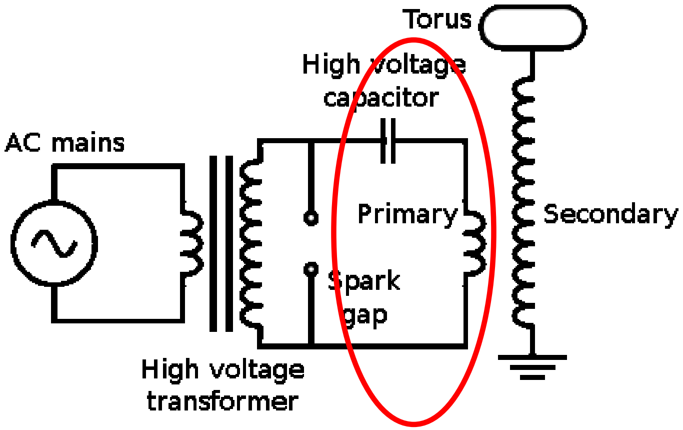
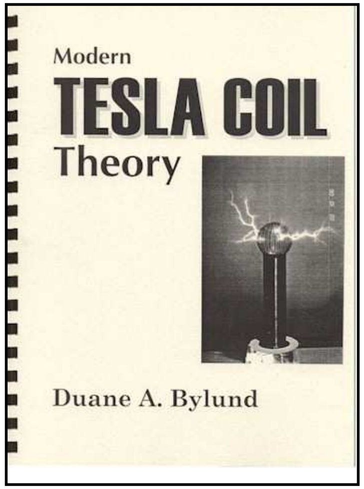
Tesla coil

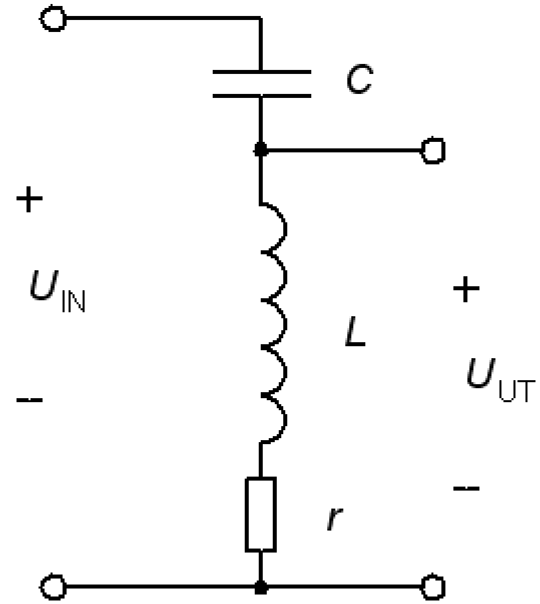
Many builds "Tesla" coils to gain some excitement in life…





Inductor quality factor *Q*

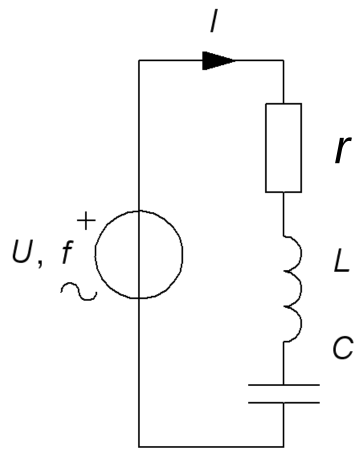
Usually it is the internal resistance of the coil which is the resistor in the RLC circuit. The higher the coil AC resistance ωL is in relation to the DC resistance r, the larger the voltage across the coil at a resonance get. This ratio is called the coil quality factor Q.
(or Q-factor).



$$Q = \frac{X_L}{r} = \frac{\omega L}{r} \quad \Rightarrow \quad \boxed{U_{UT} \approx Q \cdot U_{IN}}$$

Series resonance

$$\underline{U} = \underline{I} \cdot \left(r + \mathrm{j}\omega L + \frac{1}{\mathrm{j}\omega C} \right) = \underline{I} \cdot \left(r + \mathrm{j}(\omega L - \frac{1}{\omega C}) \right)$$



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Series resonance

=0

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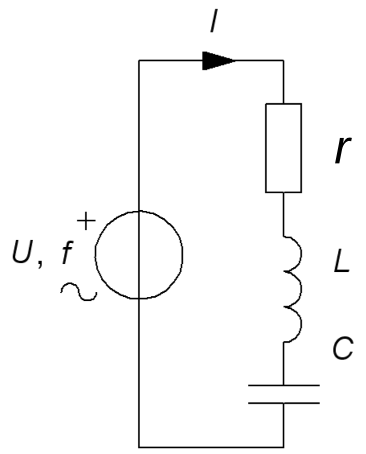

The Impedance is real when the imaginary part is ”0”. This will happen at angular frequency ω_0 (frequency f_0).

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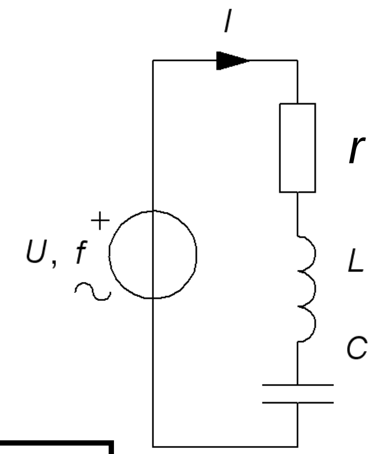

The Impedance is real when the imaginary part is ”0”.
This will happen at angular frequency ω_0 (frequency f_0).

$$\mathrm{Im}[\underline{Z}] = \omega L - \frac{1}{\omega C} = 0 \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}} \qquad \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

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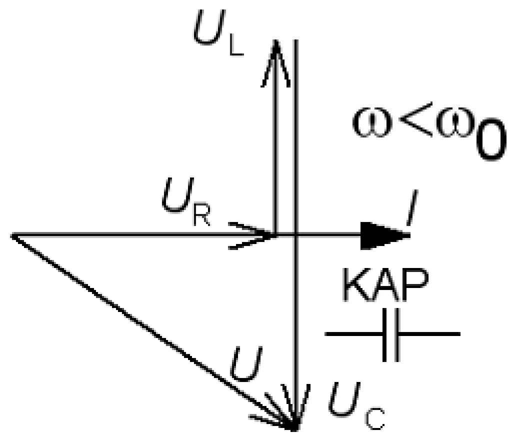
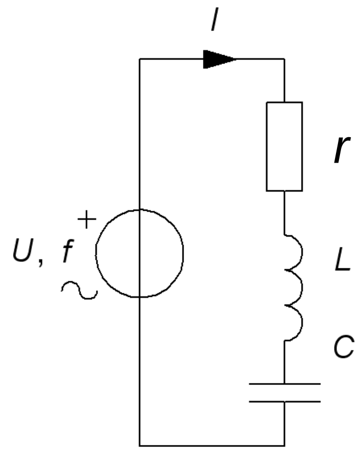
Series resonance phasor diagram

$$\underline{U} = \underline{I} \cdot \left(r + \mathrm{j}(\omega L - \frac{1}{\omega C}) \right)$$





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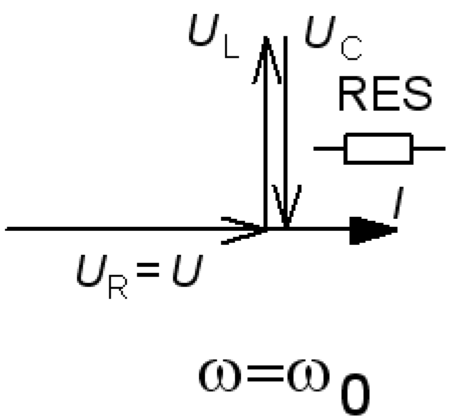

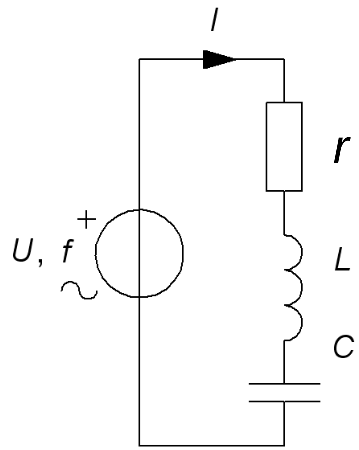

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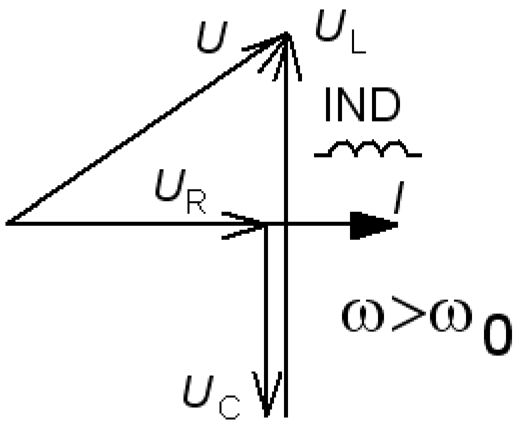
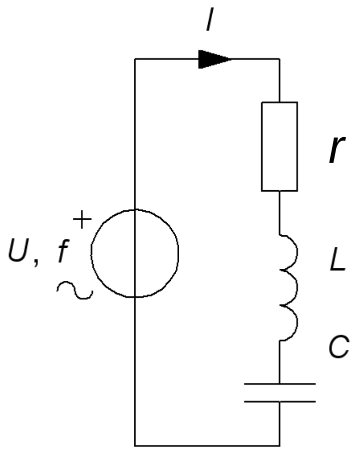
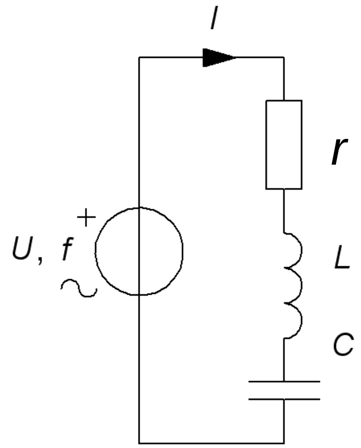




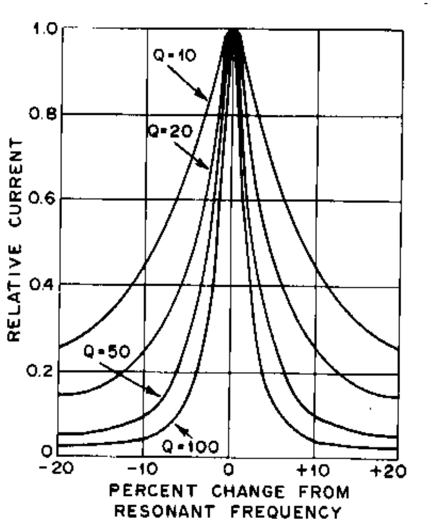
Series resonance circuit *Q*



It is the resistance of the resonant circuit, usually coil internal resistance, which determines how pronounced resonance phenomenon becomes. It is customary to "normalize" the relationship between the different variables by introducing the resonance angular frequency ω_0 together with the peak current I_{max} in the function $I(\omega)$ with parameter Q :

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q = \frac{\omega_0 L}{r}$$

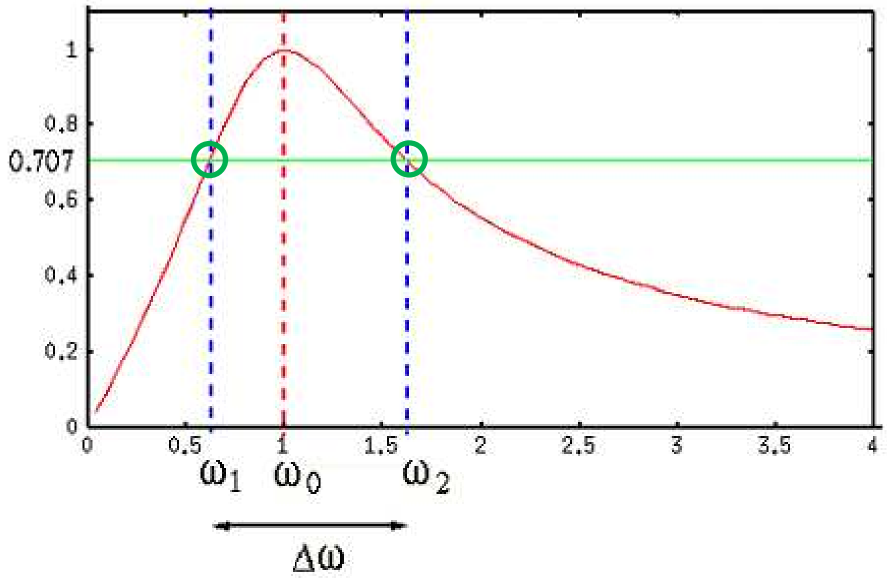
$$\underline{I} = \frac{I_{\max}}{\left(1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right)}$$



Normalized chart of the series resonant circuit. A high Q corresponds to a narrow resonance peak.

Bandwidth *BW*

At two different angular frequencies becomes imaginary Im and real part Re in the denominator equal. *I* is then $I_{max}/\sqrt{2}$ (≈71%).
The **Bandwidth** *BW*=Δω is the distans between those two angular frequencies.



$$\underline{I} = \frac{I_{max}}{\left(\boxed{1} + j \boxed{Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} \right)}$$

Re = Im

The equations give :

$$BW[\text{rad/s}] = \Delta\omega = \omega_2 - \omega_1 = \frac{\omega_0}{Q} \qquad \omega_0^2 = \omega_2 \cdot \omega_1 \qquad \omega_2, \omega_1 = \omega_0 \left(\pm \frac{1}{2Q} + \sqrt{\frac{1}{(2Q)^2} + 1} \right)$$

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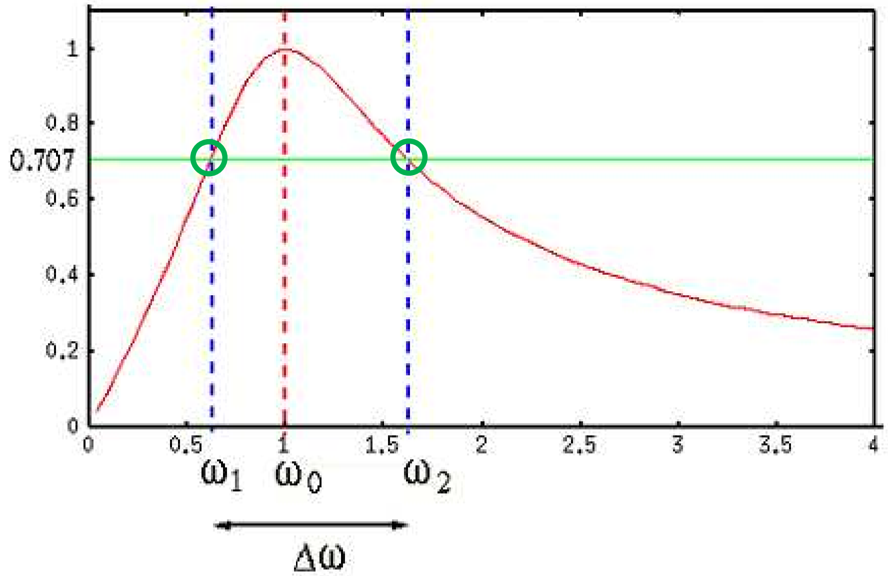
- More convenient formulas



$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$Q = \frac{2\pi f_0 L}{r}$$

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{Q} \Rightarrow \frac{\Delta f}{f_0} = \frac{1}{Q}$$

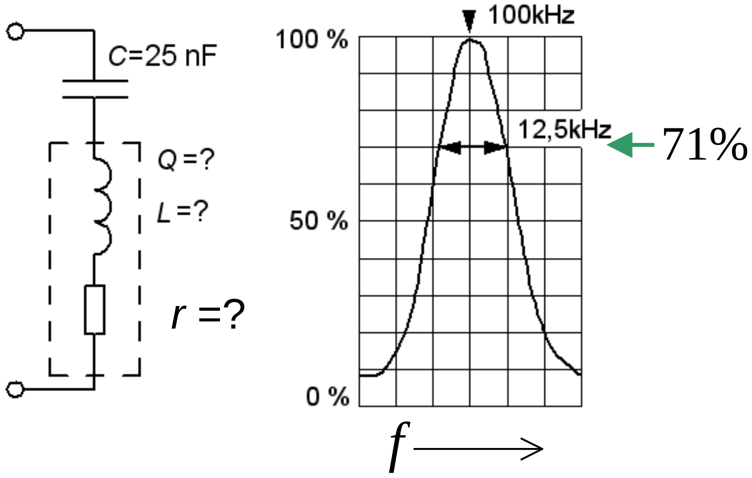
If Q is high, no significant error is done if the bandwidth is divided equally on both sides of f_0.

$$f_2, f_1 \approx f_0 \pm \frac{\Delta f}{2}$$

Example, series resonance circuit

C = 25 nF
f_0 = 100 kHz
$BW = \Delta f$ = 12,5 kHz

Q = ? L = ? r = ?



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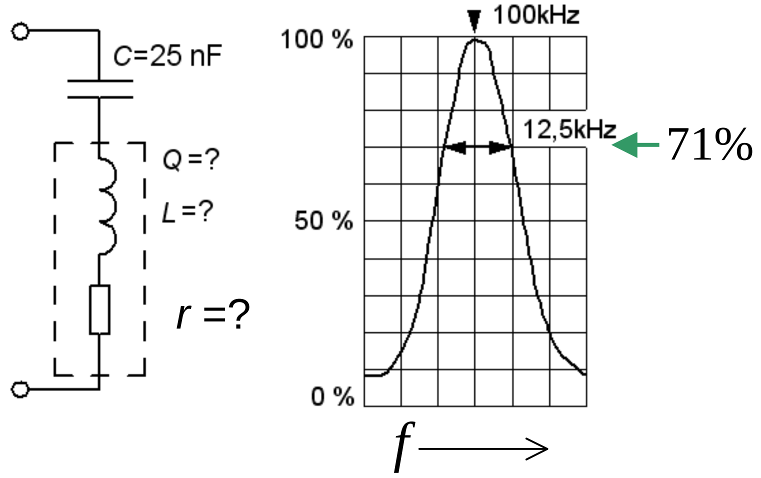
$$Q = \frac{f_0}{\Delta f} = \frac{100}{12{,}5} = 8$$



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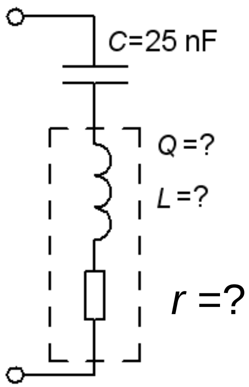

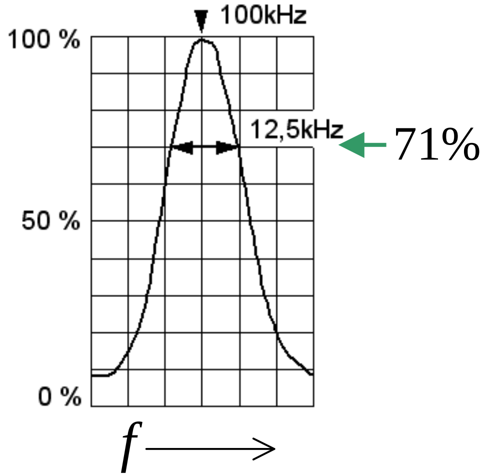

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \Rightarrow \quad L = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(2\pi \cdot 100 \cdot 10^3)^2 \cdot 25 \cdot 10^{-9}} = 0{,}1\,\text{mH}$$

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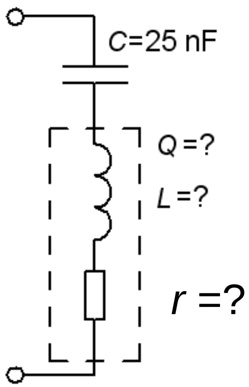

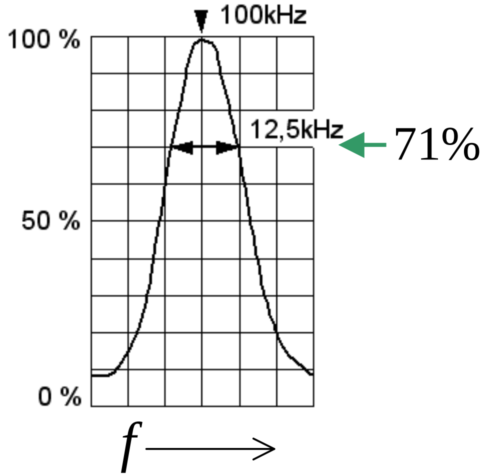

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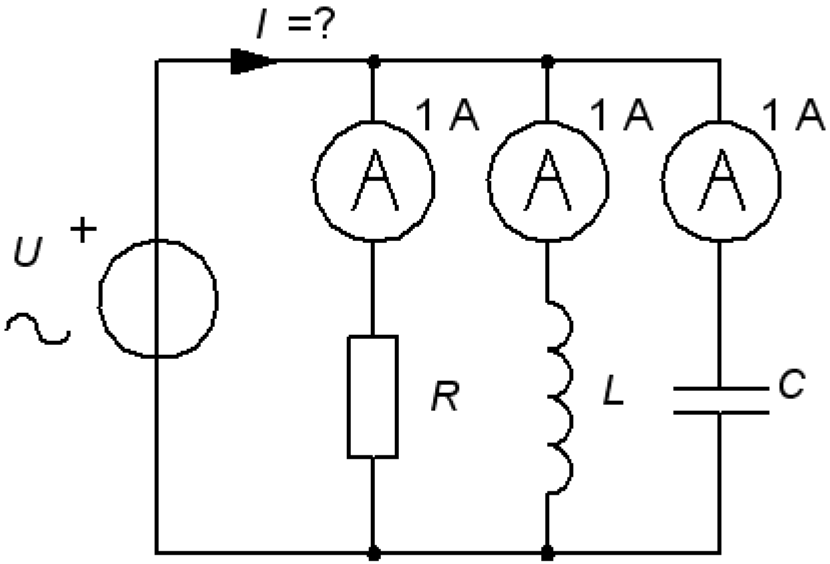
$$Q = \frac{X_L}{r} = \frac{2\pi f_0 \cdot L}{r} \quad \Rightarrow \quad r = \frac{2\pi f_0 \cdot L}{Q} = \frac{2\pi \cdot 100 \cdot 10^3 \cdot 0{,}1 \cdot 10^{-3}}{8} \approx 8\ \Omega$$

How big is *I* ? (13.2)

The three ammeters show the same, 1A, how much is the AC supply current *I* ? (*Warning*, *teaser*)



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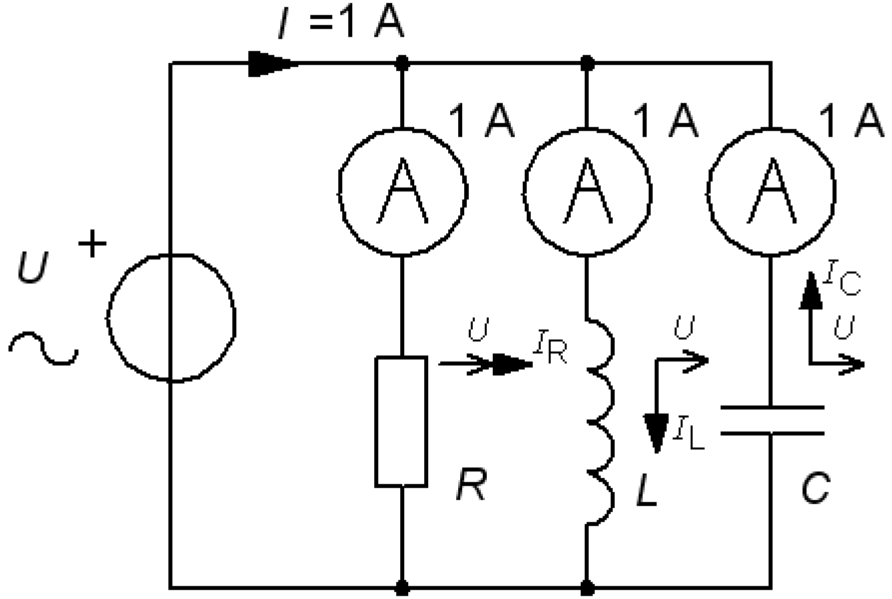

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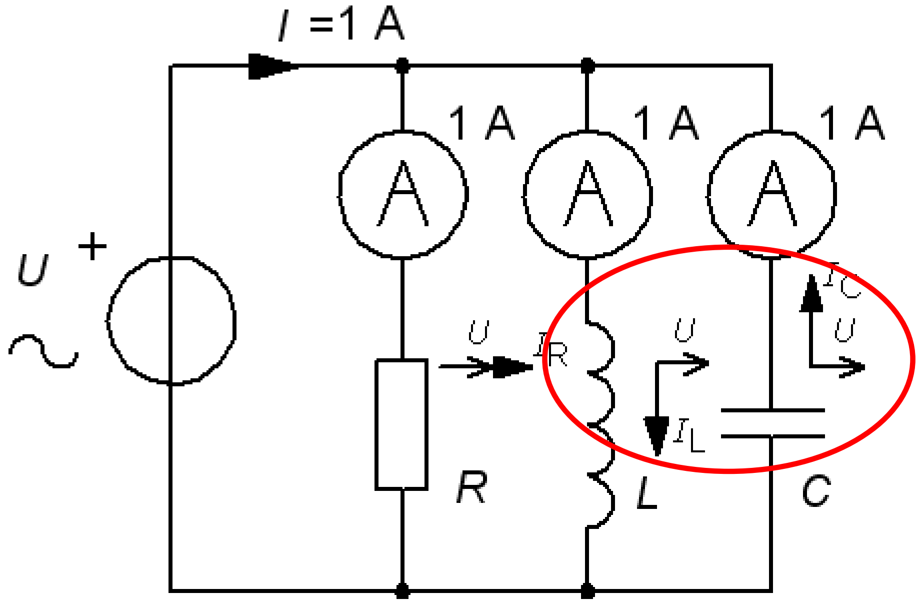
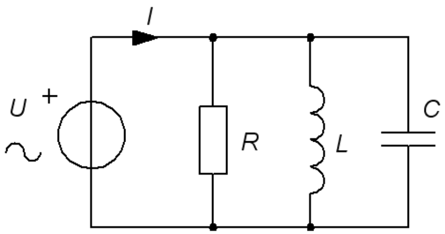


I_L and I_C becomes a circulating current decoupled from I_R. I_L, I_C can be *many times bigger* than the supply current $I = I_R$. This is parallel resonance.

Ideal parallel resonance circuit

$$\underline{Z} = R \parallel L \parallel C = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C} = \frac{1}{\frac{1}{R} + j\left(\boxed{\omega C - \frac{1}{\omega L}}\right)}$$

=0



The resonance frequency has exactly the same expression as for the series resonant circuit, but otherwise the circuit has **reverse character**, **IND** at low frequencies and **CAP** at high. At resonance, the impedance is real = *R*.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

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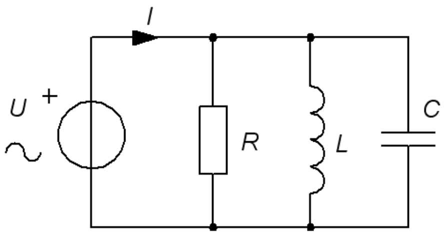
=0



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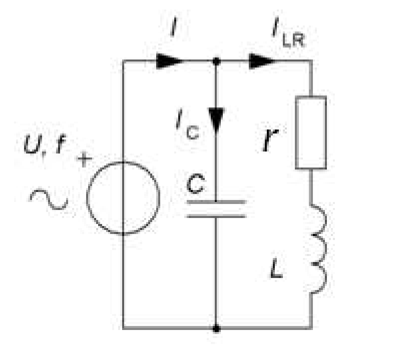
Actual parallel resonant circuit

Actual parallel resonant circuits has a series resistance inside the coil. The calculations become more complecated and the resonance frequency will also differ slightly from our formula.



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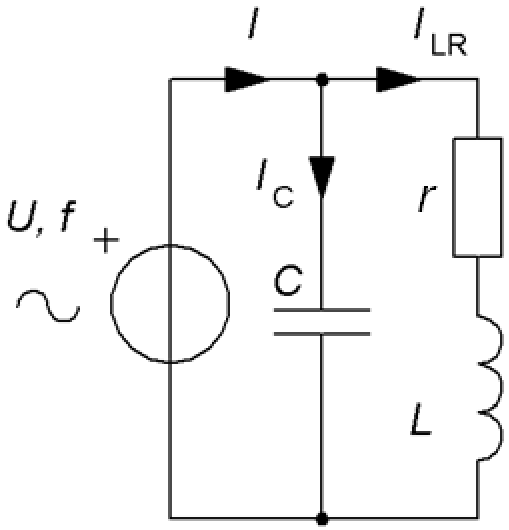
Example, actual circuit (13.3)

$$\underline{I} = \underline{I}_{\mathrm{C}} + \underline{I}_{\mathrm{LR}} = \frac{U}{\frac{1}{\mathrm{j}\omega C}} + \frac{U}{r + \mathrm{j}\omega L} \cdot \frac{(r - \mathrm{j}\omega L)}{(r - \mathrm{j}\omega L)} = U \cdot \left(\mathrm{j}\omega C + \frac{r - \mathrm{j}\omega L}{r^2 + (\omega L)^2} \right) =$$

$$= U \cdot \left(\frac{r}{r^2 + (\omega L)^2} + \mathrm{j}\left(\omega C - \frac{\omega L}{r^2 + (\omega L)^2} \right) \right)$$

=0

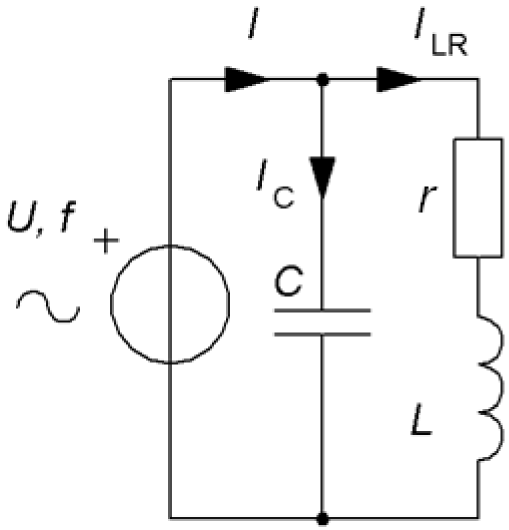


Example, actual circuit (13.3)

$$\underline{I} = \underline{I}_{C} + \underline{I}_{LR} = \frac{U}{\frac{1}{j\omega C}} + \frac{U}{r + j\omega L} \cdot \frac{(r - j\omega L)}{(r - j\omega L)} = U \cdot \left(j\omega C + \frac{r - j\omega L}{r^2 + (\omega L)^2} \right) =$$

$$= U \cdot \left(\frac{r}{r^2 + (\omega L)^2} + j\left(\omega C - \frac{\omega L}{r^2 + (\omega L)^2} \right) \right)$$

=0



$$\omega_0 C = \frac{\omega_0 L}{r^2 + (\omega_0 L)^2} \quad \Rightarrow \quad \omega_0^{\,2} = \frac{1}{LC} - \frac{r^2}{L^2} \quad \omega_0 = 2\pi f \quad \Rightarrow \quad f_0 = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{r^2}{L^2} \right)}$$

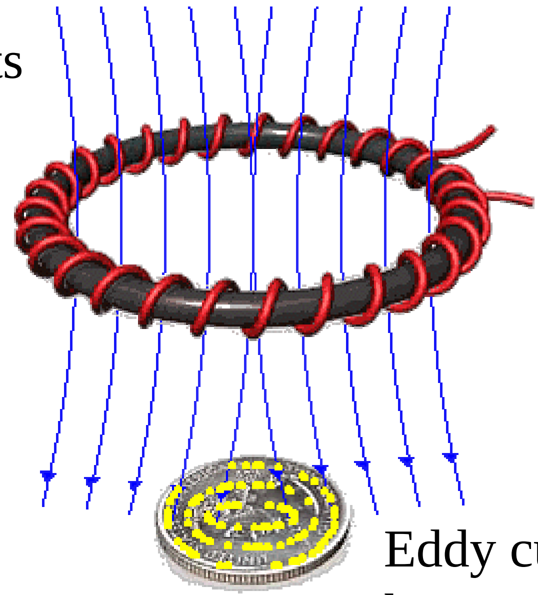
Metal Detector

$$f_0 = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{r^2}{L^2}\right)}$$

Any "losses" (even eddy-current losses in all kinds of metals) are summarized by the symbol r !

Iron objects affects the magnetic field and thus also L !

The parallel resonant frequency is affected by the coil losses. That’s how hidden treasures are found!



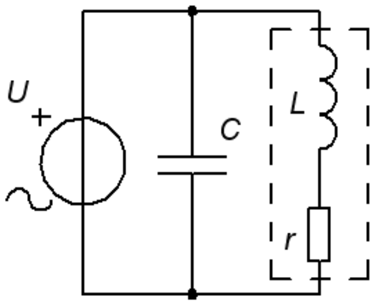
Eddy current losses

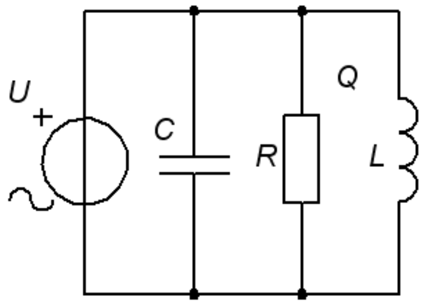
Series or parallel resistor

In manual computation for simplicity one usually uses the formulas of the ideal resonant circuit. At high Q and close to the resonance frequency f_0 the deviations becomes insignificant.

At **Q >10** are the two circuits "interchangeable".

$$\omega_0 \approx \frac{1}{\sqrt{LC}}$$





Alternative definition of Q with R_P

$$Q = \frac{\omega_0 L}{r_S} = \frac{R_P}{\omega_0 L} \Rightarrow R_P = Q^2 \cdot r_S$$

(*applies approximately for* ***Q >10***)

Example, parallel circuit

Parallel circuit.

C = 25 nF

f_0 = 100 kHz

BW = 1250 Hz

L = ? r = ?

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$$Q = \frac{f_0}{\Delta f} = \frac{100 \cdot 10^3}{1250} = 80$$

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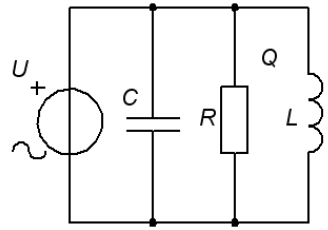

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80 > 10 justifying counting with the ideal model.

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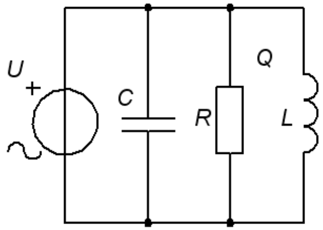

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$$Q = \frac{f_0}{\Delta f} = \frac{100 \cdot 10^3}{1250} = 80$$

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$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \Rightarrow \quad L = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(2\pi \cdot 100 \cdot 10^3)^2 \cdot 25 \cdot 10^{-9}} = 0{,}1\,\text{mH}$$

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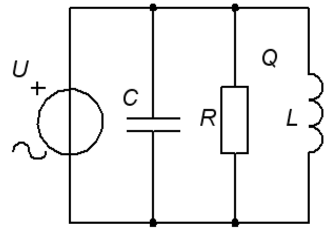

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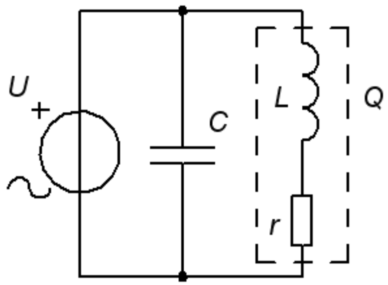
$$Q = \frac{R_\text{P}}{X_\text{L}} = \frac{R_\text{P}}{2\pi f_0 \cdot L} \quad \Rightarrow \quad R_\text{P} = 2\pi f_0 \cdot L \cdot Q = 2\pi \cdot 100 \cdot 10^3 \cdot 0{,}1 \cdot 10^{-3} \cdot 80 \approx 5027\ \Omega$$

Example, parallel circuit

Parallel circuit.
C = 25 nF
f_0 = 100 kHz
BW = 1250 Hz

Answer with a series resistor!



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Example, parallel circuit

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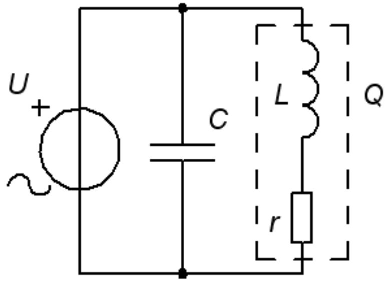
Sanswer with a series resistor!



$L = ?\quad r = ?$

$$Q = \frac{f_0}{\Delta f} = \frac{100 \cdot 10^3}{1250} = 80$$

80 > 10 justifying counting with the ideal model.

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}} \Rightarrow L = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(2\pi \cdot 100 \cdot 10^3)^2 \cdot 25 \cdot 10^{-9}} = 0{,}1\,\text{mH}$$

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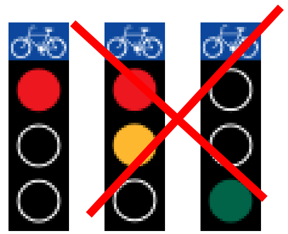
$$r_S = \frac{1}{Q^2} R_P = \frac{1}{80^2} 5027 \approx 0{,}8\ \Omega$$

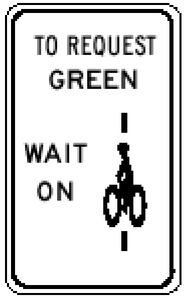
Luckily we did not have to use this formula to calculate the L

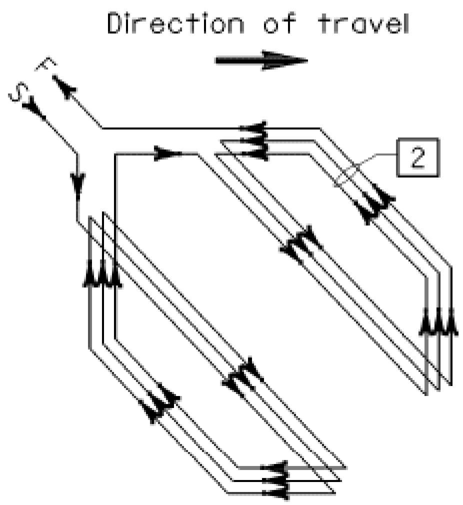
$$f_0 = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{r^2}{L^2}\right)}$$

The inductive sensor is a rugged sensor type available in many types.

Cyclists who request green?







Inductive sensor for bicycle

*Sorry! The Sensor does not work for **all** bicycles?*

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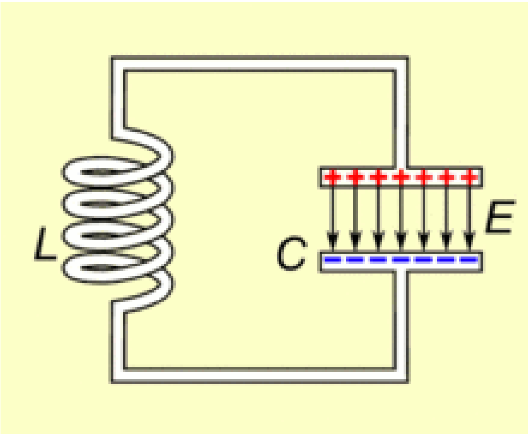

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