IE1206 Embedded Electronics

Le1 → Le2   PIC-block Documentation, Seriecom Pulse sensors
I, U, R, P, serial and parallel

Le3 → Ex1 → KC1 LAB1   *Pulse sensors, Menu program*
● **Start of programing task**

Le4 → Ex2   Kirchhoffs laws Node analysis Two-terminals R2R AD

Le5 → Ex3 → KC2 LAB2   *Two-terminals, AD, Comparator/Schmitt*

Le6 → Ex4 → Le7 → KC3 LAB3   Transients PWM
Step-up, RC-oscillator

Le8 → Ex5 → Le9   Phasor jω PWM CCP CAP/IND-sensor

Ex6 → Le10 → Le11 → KC4 LAB4   *LC-osc, DC-motor, CCP PWM*

Le12 → Ex7 → Display   LP-filter Trafo
● **Display of programing task**

Le13 → Written exam   Trafo, Ethernet contact

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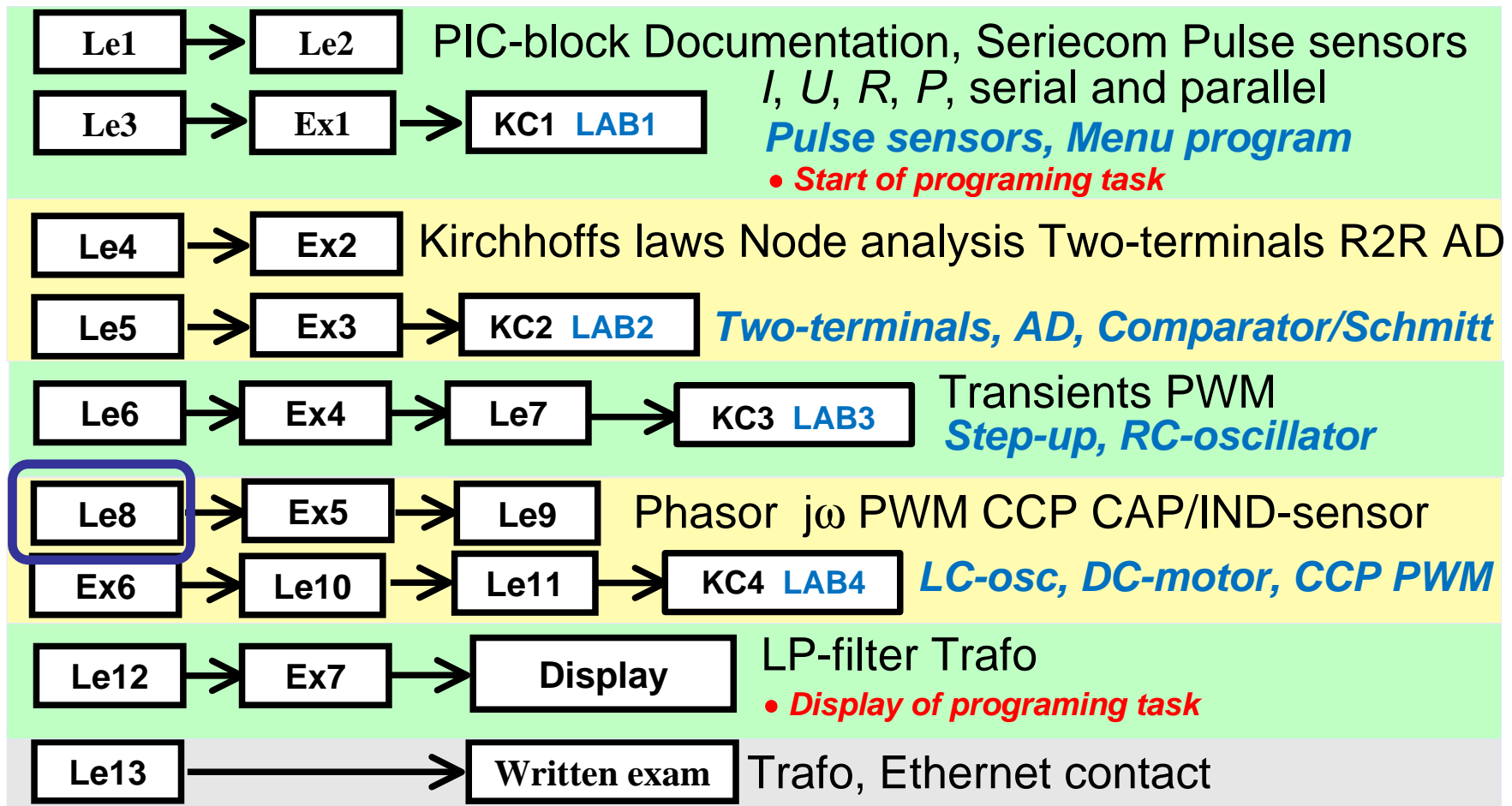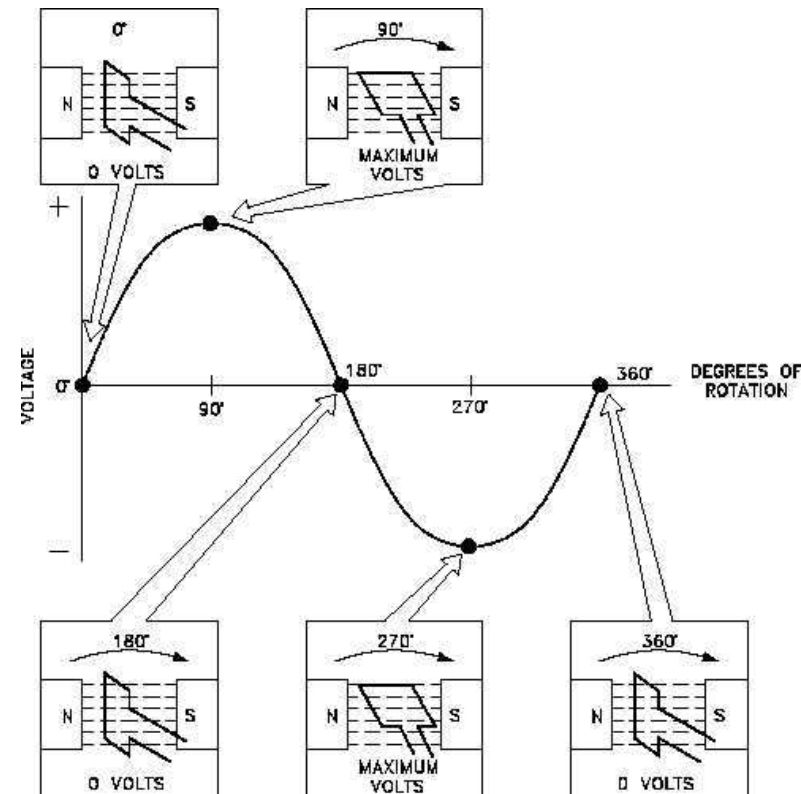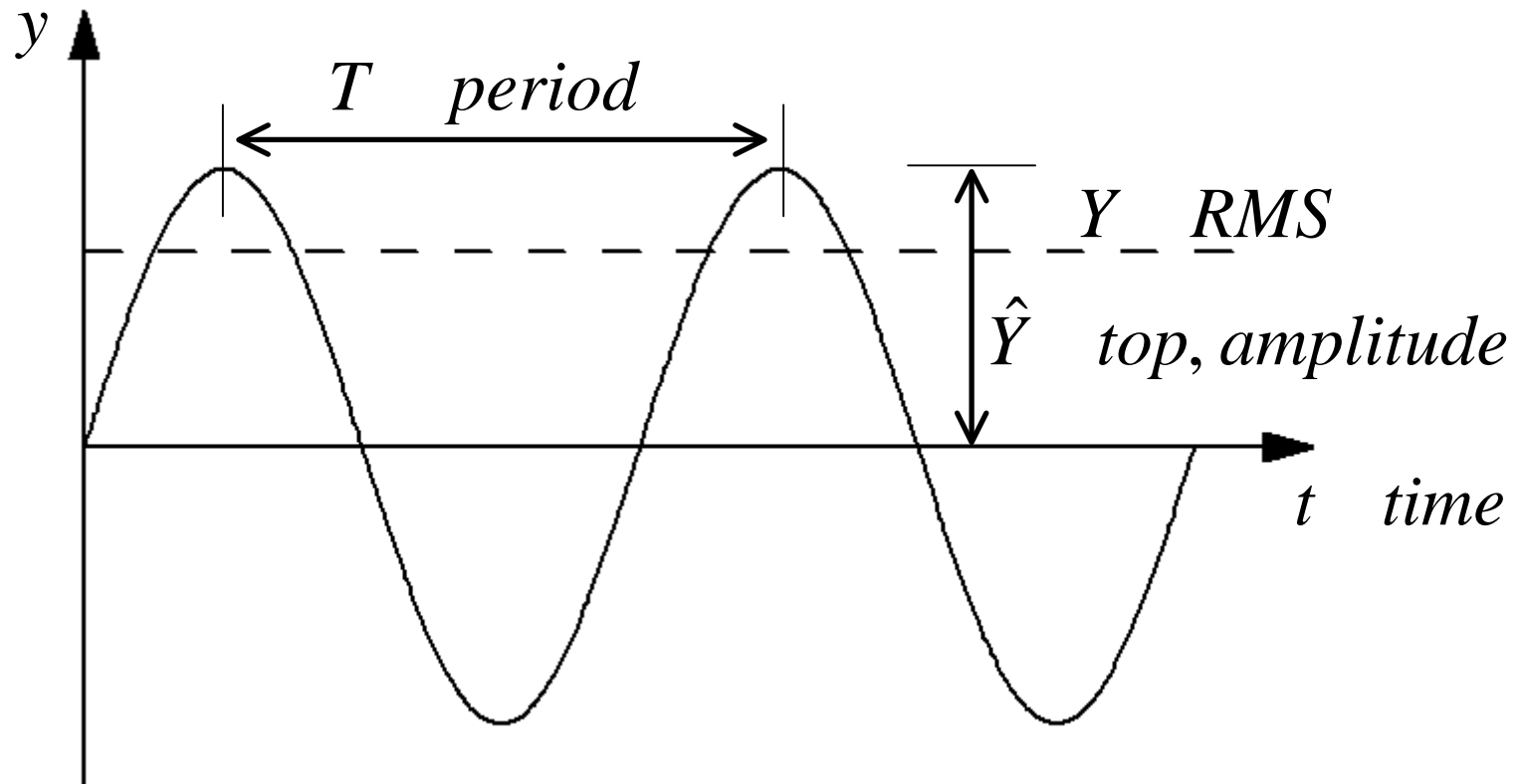
Easy to generate a sinusoidal voltage

Our entire power grid works with sinusoidal voltage.

When the loop rotates with constant speed in a magnetic field a sine wave is generated.

So much easier, it can not be …
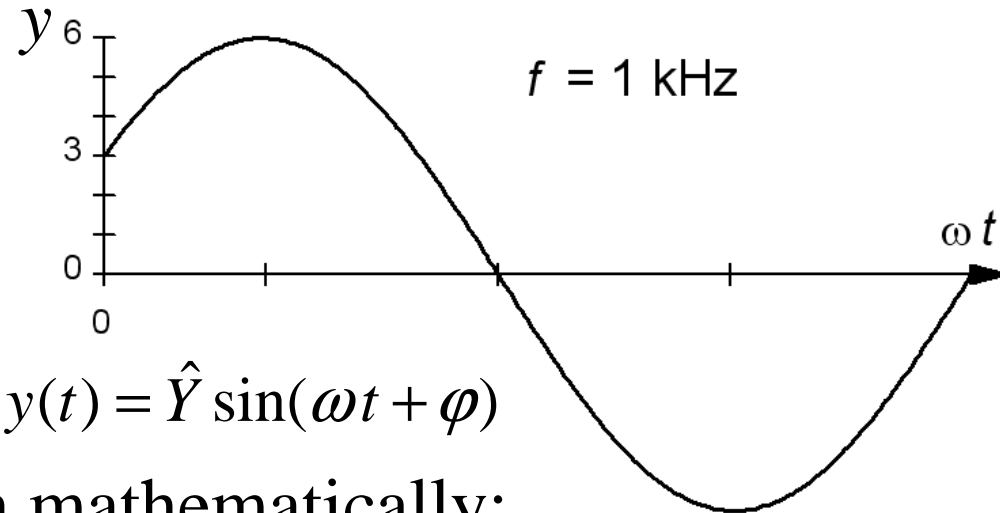

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The sine wave – what do you remember?

$$y(t) = \hat{Y}\sin(\omega t) \quad \omega = 2\pi f \quad f = \frac{1}{T} \quad Y = \frac{\hat{Y}}{\sqrt{2}}$$

(11.1) Phase φ

If a sine curve does not begin with 0 the function expression has a phase angle φ.

$f = 1$ kHz

$$y(t) = \hat{Y}\sin(\omega t + \varphi)$$

Specify this function mathematically:

$$u(t) = 6 \cdot \sin(2\pi \cdot 1000 \cdot t + \varphi)$$

$$u(0) = 3 = 6 \cdot \sin(\varphi) \quad \Rightarrow \quad \varphi = \arcsin\left(\frac{3}{6}\right) = 0{,}52 \text{ rad} \quad (= 30°)$$

$$u(t) = 6 \cdot \sin(6283 \cdot t + 0{,}52)$$

Apples and pears?

In circuit analyses it is common (eg. in textbooks) to expresses the angle of the sine function mixed in radians ω·t [rad] and in degrees φ [°].

This is obviously improper, but practical (!). The user must "convert" phase angle to radians to calculate the sine function value for any given time t.

(You have now been warned …)

$$u(t) = 6 \cdot \sin(6283 \cdot t + 30°)$$

?↑   ?↑

Conversion:
$x[°] = x[\text{rad}] \cdot 57,3$
$x[\text{rad}] = x[°] \cdot 0,017$

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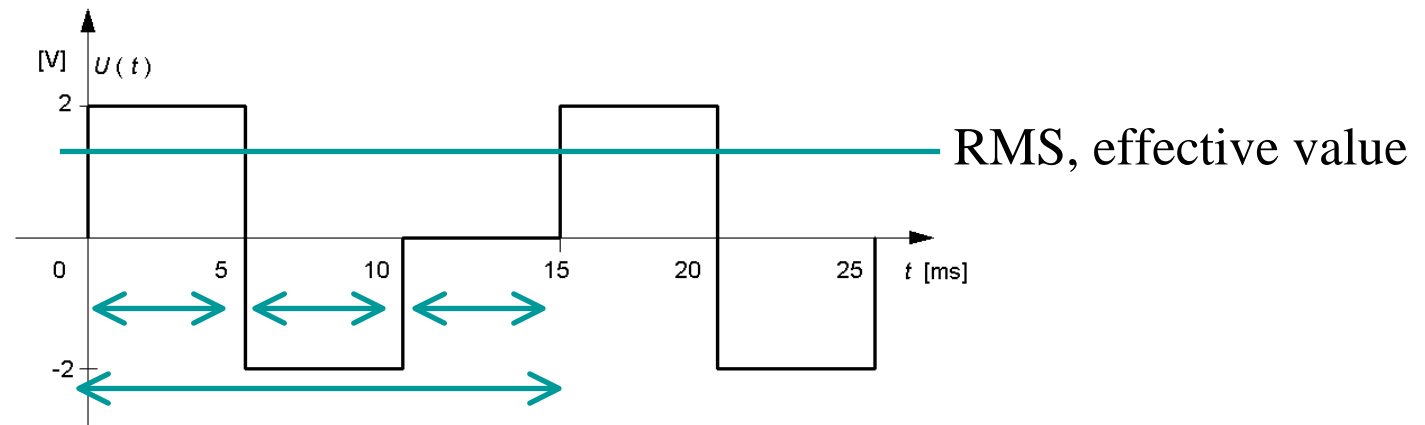# Mean and effective value

All pure AC voltages, has the **mean value** 0.

• More interesting is the **effective value** – root mean square, rms.

$$U_{\text{med}} = \tfrac{1}{T}\int_0^T u(t)\,\mathrm{d}t = \boxed{0}$$

$$U = \sqrt{\dfrac{\displaystyle\int_0^T u(t)^2\,\mathrm{d}t}{T}}$$

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(11.2) Example. RMS.

The rms value is what is normally used for an alternating voltage **U**. 1,63 V effective value gives the *same* power in a resistor as a 1,63 V pure DC voltage would do.

RMS, effective value

$$U = \sqrt{\frac{\int_0^T u(t)^2 \, \mathrm{d}t}{T}} = \sqrt{\frac{(2^2 + (-2)^2 + 0) \cdot 5 \cdot 10^{-3}}{15 \cdot 10^{-3}}} = \sqrt{\frac{8 \cdot 5 \cdot 10^{-3}}{15 \cdot 10^{-3}}} = 1{,}63 \text{ V}$$
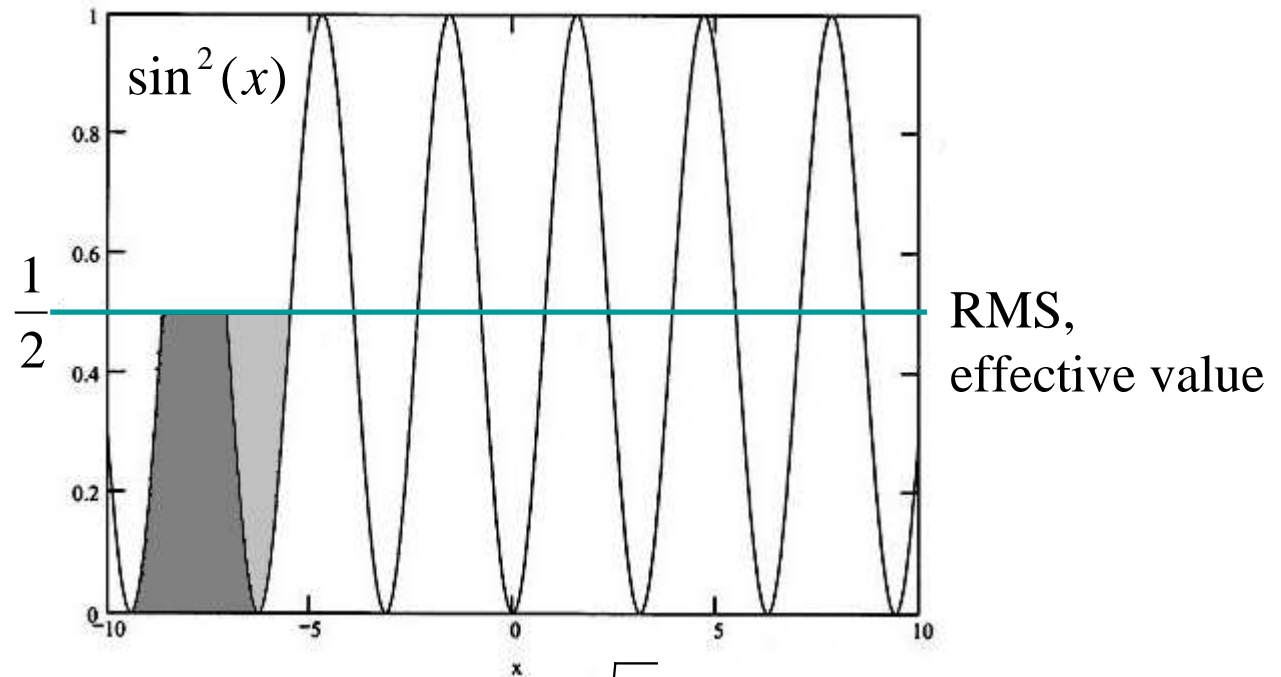
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Sine wave effective value

Ex. 11.3

sin² has the
mean value **½**

Therefore:

$$U = \frac{\hat{U}}{\sqrt{2}}$$

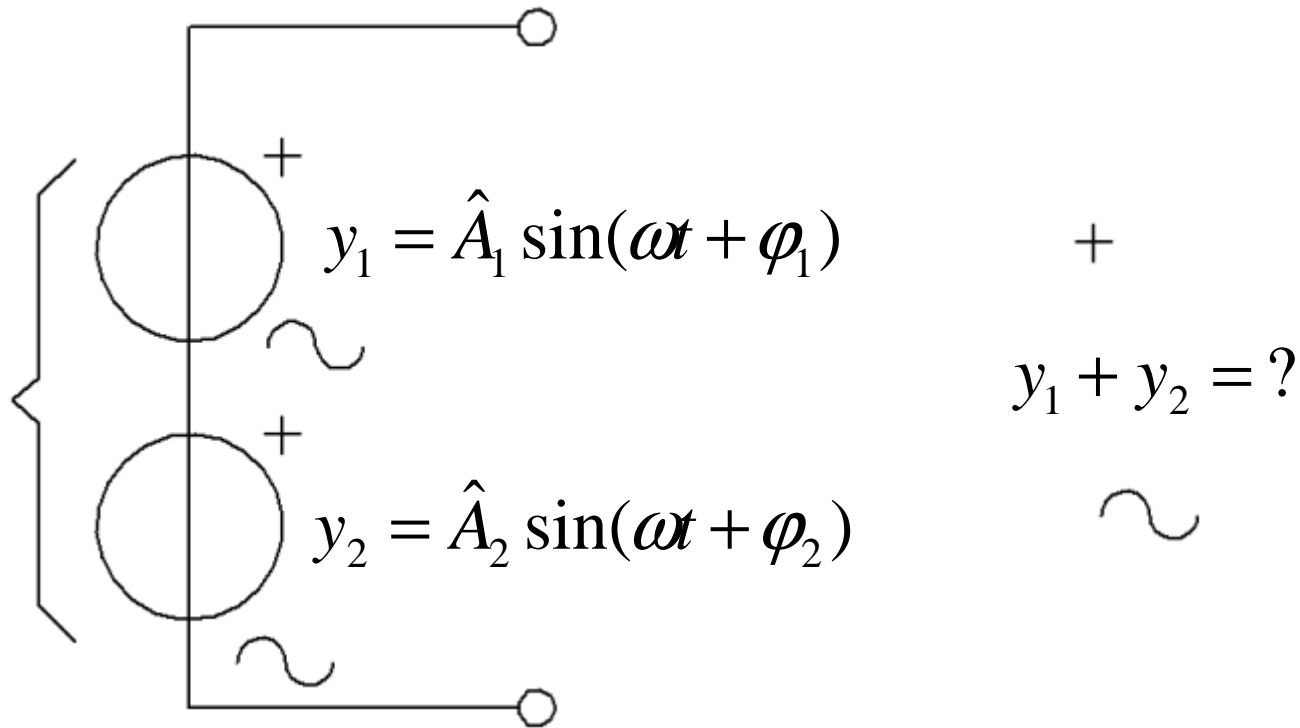$\sin^2(x)$

$\dfrac{1}{2}$

RMS,
effective value

$$\sqrt{\int \sin^2(x)\, dx} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

- Effective value is often called **RMS** (**R**oot **M**ean **S**quare).

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Addition of sinusoidal quantities

$$y_1 = \hat{A}_1 \sin(\omega t + \varphi_1)$$

$$y_2 = \hat{A}_2 \sin(\omega t + \varphi_2)$$

$$y_1 + y_2 = ?$$

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Addition of sinusoidal quantities

When we shall apply the circuit laws on AC circuits, we must add the sines. The sum of two sinusoidal quantities of the same frequency is always a new sine of this frequency, but with a new amplitude and a new phase angle.

(Ooops! The result of the rather laborious calculations are shown below).

$$y_1(t) = \hat{A}_1 \cdot \sin(\omega t + \varphi_1) \qquad y_2(t) = \hat{A}_2 \cdot \sin(\omega t + \varphi_2) \qquad y(t) = y_1(t) + y_2(t) =$$

$$= \sqrt{\hat{A}_1^2 + \hat{A}_2^2 + 2\hat{A}_1\hat{A}_2 \cos(\varphi_1 - \varphi_2)} \cdot \sin\left(\omega t + \arctan \frac{\hat{A}_1 \sin(\varphi_1) + \hat{A}_2 \sin(\varphi_2)}{\hat{A}_1 \cos(\varphi_1) + \hat{A}_2 \cos(\varphi_2)} \right)$$

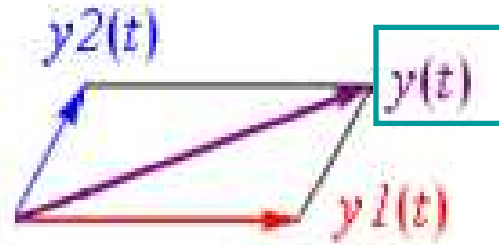# Sine wave as a pointer

A sinusoidal voltage or current,

$$y(t) = \hat{Y} \cdot \sin(\omega \cdot t)$$

can be represented by a pointer that rotates (counterclockwise) with the angular velocity ω [rad/sec] .

[Wikipedia Phasors](#)

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Simpler with vectors

If you ignore the "revolution" and adds the pointers with vector addition, as they stand at the time $t = 0$, it then becomes a whole lot easier!

[Wikipedia Phasors](http://en.wikipedia.org/wiki/Phasors)

`http://en.wikipedia.org/wiki/Phaso`

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Pointer with complex numbers

A AC voltage 10 V that has the phase 30° is usually written:

$10 \angle 30°$ (**Phasor**)

Once the vector additions require more than the most common geometrical formulas, it is instead preferable to represent pointers with complex numbers.

Imaginary axis

Real axis

$$\underline{z} = a + jb$$
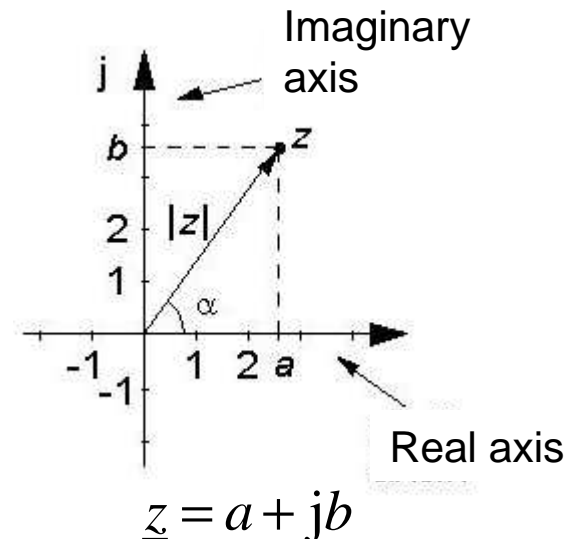
$$10\angle 30° \iff 10 \cdot e^{j30°} \iff 10 \cdot \cos 30° + 10 \cdot j \cdot \sin 30°$$

In electricity one uses **j** as imaginary unit, as **i** is already in use for current.

Phasor

Sinusoidal alternating quantities can be represented as pointers, phasors.

$$\boxed{\text{"amount"} \angle \text{"phase"}}$$

A pointer (phasor) can either be viewed as a vector expressed in polar coordinates, or as a complex number.

It is important to be able to describe alternating current phenomena without necessarily having to require that the audience has a knowledge of complex numbers - hence the vector method.

The complex numbers and $j\omega$-method are powerful tools that facilitate the processing of AC problems. They can be generalized to the Fourier transform and Laplace transform, so the electro engineer's use of complex numbers is extensive.

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peak/effective value - phasor
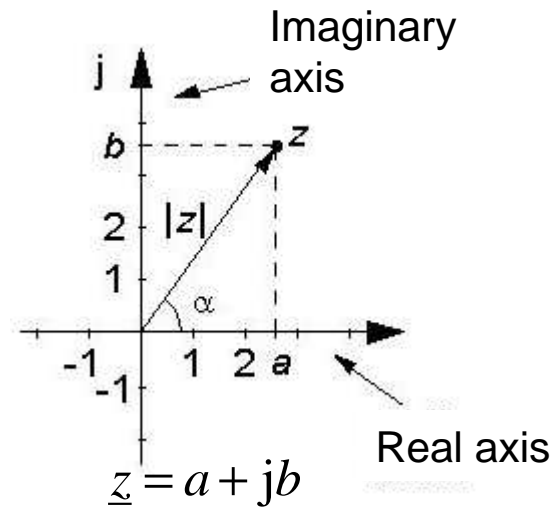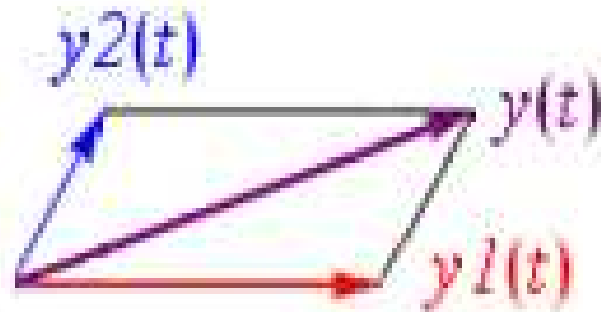
The phasor lengths corresponds to sine peak values, but since the effective value only is the peak value scaled by $1/\sqrt{2}$ so it does not matter if you count with peak values or effective values - as long as you are consistent!

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The inductor and capacitor counteracts changes

The inductor and capacitor counteracts changes, such as when connecting or disconnecting a source to a circuit.
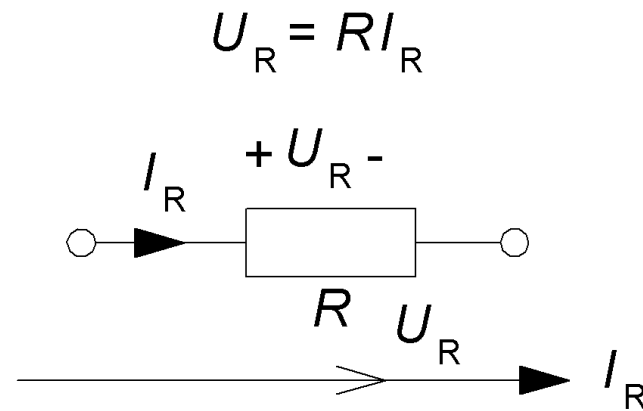
What if the source then is sinusoidal AC – which is then changing continuously?

?
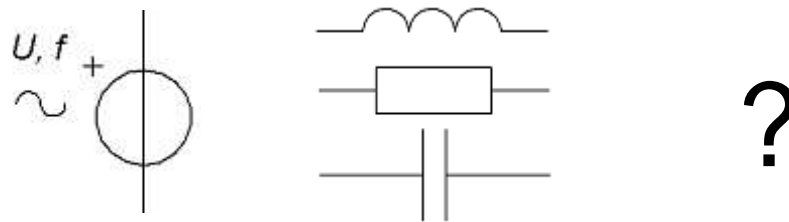
Alternating current through resistor

A sinusoidal current $i_R(t)$ through a resistor R provides a proportional sinusoidal voltage drop $u_R(t)$ according to Ohm's law. The current and voltage are in phase. No energy is stored in the resistor.

Phasors U_R and I_R become parallel to each other.

$$U_R = RI_R$$

$$i_R(t) = \hat{I}_R \cdot \sin(\omega t) \qquad u_R(t) = i_R(t) \cdot R \quad \Rightarrow \quad u_R(t) = R \cdot \hat{I}_R \cdot \sin(\omega t)$$
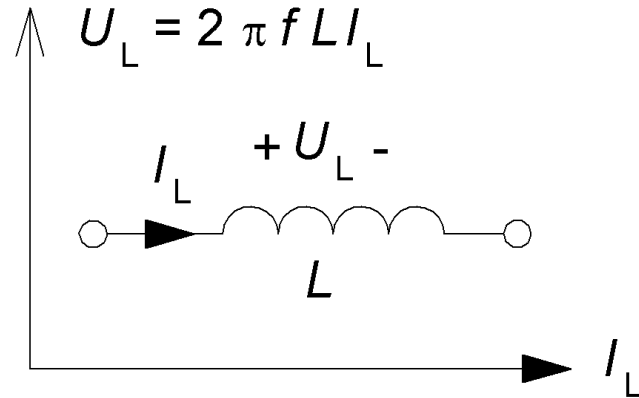
$$\boxed{U_R = R \cdot I_R}$$ • *Vector phasor*

$$\boxed{\underline{U}_R = R \cdot \underline{I}_R}$$ • *Complex phasor*

The phasor may be a peak pointer or effective value pointer as long as you do not mix different types.

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Alternating current through inductor

A sinusoidal current $i_L(t)$ through an inductor provides, due to self-induction, a votage drop $u_L(t)$ which is 90° *before* the current. Energy stored in the magnetic field is used to provide this voltage.

$U_L = 2\,\pi\,f\,L\,I_L$

I_L $+\,U_L$ -

L

I_L

The phasor U_L will be $\omega L \cdot I_L$ and it is 90° before I_L. The entity ωL is the "amount" of the inductor's AC resistance, **reactance** X_L [Ω].

$$i_L(t) = \hat{I}_L \cdot \sin(\omega t) \qquad u_L(t) = L\frac{di_L(t)}{dt} \quad \Rightarrow \quad u_L(t) = \omega L \cdot \hat{I}_L \cdot \cos(\omega t) = \omega L \cdot \hat{I}_L \cdot \sin(\omega t + \frac{\pi}{2})$$
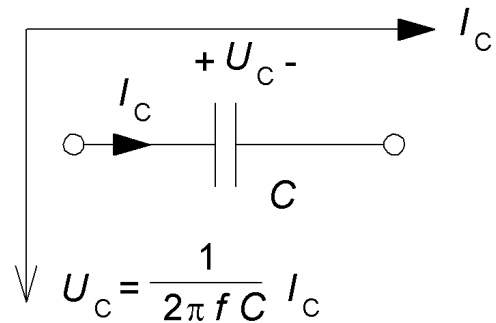
$\boxed{U_L = \omega L \cdot I_L}$ • *Vector phasor*

When using complex pointers one multiplies ωL with "j", this *rotates* the voltage pointer +90° (in complex plane). *The method automatically keeps track of the phase angles!*

$\boxed{\underline{U}_L = j\omega L \cdot \underline{I}_L = j X_L \cdot \underline{I}_L}$ • *Complex phasor*

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Alternating current through capacitor

A sinusoidal current $i_C(t)$ throug a capacitor will charge it with the "voltage drop" $u_C(t)$ that lags 90° *behind* the current. Energy is storered in the electric field.

I_C

$+ U_C -$

I_C

C

U_C $U_C = \dfrac{1}{2\pi f C} I_C$

Phasor U_C is $I_C/(\omega C)$ and it lags 90° *after I_C*. The entity $1/(\omega C)$ is the "amount" of the capacitor's AC resistance, **reactance** X_C [Ω].
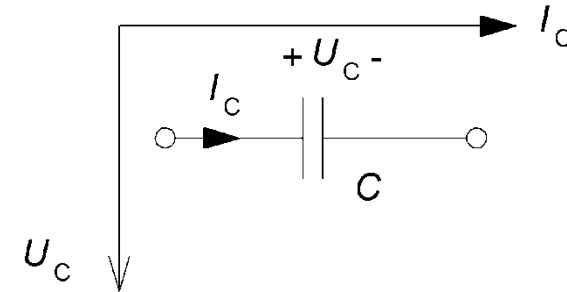
$$\boxed{U_C = \dfrac{1}{\omega C} \cdot I_C}$$

• *Vector phasor*

$$U = \frac{Q}{C} \quad \Rightarrow \quad \frac{\mathrm{d}u_C(t)}{\mathrm{d}t} = \frac{1}{C} \cdot \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{1}{C} \cdot i_C(t) \quad \Rightarrow \quad u_C(t) = \frac{1}{C} \cdot \int i_C(t)\,\mathrm{d}t$$

$$i_C(t) = \hat{I}_C \cdot \sin(\omega t) \quad \Rightarrow \quad u_C(t) = -\frac{1}{\omega C} \cdot \hat{I}_C \cdot \cos(\omega t) = \frac{1}{\omega C} \cdot \hat{I}_C \cdot \sin(\omega t - \frac{\pi}{2})$$
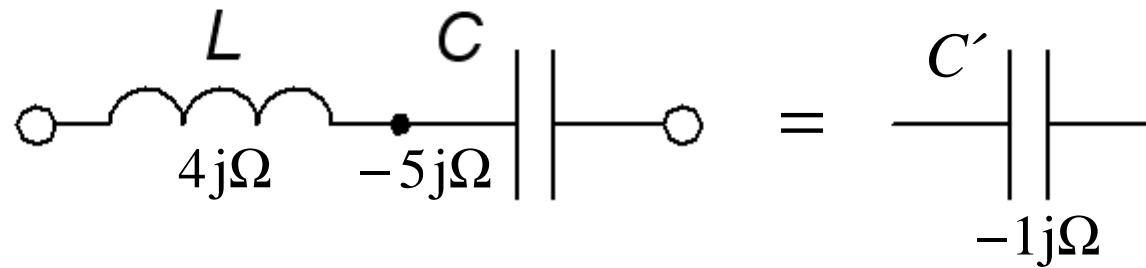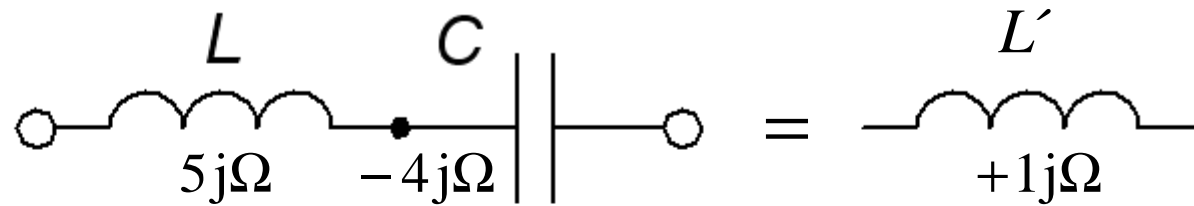
Complex phasor and the sign of reactance

If you use complex phasor you get the -90° phase by dividing $(1/\omega C)$ with "**j**".

The method with complex pointer *automatically* keeps track of the phase angles if we consider the capacitor reactance X_C as **negative**, and hence the inductor reactance X_L as **positive**.
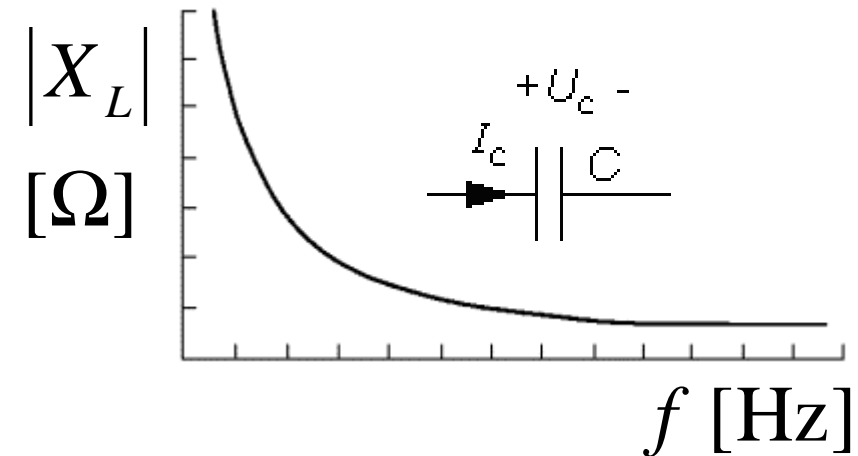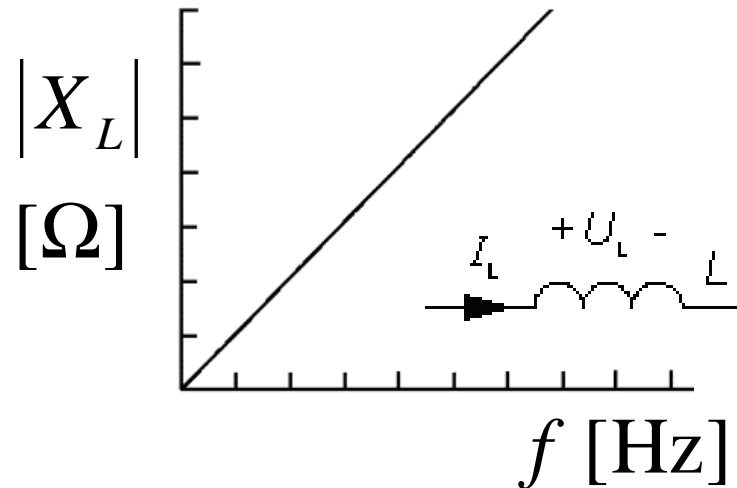
$$\underline{U}_C = \frac{1}{j\omega C} \cdot \underline{I}_C = j\frac{-1}{\omega C} \cdot \underline{I}_C \quad \Rightarrow \quad X_C = -\frac{1}{\omega C} \qquad \bullet \ Complex\ phasor$$

L+C in series

L C

$5\mathrm{j}\Omega$ $-4\mathrm{j}\Omega$ $=$ L' $+1\mathrm{j}\Omega$

L C

$4\mathrm{j}\Omega$ $-5\mathrm{j}\Omega$ $=$ C' $-1\mathrm{j}\Omega$

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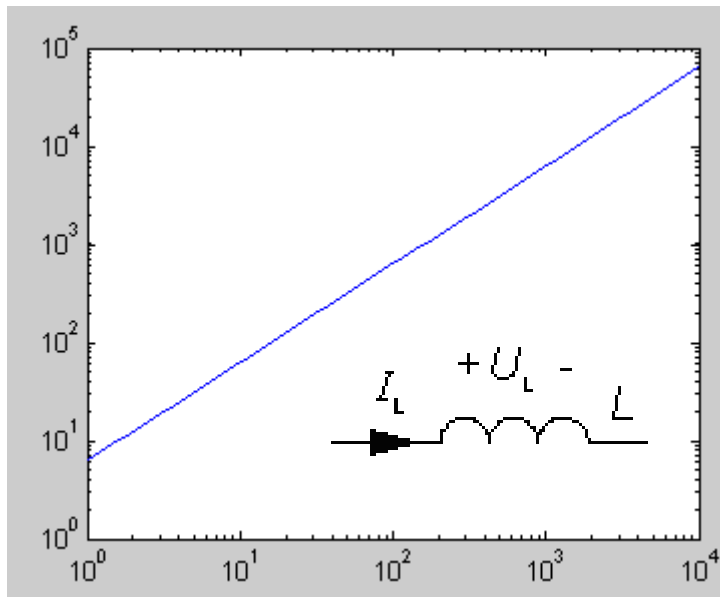# Reactance frequency dependency

$$\left|X_L\right| = \omega \cdot L \qquad \left|X_C\right| = \frac{1}{\omega \cdot C}$$
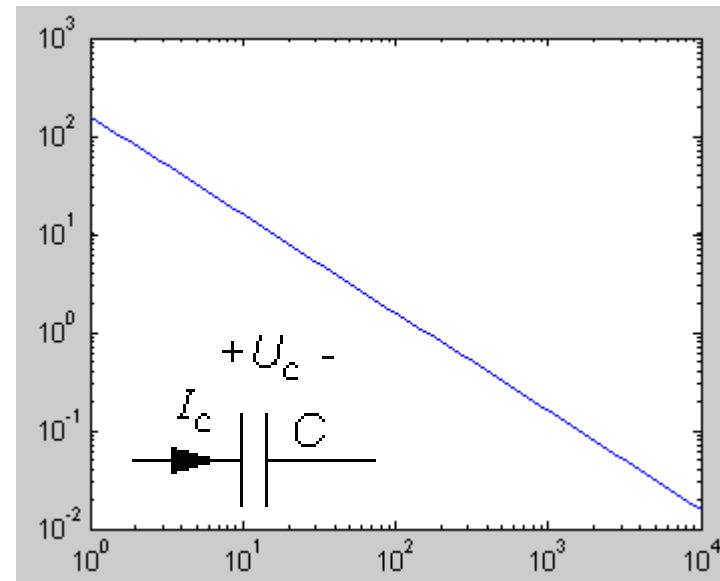
$$\omega = 2\pi\, f$$

LOG – LOG plot

$\log(|X_L|) - scale\ [\Omega]$

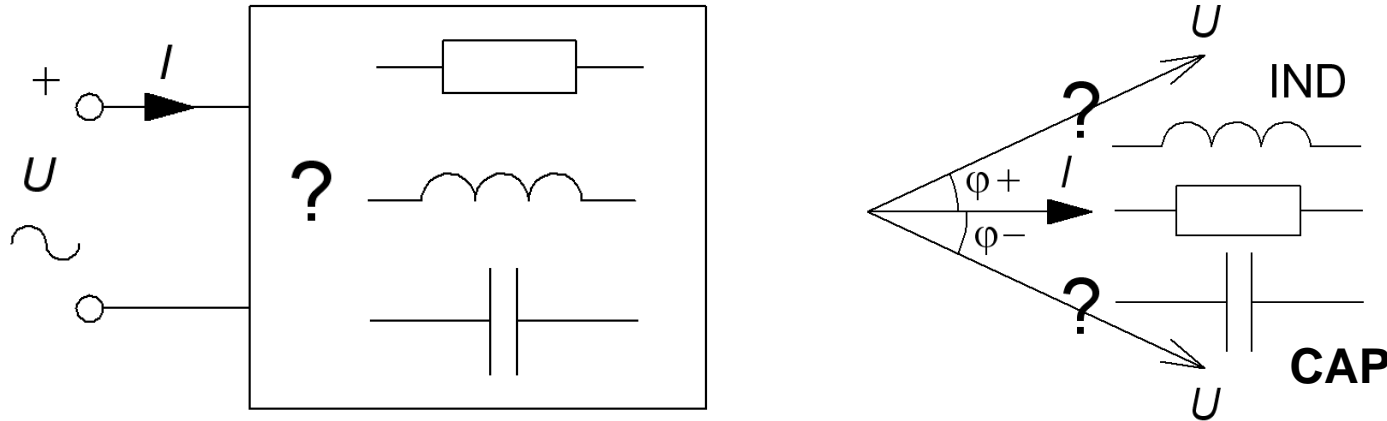$\log(f) - scale\ [\text{Hz}]$

$\log(|X_C|) - scale\ [\Omega]$

$\log(f) - scale\ [\text{Hz}]$

Often electronics engineers use log-log scale. The inductor and capacitor reactances will then both be "linear" relationship in such charts.

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R L C

In general, our circuits are a mixture of different *R L* and *C*.
The phase between *I* and *U* is then not ±90° but can have any
intermediate value. Positive phase means that the inductances
dominates over capacitances, we have inductive character **IND**.
Negative phase means that the capacitance dominates over the
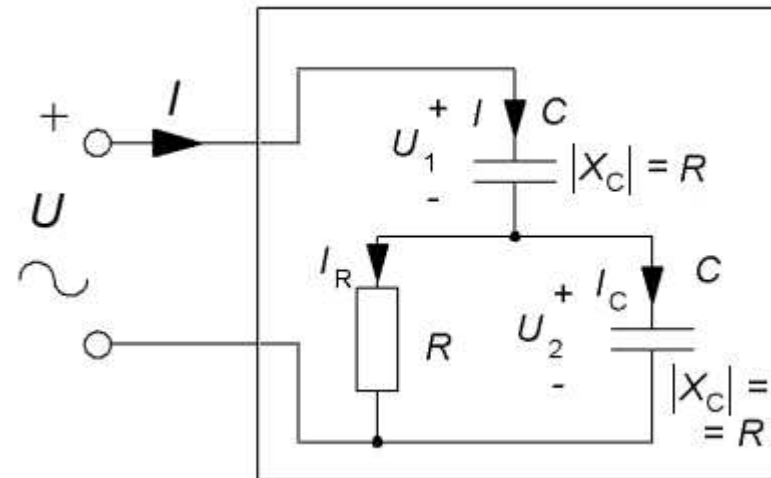inductances, we have capacitive character **CAP**.

The ratio between the voltage *U* and current *I*, the AC resistance,
is called **impedace** *Z* [Ω]. We then have **OHM′s AC law**:

$$Z = \frac{U}{I}$$

Phasor diagrams
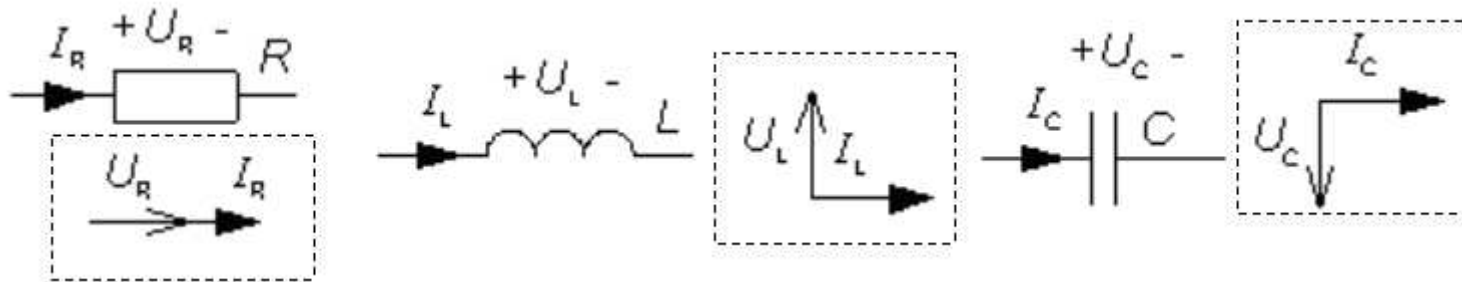
In order to calculate the AC resistance, the impedance, Z, of a composite circuit one must add currents and voltages phasors to obtain the total current I and the total voltage U.
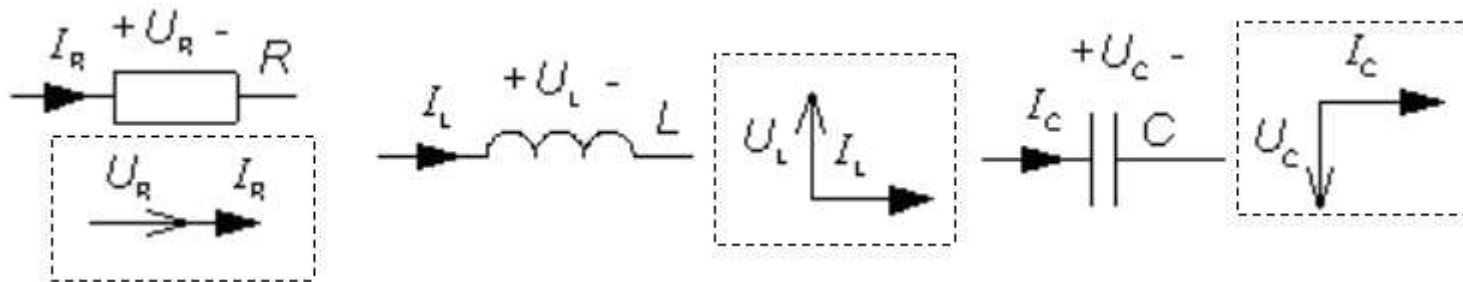
$$Z = \frac{U}{I}$$

The phasor diagram is our "blind stick" in to the AC World!

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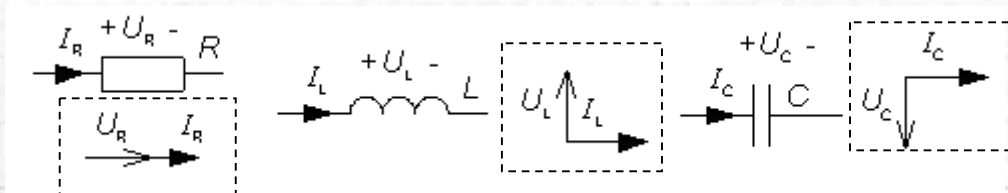
Ex. Phasor diagram (11.5)

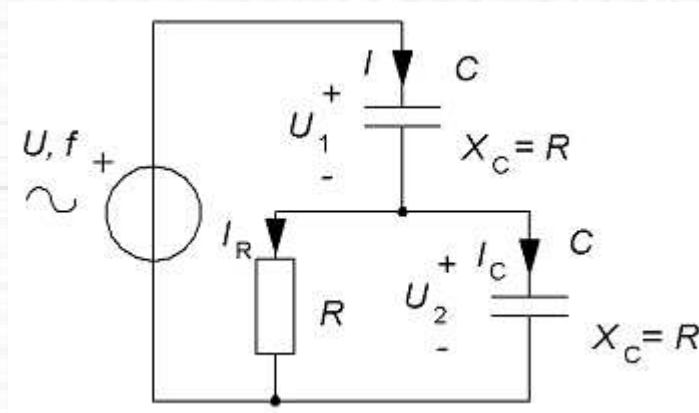Elementary diagrams for *R L* and *C*

At a certain frequency f the capacitor has the reactance $|X_C|$ and the resistor R has the same amount (absolute value), R [Ω].

Use the elementary diagrams for *R* and *C* as building blocks to draw the whole circuit phasor diagram (for this actual frequency f).

$$|X_C| = \frac{1}{2\pi f C} = R$$

$$|X_C| = \frac{1}{2\pi f C} = R$$
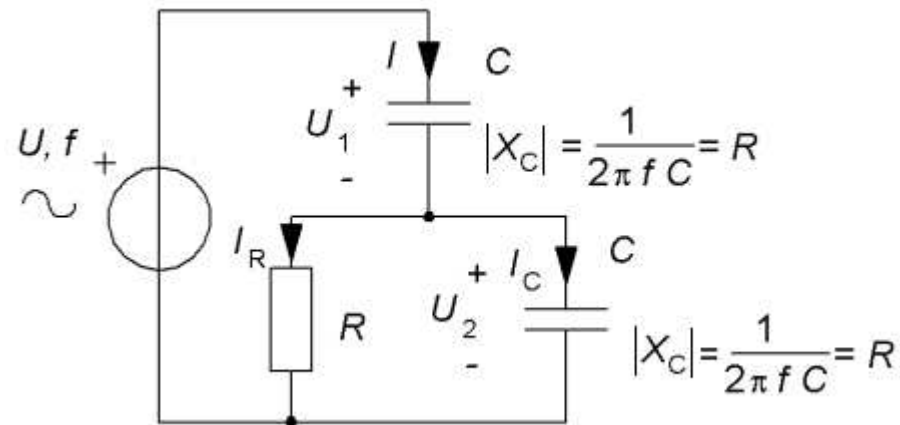
Try it your self …

Example. Phasor diagram.
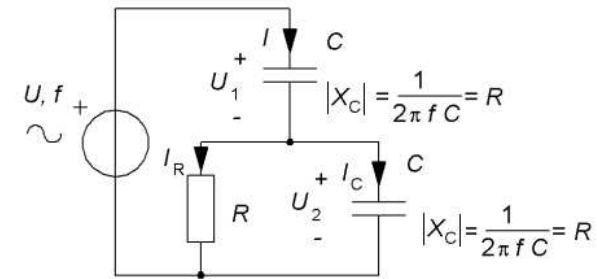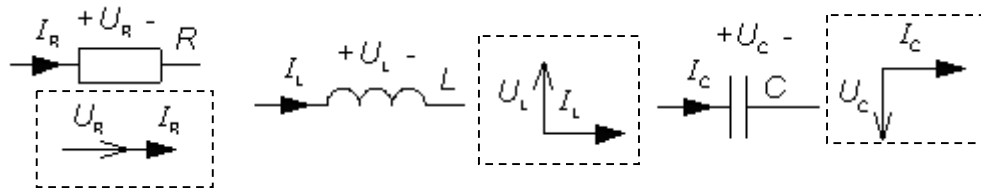
1) U_2 reference phase (= horizontal)

2) $\bar{I}_R \| \bar{U}_2$

3) $\bar{I}_C \perp \bar{U}_2$ $\quad I_C = I_R = \dfrac{U_2}{R}$

4) $\bar{I} = \bar{I}_R + \bar{I}_C$ $\quad I = \sqrt{2} \cdot I_C$

5) $\bar{U}_1 \perp \bar{I}$

$\quad U_1 = I \cdot R = I_C \cdot \sqrt{2} \cdot R \;\; \Rightarrow \;\; U_1 = \sqrt{2} \cdot U_2$

6) $\bar{U} = \bar{U}_1 + \bar{U}_2$

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$|X_C| = \dfrac{1}{2\pi f C} = R$

$|X_C| = \dfrac{1}{2\pi f C} = R$

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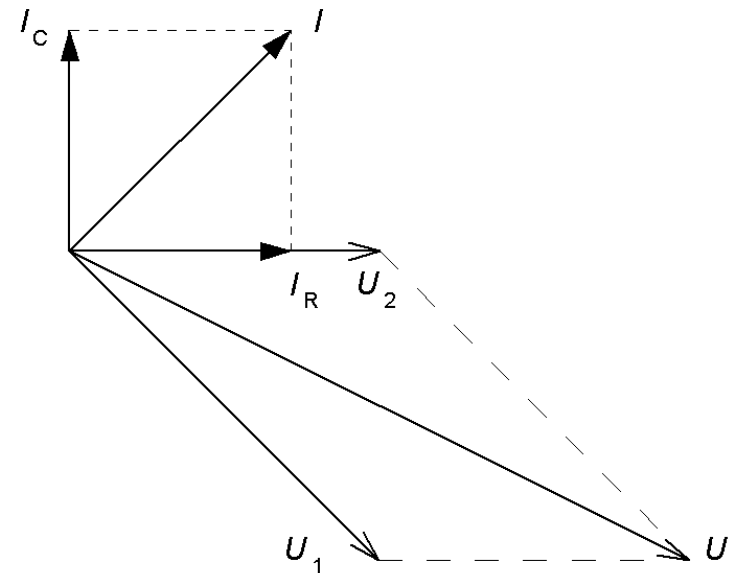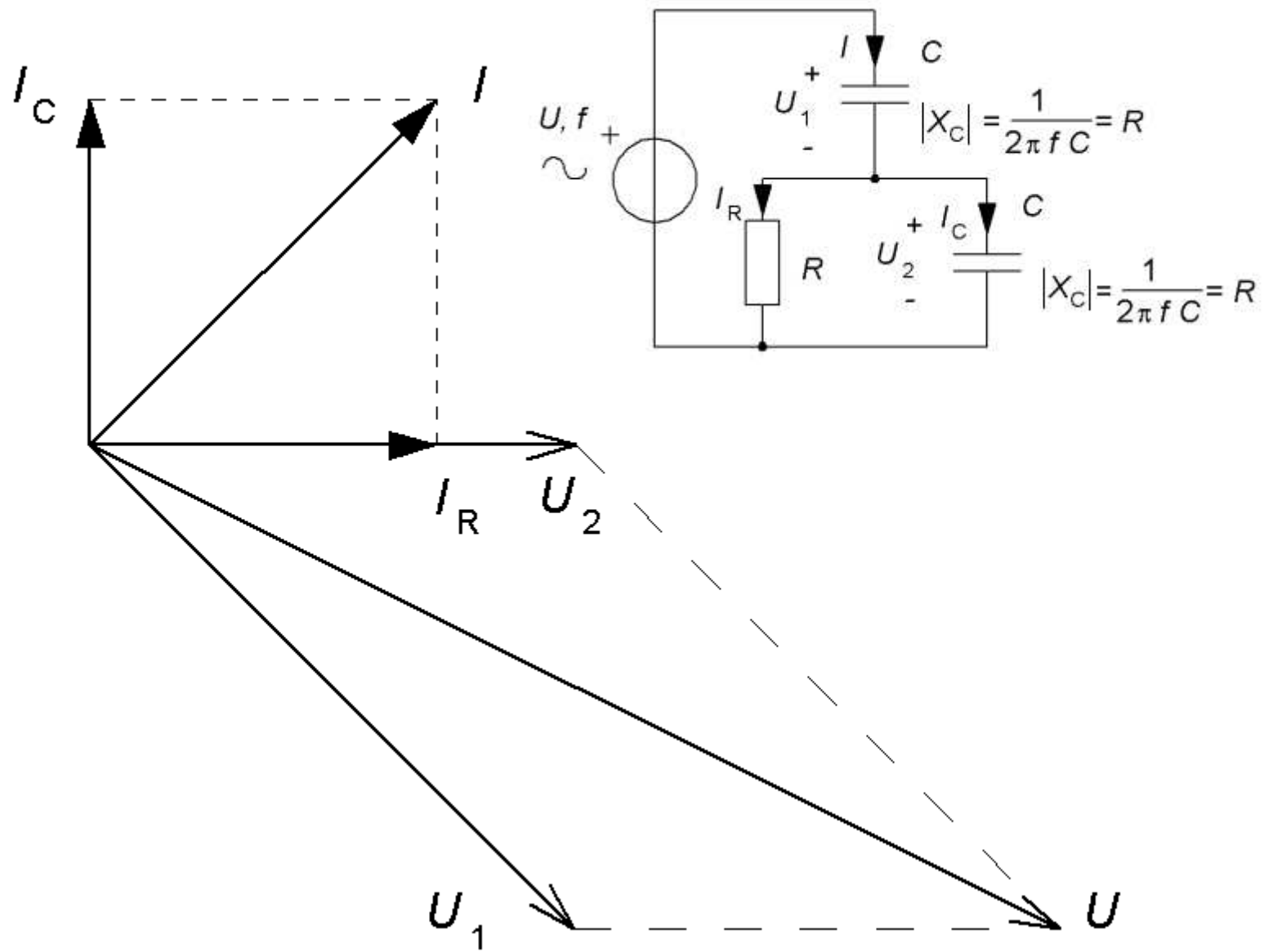
Impedance Z

The circuit AC resistance, impedance **Z**, one get as the ratio between the length of **U** and **I** phasors. The impedance phase **φ** is the angle between **U** and **I** phasors.

The current is before the voltage in phase, so the circuit has a capacitive character, **CAP**.

(*Something else had hardly been to wait since there are no coils in the circuit*)

$$|X_C| = \frac{1}{2\pi f C} = R$$

$$|X_C| = \frac{1}{2\pi f C} = R$$

$$Z = \frac{U}{I}$$

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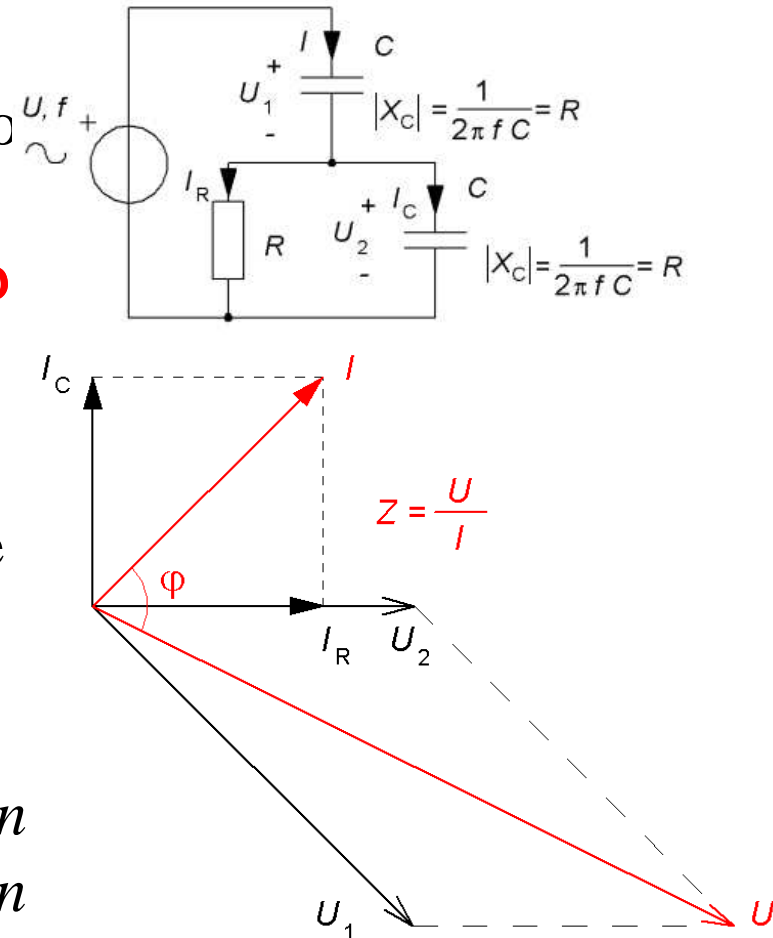
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Complex phasors, jω-method

Complex phasors. OHM's law for R L and C.

$$\underline{U}_R = \underline{I}_R \cdot R \qquad\qquad \varphi = \arg(R) = 0°$$

$$\underline{U}_L = \underline{I}_L \cdot j X_L = \underline{I}_L \cdot j\omega L \qquad \varphi = \arg(j\omega L) = +90°$$

$$\underline{U}_C = \underline{I}_C \cdot j X_C = \underline{I}_C \cdot \frac{1}{j\omega C} \qquad \varphi = \arg(\frac{1}{j\omega C}) = -\arg(j\omega C) = -90°$$

Complex phasors. OHM's law for Z.

$$\boxed{\underline{U} = \underline{I} \cdot \underline{Z}} \qquad Z = \frac{U}{I} \qquad\qquad \varphi = \arg(\underline{Z}) = \arg\left(\frac{\underline{U}}{\underline{I}}\right) = \arg(\underline{U}) - \arg(\underline{I})$$

$$\mathrm{Re}[\underline{U}] = \mathrm{Re}[\underline{I} \cdot \underline{Z}] \qquad\qquad \arg(\underline{Z}) = \arctan\left(\frac{\mathrm{Im}[\underline{Z}]}{\mathrm{Re}[\underline{Z}]}\right) = \arctan\left(\frac{X}{R}\right)$$

$$\mathrm{Im}[\underline{U}] = \mathrm{Im}[\underline{I} \cdot \underline{Z}]$$

In fact, there will be four useful relationships!
• Re, • Im, • Abs, • Arg

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Ex. Complex phasors.

$$U = 20 \text{ V} \quad C = 320 \text{ μF} \quad R = 10 \text{ Ω} \quad f = 50 \text{ Hz}$$

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Ex. Complex phasors.

$$U = 20 \text{ V} \quad C = 320 \text{ μF} \quad R = 10 \text{ Ω} \quad f = 50 \text{ Hz}$$

$$\frac{1}{j\omega C} = \frac{1}{j2\pi \cdot f \cdot C} = \frac{1}{j2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10 \, j$$

Ex. Complex phasors.

$$U = 20 \text{ V} \quad C = 320 \text{ μF} \quad R = 10 \text{ Ω} \quad f = 50 \text{ Hz}$$

$$\frac{1}{\text{j}\omega C} = \frac{1}{\text{j}2\pi \cdot f \cdot C} = \frac{1}{\text{j}2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10\,\text{j}$$

$$\underline{Z}_{\text{R//C}} = \frac{R \cdot \dfrac{1}{\text{j}\omega C}}{R + \dfrac{1}{\text{j}\omega C}} = \frac{10 \cdot (-10\,\text{j})}{10 - 10\,\text{j}} \cdot \frac{(10 + 10\,\text{j})}{(10 + 10\,\text{j})} = 5 - 5\,\text{j}$$

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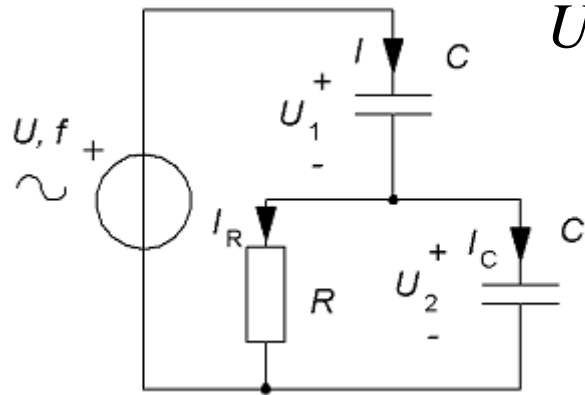
Ex. Complex phasors.

$$U = 20 \text{ V} \quad C = 320 \text{ } \mu\text{F} \quad R = 10 \text{ } \Omega \quad f = 50 \text{ Hz}$$

$$\frac{1}{\text{j}\omega C} = \frac{1}{\text{j}2\pi \cdot f \cdot C} = \frac{1}{\text{j}2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10\,\text{j}$$

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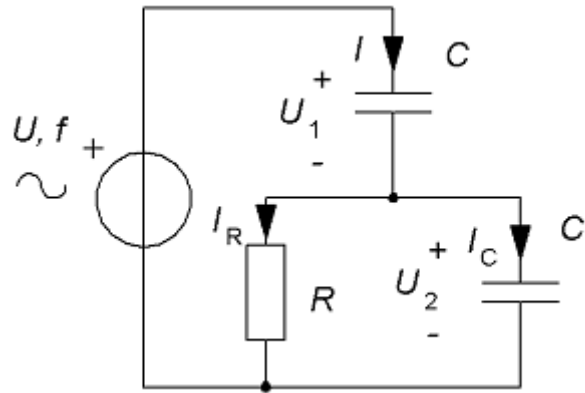William Sandqvist william@kth.se

Ex. Complex phasors. *I*

$$U = 20 \text{ V} \quad C = 320 \text{ }\mu\text{F} \quad R = 10 \text{ }\Omega \quad f = 50 \text{ Hz}$$

$$\frac{1}{\text{j}\omega C} = \frac{1}{\text{j}2\pi \cdot f \cdot C} = \frac{1}{\text{j}2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10\,\text{j}$$

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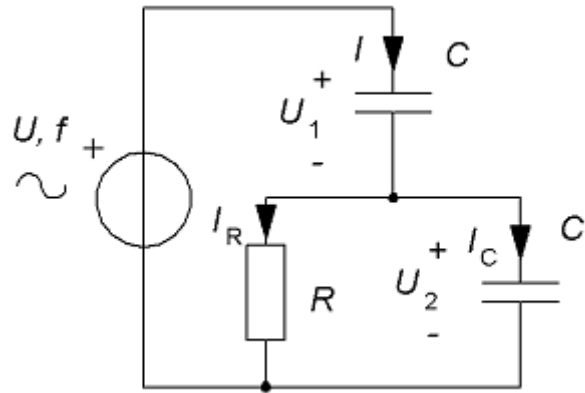William Sandqvist william@kth.se
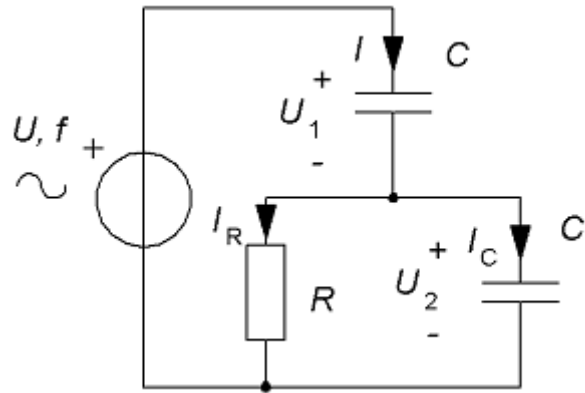
Ex. Complex phasors. *I*

$$U = 20 \text{ V} \quad C = 320 \text{ μF} \quad R = 10 \text{ Ω} \quad f = 50 \text{ Hz}$$

$$\frac{1}{j\omega C} = \frac{1}{j2\pi \cdot f \cdot C} = \frac{1}{j2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10\,j$$

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$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{\underline{U}}{\dfrac{1}{j\omega C} + \underline{Z}_{R//C}} = \frac{20}{-10\,j + (5 - 5\,j)} = \frac{4}{1 - 3\,j} \cdot \frac{(1 + 3\,j)}{(1 + 3\,j)} = 0{,}4 + 1{,}2\,j$$

$$I = |0{,}4 + 1{,}2\,j| = \sqrt{0{,}4^2 + 1{,}2^2} = 1{,}26$$

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Ex. Complex phasors. U_1

$$U = 20 \text{ V} \quad C = 320 \text{ μF} \quad R = 10 \text{ Ω} \quad f = 50 \text{ Hz}$$
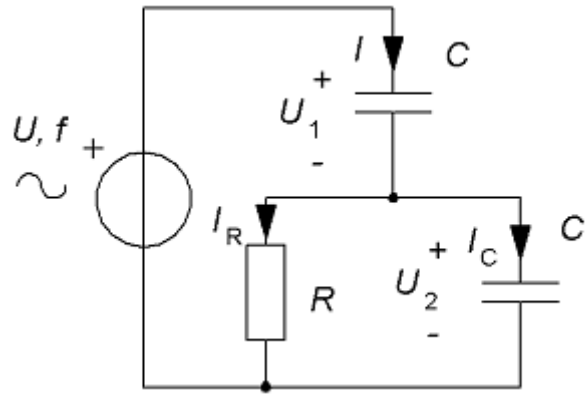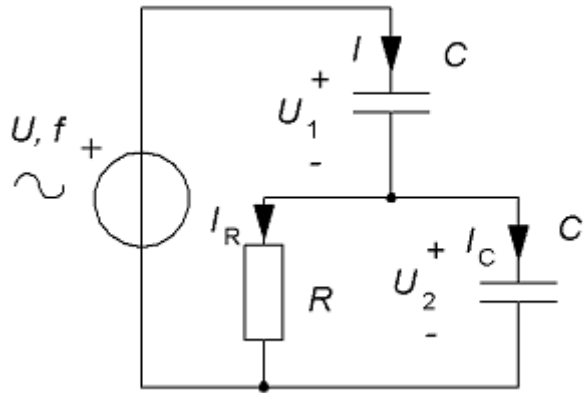
$$\frac{1}{j\omega C} = \frac{1}{j2\pi \cdot f \cdot C} = \frac{1}{j2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10\,j$$

$$\underline{Z}_{R//C} = \frac{R \cdot \dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \frac{10 \cdot (-10\,j)}{10 - 10\,j} \cdot \frac{(10 + 10\,j)}{(10 + 10\,j)} = 5 - 5\,j$$

Ex. Complex phasors. U_1

$$U = 20 \text{ V} \quad C = 320 \text{ μF} \quad R = 10 \text{ Ω} \quad f = 50 \text{ Hz}$$

$$\frac{1}{\mathrm{j}\omega C} = \frac{1}{\mathrm{j}2\pi \cdot f \cdot C} = \frac{1}{\mathrm{j}2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10\,\mathrm{j}$$

$$\underline{Z}_{R//C} = \frac{R \cdot \dfrac{1}{\mathrm{j}\omega C}}{R + \dfrac{1}{\mathrm{j}\omega C}} = \frac{10 \cdot (-10\,\mathrm{j})}{10 - 10\,\mathrm{j}} \cdot \frac{(10 + 10\,\mathrm{j})}{(10 + 10\,\mathrm{j})} = 5 - 5\,\mathrm{j}$$

$$\underline{U}_1 = \underline{I} \cdot \frac{1}{\mathrm{j}\omega C} = (0{,}4 + 1{,}2\,\mathrm{j}) \cdot (-10\,\mathrm{j}) = 12 - 4\,\mathrm{j}$$

$$U_1 = |12 - 4\,\mathrm{j}| = \sqrt{12^2 + (-4)^2} = 12{,}65$$

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Ex. Complex phasors. U_2

$$U = 20\text{ V} \quad C = 320\text{ }\mu\text{F} \quad R = 10\text{ }\Omega \quad f = 50\text{ Hz}$$

$$\frac{1}{\text{j}\omega C} = \frac{1}{\text{j}2\pi \cdot f \cdot C} = \frac{1}{\text{j}2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10\,\text{j}$$

$$\underline{Z}_{\text{R//C}} = \frac{R \cdot \dfrac{1}{\text{j}\omega C}}{R + \dfrac{1}{\text{j}\omega C}} = \frac{10 \cdot (-10\,\text{j})}{10 - 10\,\text{j}} \cdot \frac{(10 + 10\,\text{j})}{(10 + 10\,\text{j})} = 5 - 5\,\text{j}$$

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Ex. Complex phasors. U_2

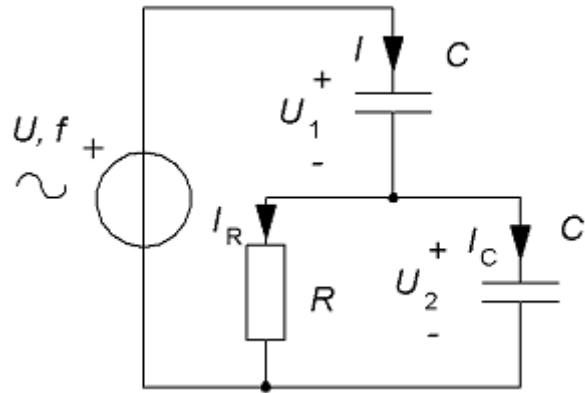$$U = 20 \text{ V} \quad C = 320 \ \mu\text{F} \quad R = 10 \ \Omega \quad f = 50 \text{ Hz}$$

$$\frac{1}{\text{j}\omega C} = \frac{1}{\text{j}2\pi \cdot f \cdot C} = \frac{1}{\text{j}2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10\,\text{j}$$

$$\underline{Z}_{\text{R}//\text{C}} = \frac{R \cdot \dfrac{1}{\text{j}\omega C}}{R + \dfrac{1}{\text{j}\omega C}} = \frac{10 \cdot (-10\,\text{j})}{10 - 10\,\text{j}} \cdot \frac{(10 + 10\,\text{j})}{(10 + 10\,\text{j})} = 5 - 5\,\text{j}$$

Voltage divider:

$$\underline{U}_2 = \underline{U} \cdot \frac{\underline{Z}_{\text{R}//\text{C}}}{\dfrac{1}{\text{j}\omega C} + \underline{Z}_{\text{R}//\text{C}}} = 20 \cdot \frac{5 - 5\,\text{j}}{-10\,\text{j} + (5 - 5\,\text{j})} = 20 \cdot \frac{1 - \text{j}}{1 - 3\,\text{j}} \cdot \frac{(1 + 3\,\text{j})}{(1 + 3\,\text{j})} = 8 + 4\,\text{j}$$

$$U_2 = |8 + 4\,\text{j}| = \sqrt{8^2 + 4^2} = 8{,}94$$

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Ex. Complex phasors. I_C

$$U = 20 \text{ V} \quad C = 320 \text{ } \mu\text{F} \quad R = 10 \text{ } \Omega \quad f = 50 \text{ Hz}$$

$$\frac{1}{j\omega C} = \frac{1}{j2\pi \cdot f \cdot C} = \frac{1}{j2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10\,j$$
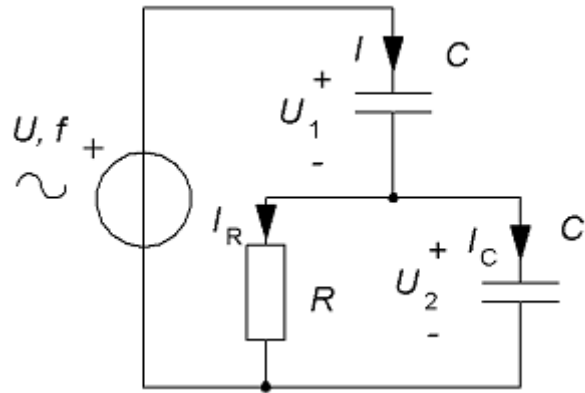
$$\underline{Z}_{R//C} = \frac{R \cdot \dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \frac{10 \cdot (-10\,j)}{10 - 10\,j} \cdot \frac{(10 + 10\,j)}{(10 + 10\,j)} = 5 - 5\,j$$

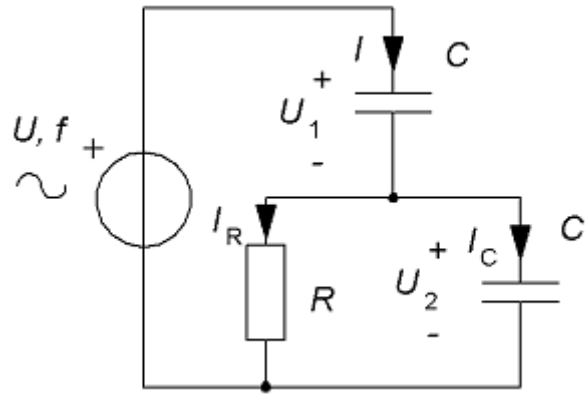# Ex. Complex phasors. I_C

$$U = 20 \text{ V} \quad C = 320 \text{ μF} \quad R = 10 \text{ Ω} \quad f = 50 \text{ Hz}$$

$$\frac{1}{j\omega C} = \frac{1}{j2\pi \cdot f \cdot C} = \frac{1}{j2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10\,j$$

$$\underline{Z}_{R//C} = \frac{R \cdot \dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \frac{10 \cdot (-10\,j)}{10 - 10\,j} \cdot \frac{(10 + 10\,j)}{(10 + 10\,j)} = 5 - 5\,j$$

$$\underline{I}_C = \cdot \frac{\underline{U}_2}{\dfrac{1}{j\omega C}} = \frac{8 + 4\,j}{-10\,j} = -0,4 + 0,8\,j$$

$$I_C = |0,4 + 0,8\,j| = \sqrt{0,4^2 + 0,8^2} = 0,89$$

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Ex. Complex phasors. I_R

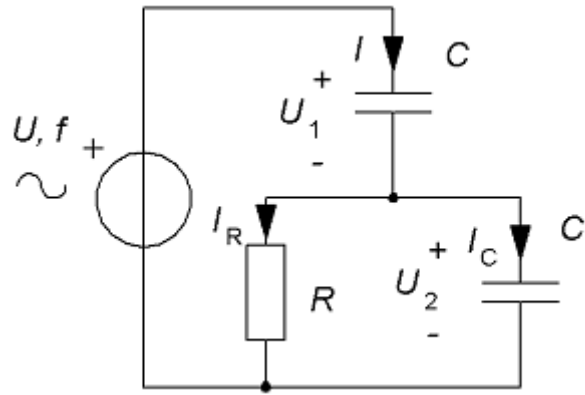$$U = 20 \text{ V} \quad C = 320 \text{ µF} \quad R = 10 \text{ } \Omega \quad f = 50 \text{ Hz}$$

$$\frac{1}{j\omega C} = \frac{1}{j2\pi \cdot f \cdot C} = \frac{1}{j2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10\,j$$

$$\underline{Z}_{R//C} = \frac{R \cdot \dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \frac{10 \cdot (-10\,j)}{10 - 10\,j} \cdot \frac{(10 + 10\,j)}{(10 + 10\,j)} = 5 - 5\,j$$

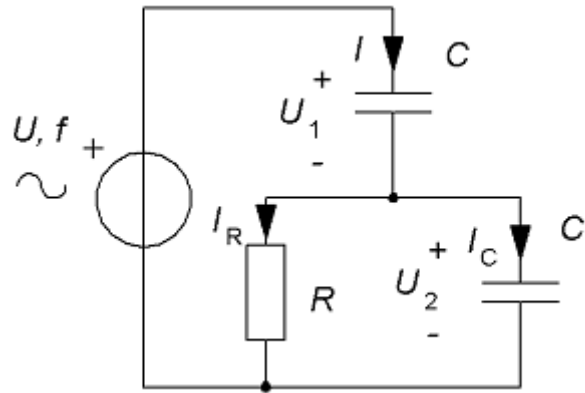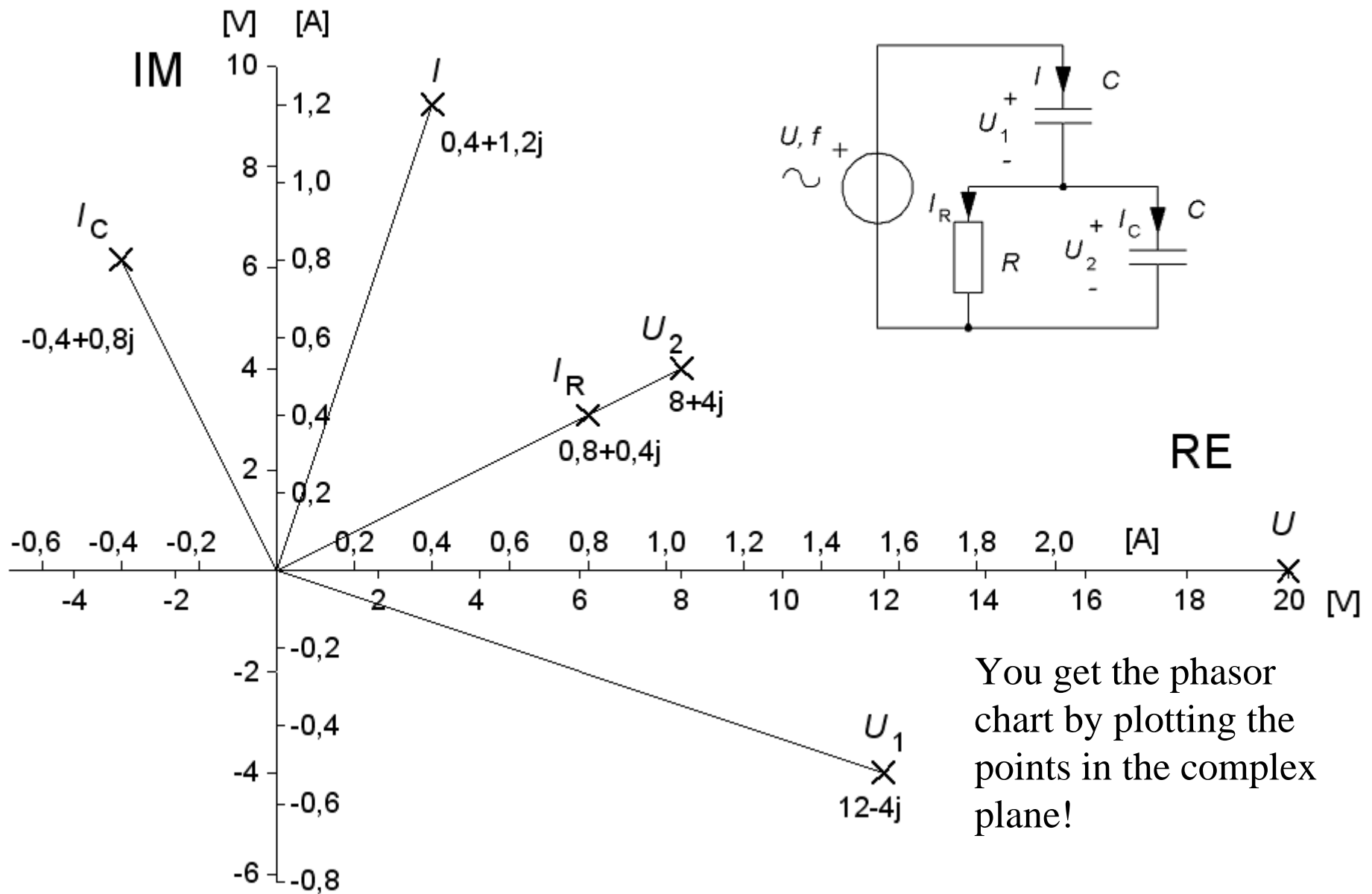William Sandqvist william@kth.se

Ex. Complex phasors. I_R

$$U = 20 \text{ V} \quad C = 320 \ \mu\text{F} \quad R = 10 \ \Omega \quad f = 50 \text{ Hz}$$

$$\frac{1}{j\omega C} = \frac{1}{j2\pi \cdot f \cdot C} = \frac{1}{j2\pi \cdot 50 \cdot 320 \cdot 10^{-6}} = -10 \text{ j}$$

$$\underline{Z}_{R//C} = \frac{R \cdot \dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \frac{10 \cdot (-10 \text{ j})}{10 - 10 \text{ j}} \cdot \frac{(10 + 10 \text{ j})}{(10 + 10 \text{ j})} = 5 - 5 \text{ j}$$

$$\underline{I}_R = \frac{\underline{U}_2}{R} = \frac{8 + 4 \text{ j}}{10} = 0{,}8 + 0{,}4 \text{ j}$$

$$I_R = |0{,}8 + 0{,}4 \text{ j}| = \sqrt{0{,}8^2 + 0{,}4^2} = 0{,}89$$

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You get the phasor chart by plotting the points in the complex plane!

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Vrida diagrammet ...

When we draw the phasor diagram it was natural to have U_2 as reference phase (=horizontal), with the jω-method U was the natural choice of reference phase (=real).

Because it is easy to rotate the chart, so, in practice, we have the freedom of choosing any entity as the reference.

$$\arg(\underline{U}_2) = \arg(8 + 4\,\mathrm{j}) = \arctan\left(\frac{4}{8}\right) = 26{,}7°$$

$$\times(\cos(-26{,}7°) + \mathrm{j}\cdot\sin(-26{,}7°))$$
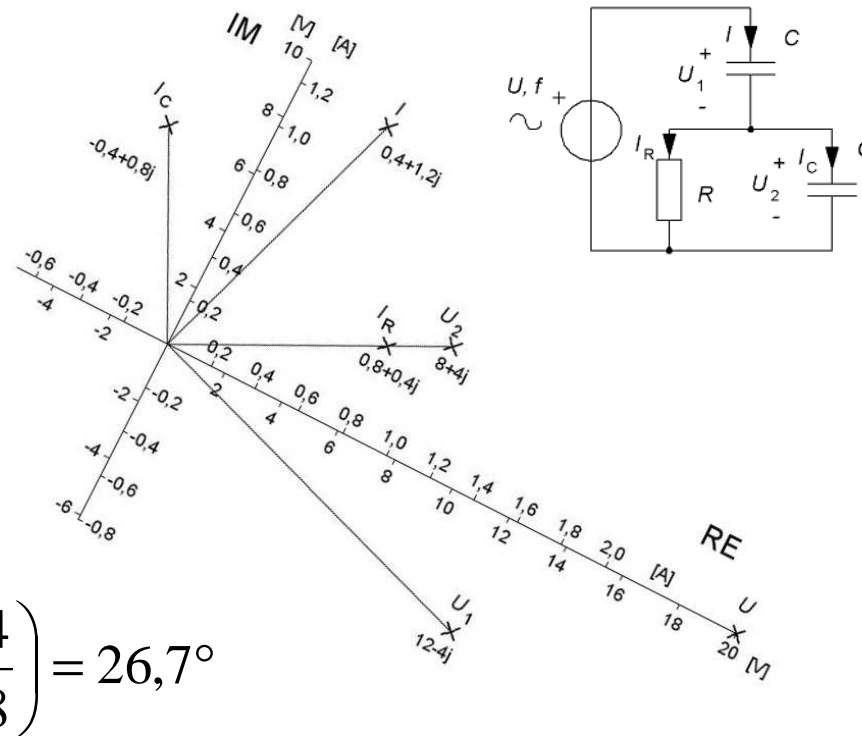
Multiply the all complex numbers by this factor and rhe rotation will take effect!

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Summary

Sinusoidal alternating quantities can be represented as pointers, phasors,

"amount" $\angle$ "phase".

A pointer (phasor) can either be seen as a vector expressed in polar coordinates, or as a complex number.

Calculations are usually best done with the complex method, while phasor diagrams are used to visualize and explain alternating current phenomena.

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Notation

x Instant value

$\hat{X}$ Top value

$\underline{X}$ Complex phasor

$X \left| \underline{X} \right|$ Absolute value, the amount

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