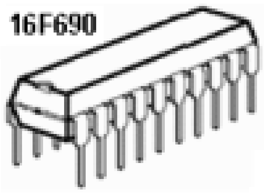
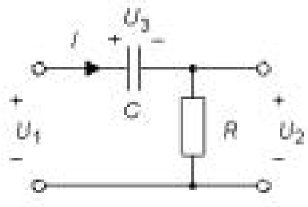
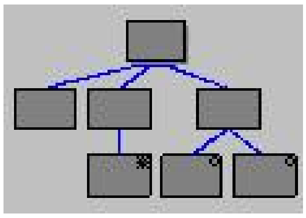
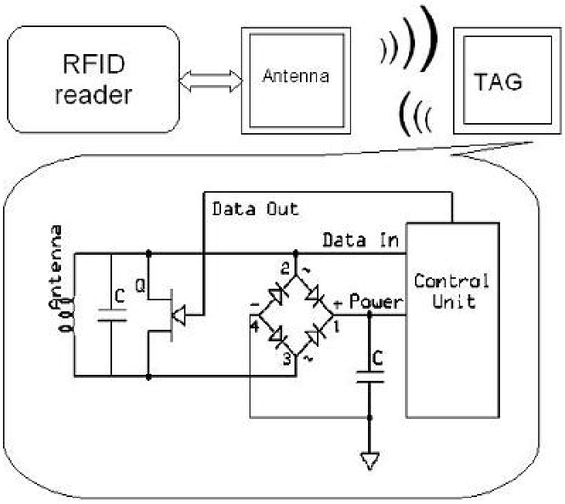
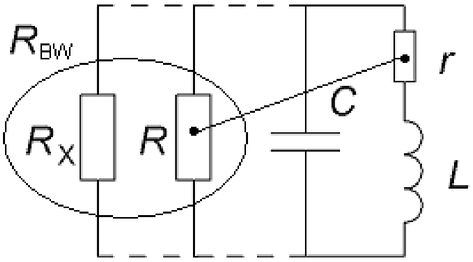
Embedded Electronics exercises











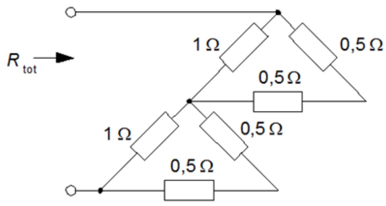
Equivalent resistance, Resistivity and resistor temperature dependens, Serial – parallel circuits, Batteries, Kirchoff's current law/voltage law, Kirchoff's laws, Node analysis – potential, Thevenin and Norton Equivalents, Transients, Capacitance, Magnets – Inductance, Phasors, phasor charts jω-methood, Resonance, Filters, Transformer, Inductive coupling.

Equivalent resistance

1.1

How much will the equivalent resistance R_{tot} for this circuit be ? (Given resistors with resistance values 1 Ω and 0,5 Ω connected as shown).

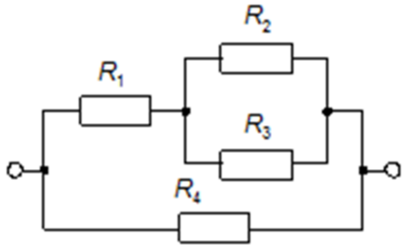
R_{tot} = ? [Ω]



1.2

How much will the equivalent resistance R_{tot} for this circuit be?
Given:
R_1 = 1 Ω
R_2 = 21 Ω
R_3 = 42 Ω
R_4 = 30 Ω

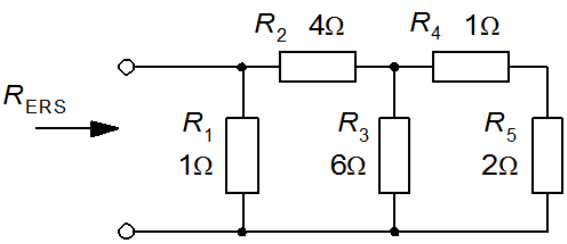
R_{tot} = ? [Ω]



1.3

How much will the equivalent resistance R_{ERS} for this circuit be.
R_1 = 1 Ω, R_2 = 4 Ω, R_3 = 6 Ω, R_4 = 1 Ω, R_5 = 2 Ω

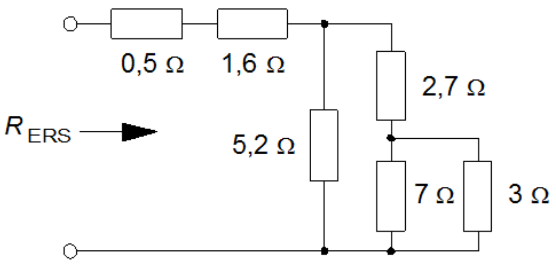
R_{ERS} = ? [Ω]



1.4

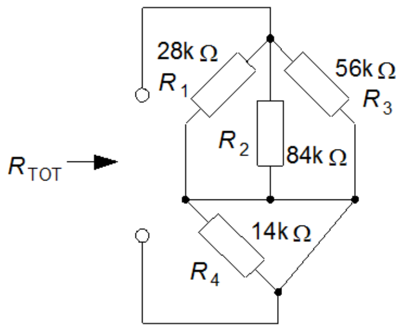
Calculate the equivalent resistance R_{ERS} for this circuit.
Resistors have the values 0,5 Ω, 1,6 Ω, 5,2 Ω, 2,7 Ω, 7 Ω and 3 Ω.

R_{ERS} = ? [Ω]



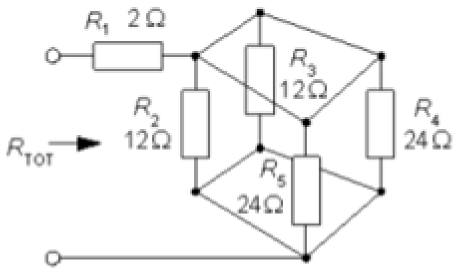
1.5

How much will the equivalent resistance R_{tot} for this circuit with 4 resistors?



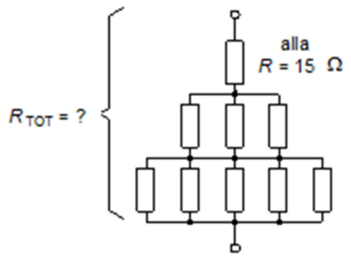
1.6

How much will the equivalent resistance R_{tot} for this circuit consisting of 5 soldered resistors?



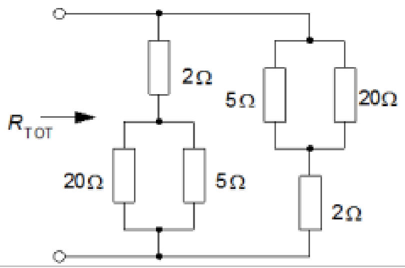
1.7

One builds a "pyramid" of resistors R = 15 Ω. Se the figure.
How much will the equivalent resistance R_{TOT} be?



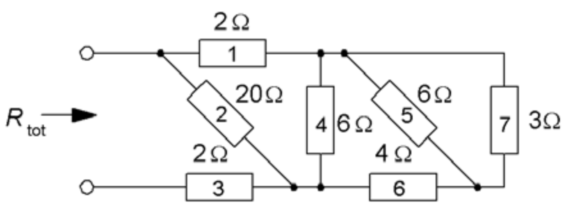
1.8

How much will the equivalent resistance R_{TOT} for this circuit with 6 resistors?
R_{TOT} = ? [Ω]

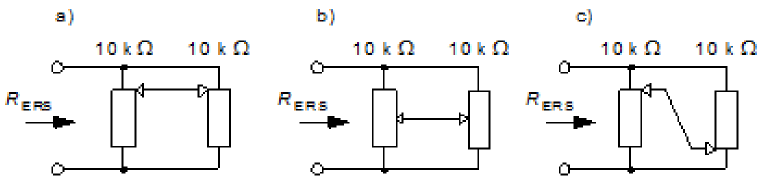


1.9

Set up an expression, and calculate the replacement resistance R_{tot}. (use || to to denote the parallel connection).
R_1 = 2Ω, R_2 = 20Ω, R_3 = 2Ω, R_4 = 6Ω, R_5 = 6Ω, R_6 = 4Ω, R_7 = 3Ω.



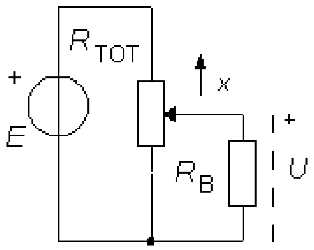
1.10

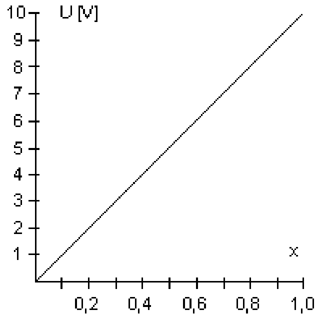


Two potentiometers with total resistance of 10 kΩ are connected as shown. How large is the replacement resistance when:

a) both potentiometers are in the upper position. R_{ERS} = ? [Ω]
b) both potentiometers are in the middle position. R_{ERS} = ? [Ω]
c) one potentiometer is in the upper position, the other in the lower end position. R_{ERS} = ? [Ω]

1.11



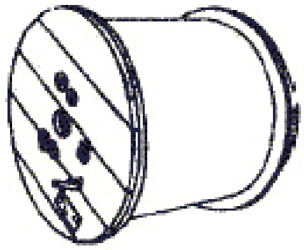


Potentiometer with total resistance R_{TOT} = 10 kΩ is connected to a measuring instrument which has the internal resistance R_B = 1 kΩ. This is far too low load resistance, so the instrument draws current from the potentiometer sensor so that the linearity is lost.
The graph shows the ideal relationship between *U* and *x*. Sketch (rough) in the diagram how the deviation from the ideal curve will become. Suppose *E* = 10 V and that 0 < *x* < 1.

Resistor resistivity and temperature dependency

2.1

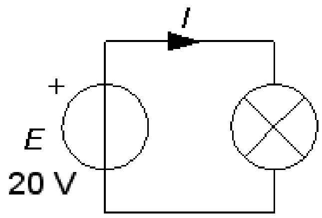
How long is the cable? An electrical installation company usually give their trainees following mission – in the store is a large and heavy cable on a reel, how long is the cable?



A cable consists of two conductors. A leader and a return conductor. The two leaders in the cable end that is wrapped in the back of the roll has been stripped and twisted together. The second cable end is directly accessible. On the cable reel side are stamped conductors cross-sectional area $\boldsymbol{A} = 2{,}5\ \text{mm}^2$.
The resistivity of copper $\rho = 0{,}018$ [$\Omega\text{mm}^2/\text{m}$].
(This number is known by hart of many in the electrical industry).

2.2

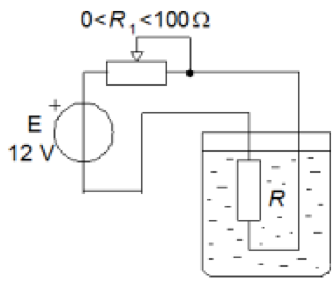
With a radiation thermometer one can measure temperature contactlessly. In order to check such a thermometer one directs it against a bright light bulb and then it showed the temperature 280 °C. The bulb had a Wolfram Wire and were fed with voltage of 20 V. It consumed the current 0.11 A.
Previously, the cold lamp resistance had been measured at room temperature 22 °C to 98 Ω. The temperature coefficient for Wolfram is $\alpha = 4{,}5 \cdot 10^{-3}$.



Calculate the filament temperature of the lamp [°C] and compare it to the temperature reading. Did the radiation thermometer showed the correct temperature value?

2.3

An immersion heater, with resistance R = 50 Ω at room temoperature 25 °C, used in conjunction with an adjustable resistor R_1, adjustable between 0 och 100 Ω.
The heater resistance wire is made of nickel. Nickel has temperature coefficient $\alpha = 6{,}7 \cdot 10^{-3}$. The two resistors are connected to a stable voltage source E = 12 V. See the figure.

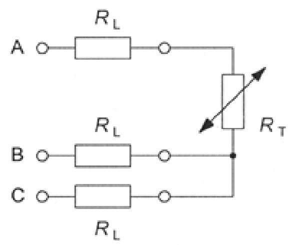


a) One adjusts R_1 until the water begins to boil (100 °C). What is the value of the resistance R then? R = ? [Ω]

b) One measures R_1 = 25 Ω.
Which heating power is supplied to the water via R? P = ? [W]

2.4

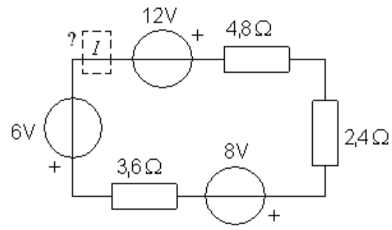
Describe the principle of three terminal sensing.



Serial – parallel circuits

3.1

Determine the current I in magnitude and direction.

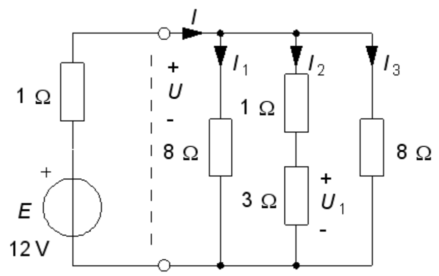


3.2

a) Calculate the resultant resistance R_{RES} for the three parallel connected branches.

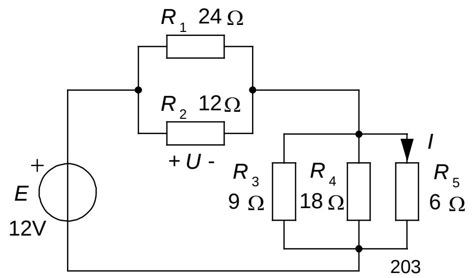
b) Calculate current I and voltage U.

c) Calculate the three currents I_1 I_2 and I_3 and the voltage U_1 over 3 Ω-resistor.



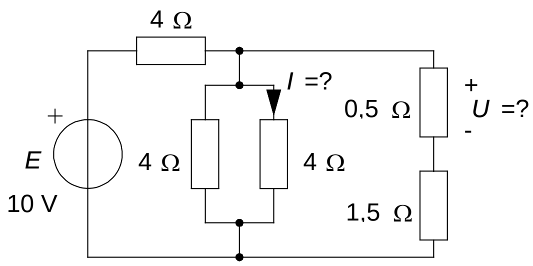
3.3

Calculate current I and voltage U for the serial-parallel circuit in the figure.



3.4

Calculate current I = ? and voltage U = ? for the serial-parallel circuit in the figure.



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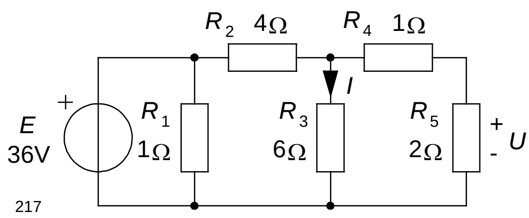
3.5

Calculate current I and voltage U for the serial-parallel circuit in the figure.



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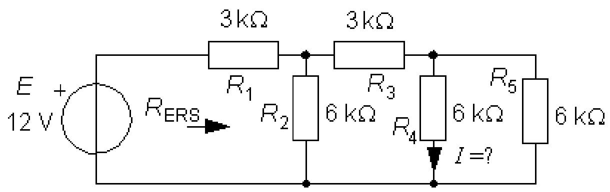
3.6

a) Set up an expression of, and calculate the replacement resistance R_{ERS}.
b) Set up an expression, and calculate the current *I*.

R_1 = 3 kΩ R_2 = 6 kΩ R_3 = 3 kΩ
R_4 = 6 kΩ R_5 = 6 kΩ

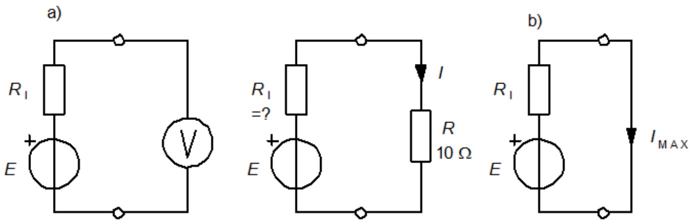


Batteries

4.1

A battery has the capacity number C_{20} = 60 Ah. The capacity number is based on laboratory measurements.
a) How long time did the discharge last, and wich discharge current was used at this laboratory measurement?
b) Suppose that the battery with the capacity number C_{20} = 60 Ah is used to a lap that consumes the current 1 A. How long will the battery last?
c) Suppose that the battery runs a start motor with the current 300 A. How long will the battery last? (expect the capacity number to be reduced by 30% at this high current).

4.2



In order to determine a battery's internal resistance R_I two measurements were made. see figure above to left.
First the battery emf was measured with a good voltmeter E = 1,4 V, and then the battery was loaded with a resistor R =10 Ω and then the current I through the resistor was measurerd tol I = 123 mA.

a) What was the battery's internal resistance? R_I = ? [Ω]

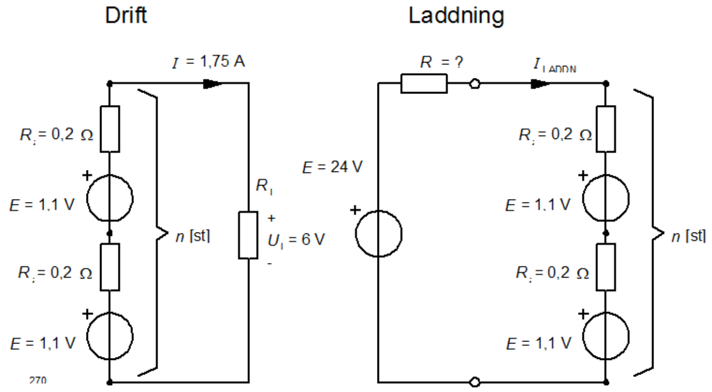
b) What is the maximum current I_{MAX} the battery could deliver if it was short circuited? I_{MAX} = ? [mA]

4.3

A battery-powered equipment is powered by a rechargeable battery. The battery consists of a number (*n* pieces) NiCad cells.

(The figure is simplified with only two of the n cells shown.) The Cells has E = 1,1 V and R_i = 0,2 Ω. The capacity number for each cell is C = 3000 mAh.

The equipment consumes 1.75 A at 6 V, how many cells are needed?



a) n = ?

The battery is charged from a 24 V battery. What a charging current I_{LADDN} should you have if you want the battery to be charged rapidly in an hour? (From empty to full, assuming that the cell E is constant during charging).

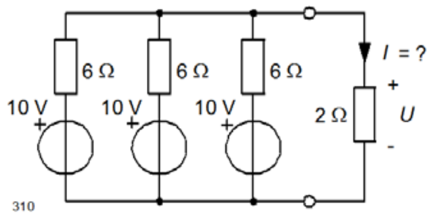
b) I_{LADDN} = ?

What value should R have in order to receive this charging current?

c) R = ?

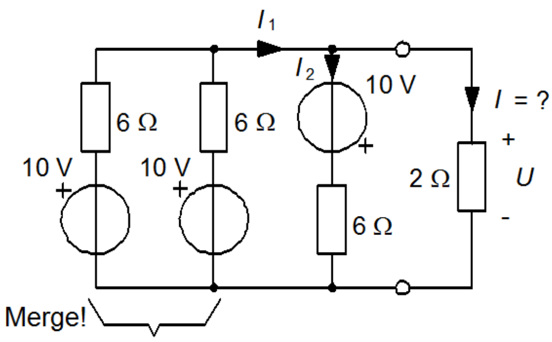
4.4

Three similar batteries E = 10 V and internal resistance 6 Ω are connected in parallel to supply current to a resistor with resistance 2 Ω.



a) How great will current I and voltage U be?
I = ? [A]
U = ? [V]

b) Accidentally one happens to turn one of the batteries wrong. Use Kirchoff's laws to determine the currents I_1, I_2, and I magnitude and direction (sign). Determine U.
The task is simplified if you "merge" the two of the batteries to one battery in a similar way as in **a**.

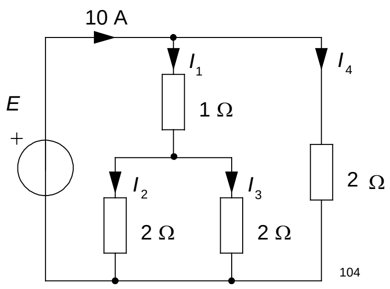


I_1= ? [A]
I_2 = ? [A]
I = ? [A]
U = ? [V]

Kirchoff’s current law and voltage law

5.1

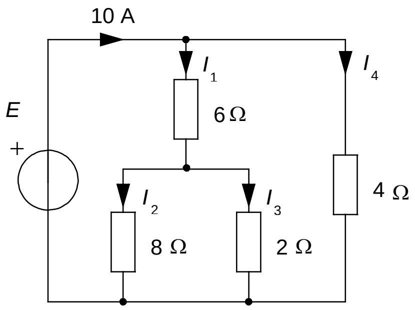
Calculate the four currents I_1 I_2 I_3 och I_4.



5.2

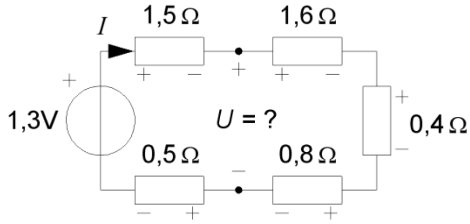
We know that the current I from, E, to circuit is 10 A. How big are the currents I_1, I_2, I_3, I_4 ? What value has E ?

I_1 = ?
I_2 = ?
I_3 = ?
I_4 = ?
E = ?



5.3

Use Kirchoff’s voltage law to calulate U = ?

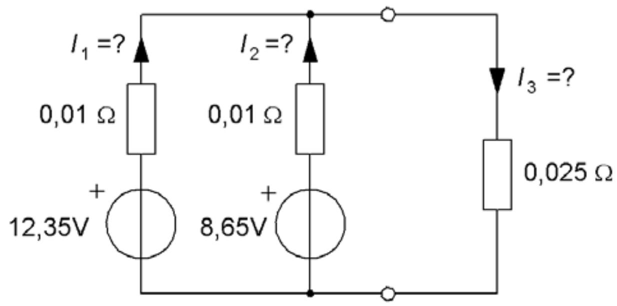


Kirchoff’s laws, equation system

6.1

Use Kirchoff's laws to determine the three currents amount and direction (sign).
I_1=?, I_2=?, I_3=?.

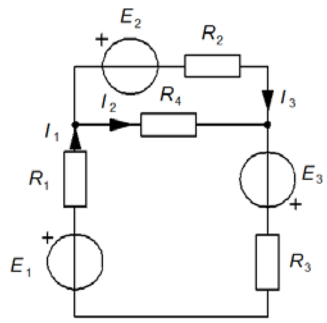
An interpretation of the circuit:
A car owner parallel connects his poor battery (8,65V) with a borrowed good battery (12,35V) to boost the starter motor (0,025Ω) a cold winter day. Having solved the task do you know if he had any advantage of the poor battery?



6.2

Use Kirchoff's laws to determine the three currents I_1, I_2, and I_3 their amount and direction (sign)..
Given:
E_1 = 5V R_1 = 1 Ω
E_2 = 21V R_2 = 2 Ω
E_3 = 4V R_3 = 2 Ω
R_4 = 15 Ω

I_1= ? [A]
I_2 = ? [A]
I_3 = ? [A]



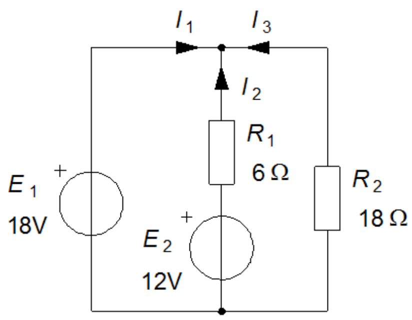
6.3

Use Kirchoff's laws to

a) Determine the voltage across R_2 (18Ω resistor).

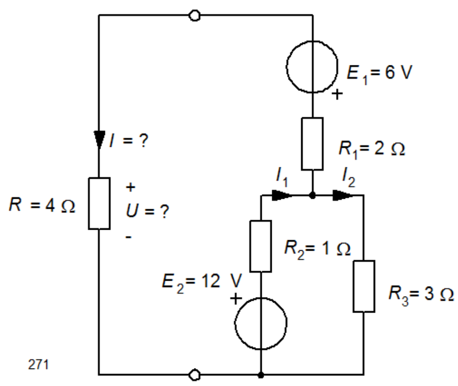
b) Determine the current I_2 to the amount and direction.

c) Determine the current I_1 to the amount and direction.



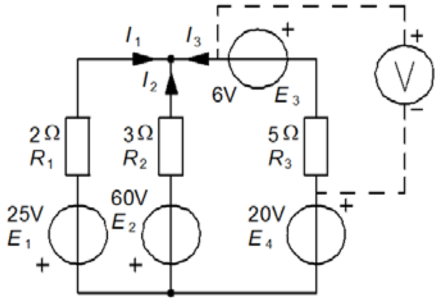
6.4

Use Kirchoff's laws to determine the current *I*:s and the voltage *U*:s sizes and directions (sign).



6.5

a) Use Kirchhoff's two laws to set up an equation system by which the three currents I_1 I_2 and I_3 can be calculated. (You need not solve the system of equations)

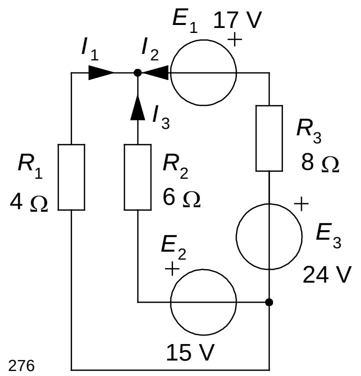


If equationsystem is solved one gets:
I_1 = 1,87 I_2 = -10,4 I_3 = 8,55 [A].

b) What does the voltmeter at the right in the figure show (give both amount and sign) [V]?

6.6

Use Kirchoff's laws to calculate the currents I_1, I_2, and I_3 to magnitude and direction (sign).



I_1 = ? [A]

I_2 = ? [A]

I_3 = ? [A]

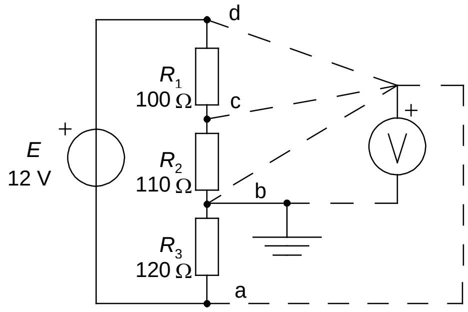
Potential, nodal analysis, depending generators

7.1

A voltage divider consists of three resistors R_1 = 100 Ω, R_2 = 110 Ω, R_3 = 120 Ω, it is fed with an emf E = 12 V.
One measures the potential (the voltage relative to ground) at the different sockets of the voltage divider.

Voltmeter negative terminal is all the time connected to the socket b, ground, while the voltmeter positive terminal medan voltmeterns pluspol in turn connected to the sockets a, b, c, and d.

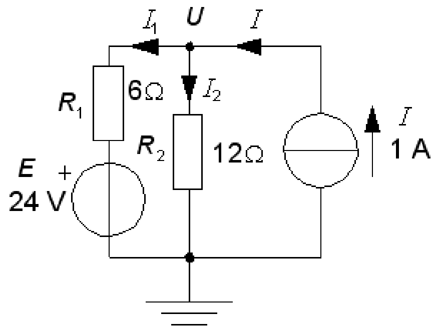
What will voltmeter show? Fill in the table below.



Socket	a)	b)	c)	d)
Voltmeter [V]				

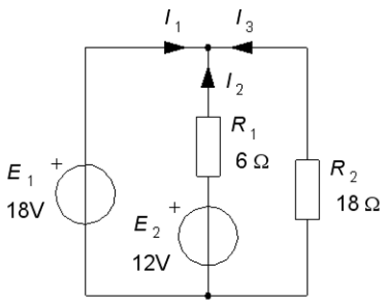
7.2

Use node analysis to calculate the currents I, I_1, and I_2.



7.3

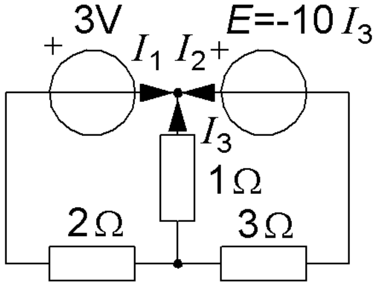
Use nodal analysis to determine the currents I_1 I_2 och I_3 to magnitude and direction.



7.4 Depending generator

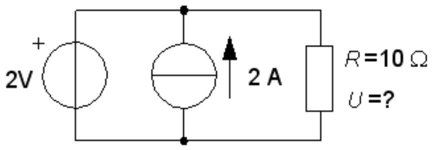
Use Kirchoff's laws to determine the three currents amount and direction (sign).
I_1=?, I_2=?, I_3=?.

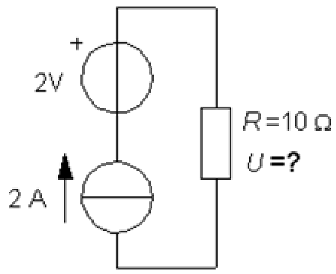
Note that E is a dependent emf.
The dependent emf E depends on the current through the 1 Ω resistor according to the relationship $E = -10 \cdot I_3$.



Thevenin and Norton Equivalent circuits

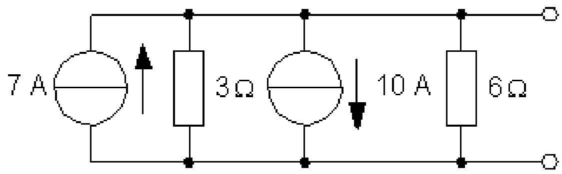
8.1

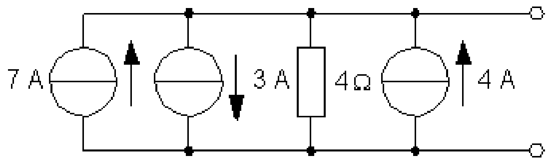




What value should voltage U get in these idealized and usually unrealistic circuits?

8.2

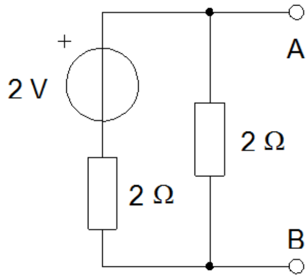




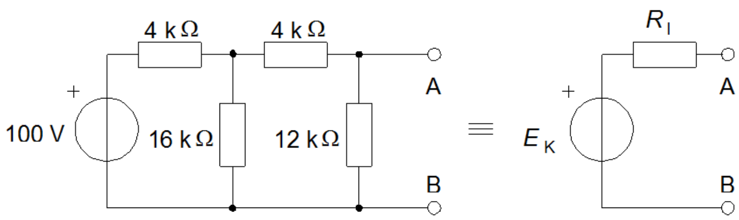
Simpyify the circuits.

8.3

Replace the given two port circuit with a simpler one having an emf in series with a resistor.



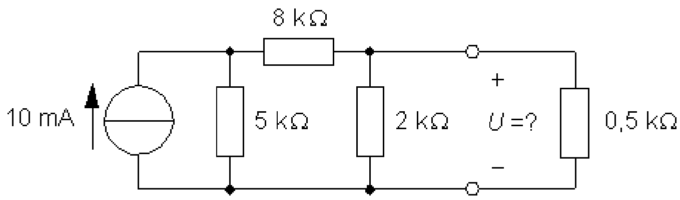
8.4



a) Determine the voltage between A and B (the open circuit voltage).
b) Determine the current that would go through a conductor with very little resistance, if it is connected directly between A and B in the figure (the so-called short-circuit current.)
c) Determine a circuit consisting of an emf E_K in series with a resitor R_I (according to figure) which is equivalent to the given circuit if viewed from points A and B.
d) Determine the maximum power which can be obtained in a resistor connected between points A and B. (Use the result from task c.)

8.5

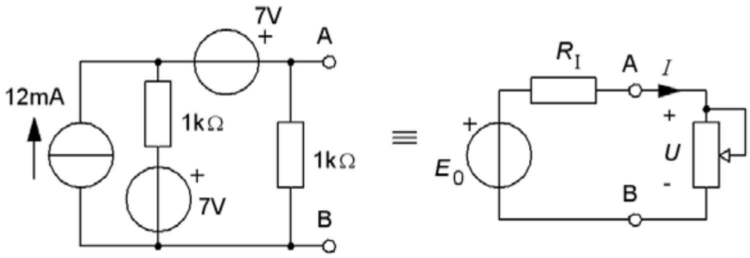
Use Norton and Thevenin equivalents to gradually reduce the circuit, and then calculate the voltage U = ?



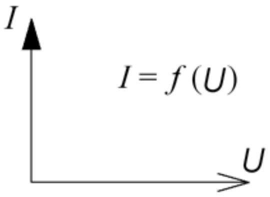
8.6

a) Develop an Thévenin equivalent , E_0 R_I, to the circuit with the two voltage sources (7V) and current source (12 mA).

E_0 = ? [V] R_I = ? [kΩ]

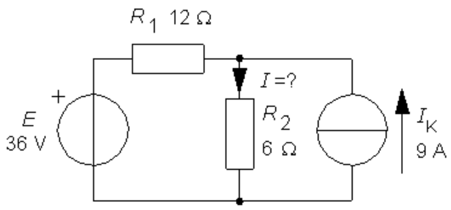


b) The circuit AB is connected to a variable resistor. One measures voltage and current to draw the circuit's characteristic curve *I* as a function of *U*. Draw this graph. grade axes.



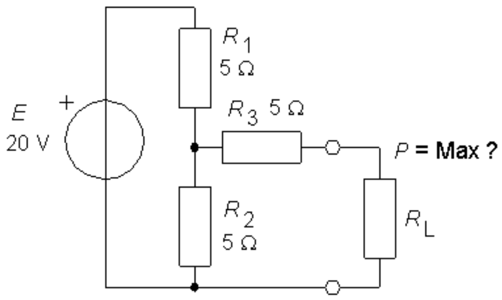
8.7 superposition

Use superposition to solve I = ?.

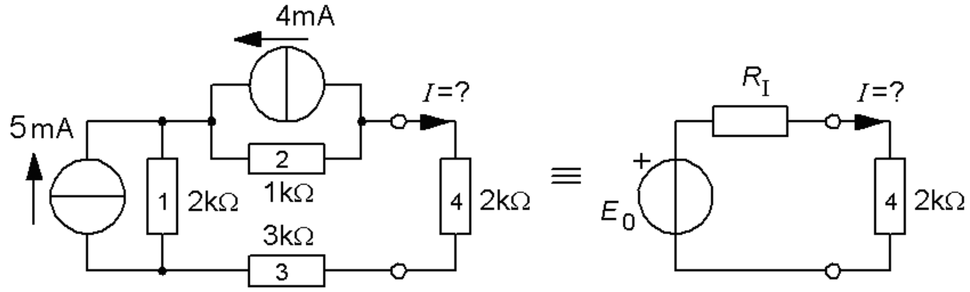


8.8

Choose the load resistor R_L to dissipate the maximum power.
How big is this power?

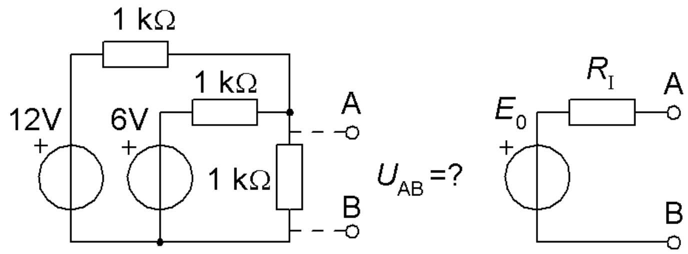


8.9



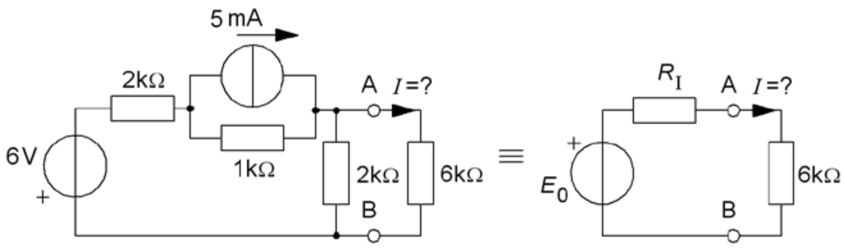
a) Develop the equivalent Thévenin circuit, E_0 R_I, to the net with the two current sources.
b) Then calculate what current I one would get if a resistor R_4 = 2 kΩ is connected to the original (or equivalent) net.

8.10



a) Develop the equivalent Thévenin circuit, E_0 R_I, to the circuit with the two emf and the three resistors.
b) How big is the voltage drop U_{AB} over 1 kΩ resistor in the original circuit?

8.11

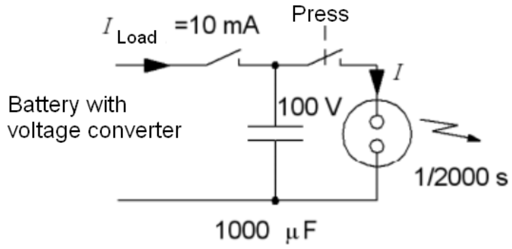


a) Develop the equivalent Thévenin circuit, E_0 R_I, to the circuit with the voltage source and the current source and the three resistors. (6 kΩ resistor is not included in the circuit)
b) How large current would flow in a 6 kΩ resistor if it is connected between A-B? Calculate current I magnitude and direction (positive current direction defined in the figure).

Capacitance, magnetism, inductance

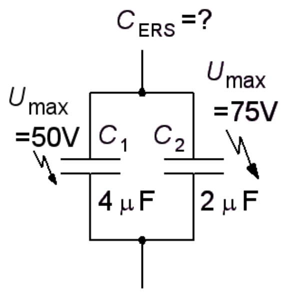
9.1
The figure shows a principle view of a camera flash.

a) How much electrical energy is stored in the capacitor *W*?
b) How big is the charge in capacitor *Q*?
c) How big is the lightning current (mean value) *I*?
d) How big is the power during flash discharge *P*?
e) How long must you wait for the next flash t_{Load}?

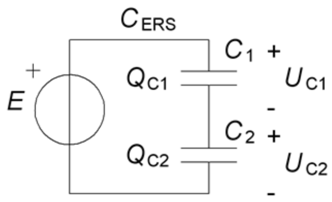


9.2
The backup capacitors of the type "Supercap" can be used as a power backup for memories - if one for example. needs to move the phone from one room to another without the phone forgetting its settings.
Make a rough estimate of how long the charge on the capacitor will last? Assume that C = 1 F and U is initially 5V. The equipment draws I = 10 mA and operates down to 2.5V.

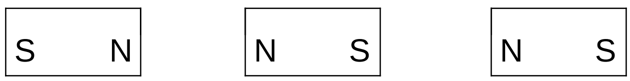
9.3
Two capacitors are parallel-connected. What about equivalent capacitance and rated voltage?
C_1 = 4 μF 50V
C_2 = 2 μF 75V



9.4
Two capacitors are connected in series. Calculate the equivalent capacitance and specify how the voltage is divided between the capacitors.
E = 10 V
C_1 = 6 μF
C_2 = 12 μF



9.5
Three permanent magnets are placed in a row as shown. Draw the magnetic force lines in the figure. Mark with arrows the magnetic field direction.



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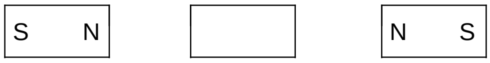
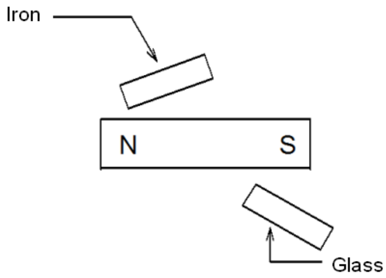
9.6

Two permanent magnets are placed on either side of an equally large piece of metal. See Fig. The metal piece, in the center, is of a material having permability $\mu_r = 1$. Draw the magnetic force lines in the figure. Mark with arrows the magnetic field direction.



9.7

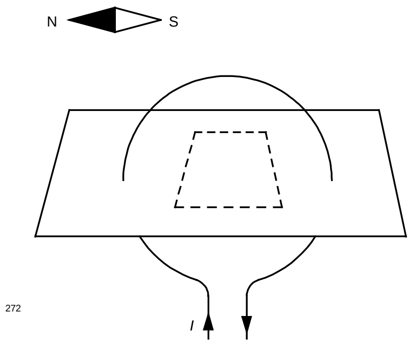
Sketch the magnet's field lines in the figure, and how these are affected by the iron piece and the glass piece in the magnet proximity. Also mark the direction of the field.



9.8

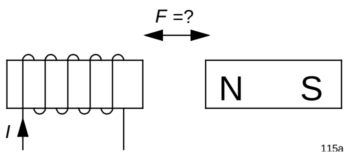
A copper wire is formed as a loop through a paper. See the figure. Through the loop, a current of a few Amperes flows in the direction of the arrows.

a) Draw the magnetic field (the magnetic lines of force). around the wires in the plane of the paper. Mark field direction of arrows.
b) Assume that a compass needle is placed in the hatched area on the paper. Draw how the compass needle will aligns itself in the magnetic field of the wire loop.



9.9

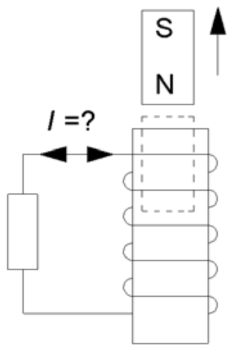
A current is flowes through the coil. Amagnet is nearby outside the coil. See the fig.Wich direction has the resulting force F, is it attractive or repulsive?



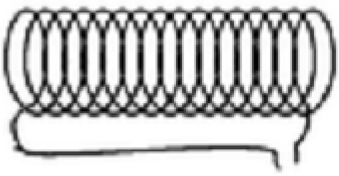
9.10

Lens law.

We draw out the magnet (as a cork from a bottle) from the coil. Which direction will the current I have?



9.11



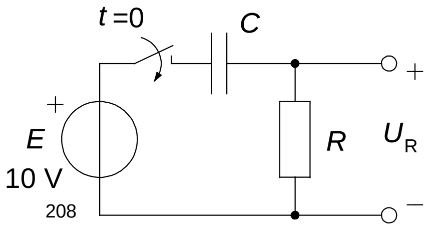
$$L = \frac{N^2 \cdot \mu \cdot A}{l} \qquad K = \frac{\mu \cdot A}{l} \qquad L = K \cdot N^2$$

Suppose that a coil is wound with N = 100 turns and then have the inductance 1 H. How many turns has to be unwound if you want to change the coil so that the inductance becomes ½ H?

Transients with RC and L/R

10.1

At time t = 0 the contact bettween voltage source E = 10 V and the capacitor C = 500 μF which is series connected to the resistor R = 500 Ω.

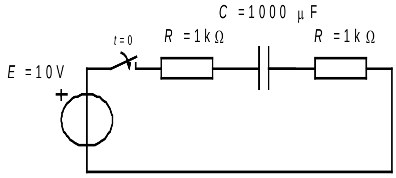


a) How long time will it take for the voltage across the resistor U_R = 2 V?

b) After how long time will the voltage across the capacitor be 2 V?

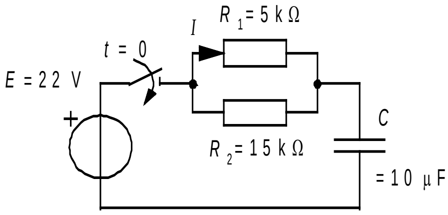
10.2

A capacitor C = 1000 μF is connected in series with two resistors with the resistance R = 1kΩ. at time t = 0 a voltage source with the constant voltage E = 10 V is connected to the circuit.
At what time (t = ?) has the three components the same voltage across them?



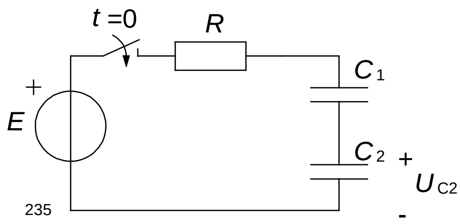
10.3

A capacitor, C = 10μF, is charged from a constant voltage source E = 22 V. The charging current is limited by two parallel connected resistors R_1 = 5 kΩ and R_2 = 15 kΩ. The charging is started by the switch closing at time t = 0.
a) What time constant τ has the circuit during the charge?
b) How long time will it take until the current I through R_1 has passed below 3 mA?



10.4

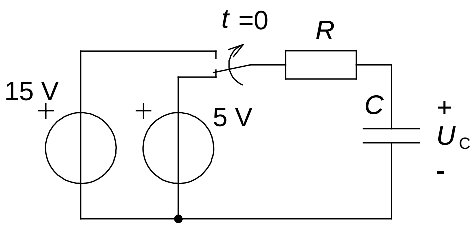
Two serieal connected capacitors, C_1 = 25μF and C_2 = 15μF, are charged from a constant voltage source E = 15 V. The charging current is limited by a resistor R = 330 kΩ. the charge is started by the switch closed at time t = 0.
a) What time constant τ has the circuit during charge?
b) How long will it take befor the voltage U_{C2} reaches 2 V?



10.5

Before time t = 0 is the capacitor via a switch connected to +5V. At the time t = 0 the switch is changed and the capacitor will be connected to +15 V. Suppose R = 2000 Ω and C = 1000 μF.

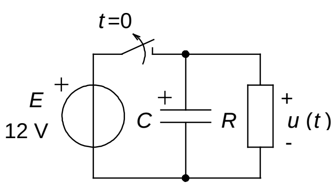
a) How long will it take after t = 0 for voltage U_C to reach +10 V?
b) How long time after t = 0 *do you estimate* dit will take for current through R to cease? (For a technican less than 1% is nothing.)



10.6

before time t = 0 a constant voltage E = 12 V is connected to R and C. At time t = 0 the connection to the voltage E is removed. Suppose R = 110 Ω and that C = 10000 μF.

a) How long time will it take after t = 0 for the voltage $u(t)$ across the resistor to drop to 2 V?
b) How long after t = 0 *do you estimate* it will take for current through R to cease? (For a technican less than 1% is nothing.)



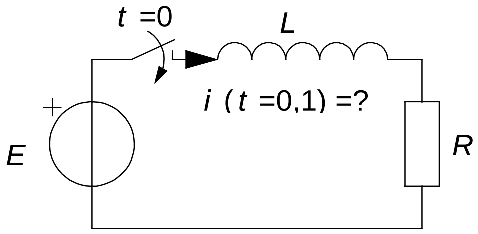
10.7

A coil with inductance L = 0,8 H and the internal resistance R = 12 Ω is connected to a constant voltage source E = 12 V with a switch. At time t = 0 the switch is closed.

a) What value will the current i through the circuit have after one tenth af a second? $i(t = 0{,}1) = ?$ [A]

b) What value will the current have after one tenth of a second if R would be twice as large $R = 2 \cdot 12 = 24\ \Omega$?

$i_{2R}\,(t = 0{,}1) = ?$ [A]



10.8

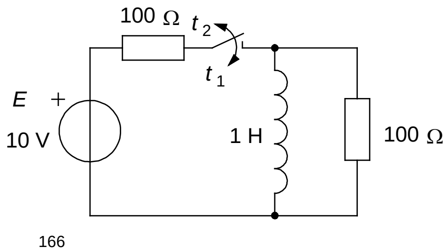
E is a constant voltage source. At time t_1 the switch is closed.

a) How large is the current through the coil in the first moment?

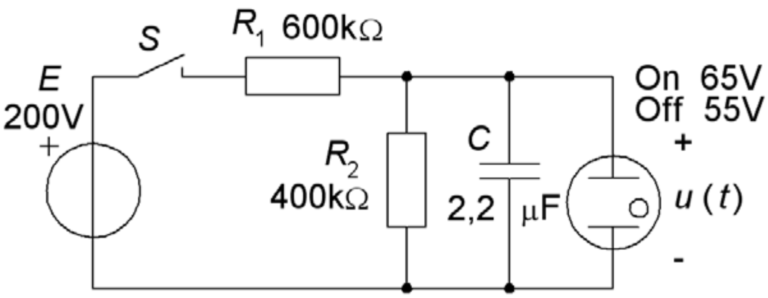
b) How large is the current through the coil after a long timel?

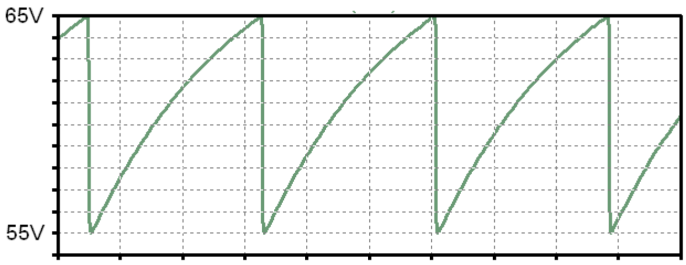
Later, at time t_2 the switch will be opened.

c) Write down the expression for the current through the coil as a function of time t for the time after t_2. (Suppose that the switch is opened at time $t = t_2 = 0$).



10.9





E = 200V R_1 = 600kΩ R_2 = 400kΩ C = 2,2µF On 65V Off 55 V

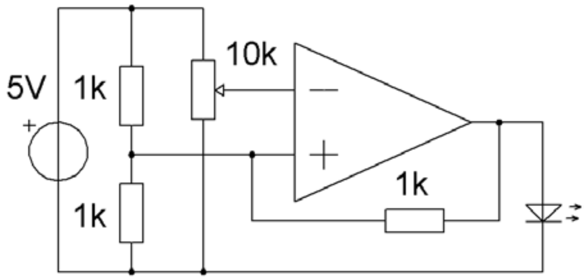
The circuit above flashes with a ***neon lamp*** (this was a common component at the time before the LEDs). Capacitor is charged. Neon lamp will ”ignite” when the voltage across it reaches 65 V. It will quickly discharge the capacitor to 55 V, and then the light will "go off".

a) From the beginning the capacitor is discharged. Calculate how long time it will take, after having hit the switch S, until the first light pulse will appear.

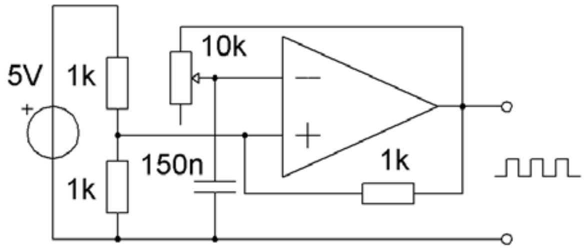
b) Then the circuit will flash at a constant rate, see the oscilloscope-graph. Calculate how long time there will be between the flashes?

c) Suppose one **removes** the resistor R_2 from the circuit. How long time will it then be between the flashes?

10.10

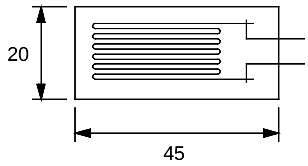


a) Calculate the Schmitt trigger trig levels.



b) Calculate the Schmitt trigger oscillator frequency.
The potentiometer is set to R = 5k.

10.11



A resistance thermometer is used for measuring the temperature of the surface of an internal combustion engine.
When the engine is hot one measures 176 Ω, and when the engine has cooled down for 10 minutes one measures 139 Ω. After a long time (very long) the engine will be cooled down to ambient temperature which is measured to be 25° C.
The temperature $\vartheta\,[°\mathrm{C}]$ during cool down follows an exponential function with a time constant τ, use the ”**equation for exponential changes**”.
For the resitance thermometer (with platinum) has the relationship:

$$R(\vartheta) = 100 \cdot (1 + 3{,}85 \cdot 10^{-3} \cdot \vartheta)\,[\,\Omega\,]$$

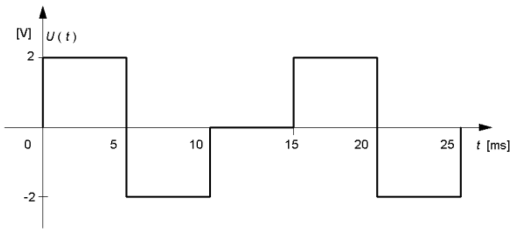
- Determine the time constant for cool down.
  τ = ? [minutes]

Alternating voltage and current, phasors

11.1
A sinusoidal entity has the maximum value A 6,0 and becomes 0 2000 times per second. The time $t = 0$ is choosen so that the entity at that time has the value 3,0 and is rising towards 6,0. State the entity

a) mathematically
b) vawe graph
c) phasor

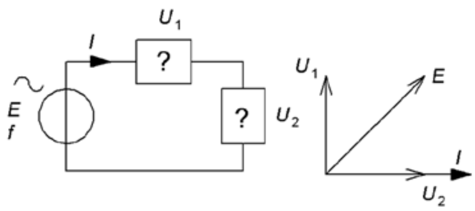
11.2
Calculate for this periodic alternating voltage its rms value – which is the equivalent DC voltage that would produce the same effect in a resistor.



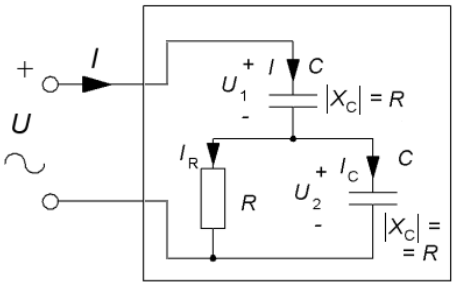
11.3
What is the relationship between the amplitude, the peak value and the RMS value of a sine voltage?

Phasor charts

11.4
Identify what components that have given the voltage phasors U_1 and U_2.

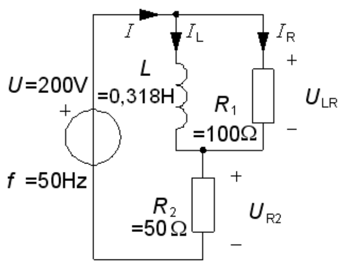


11.5
Draw the phasor chart with all volages and currents for this circuit. Estimate the impedance Z as the ratio between the lengths of U och I. Estimate the impedance phase angle φ as the angle between U and I phasors.



11.6
The circuit of the figure is supplied with a sinusoidal alternating voltage $U = 200$ V, $f = 50$ Hz. The solenoid has the inductance $L = 0{,}318$ H and the two resistors $R_1 = 100\ \Omega$ and $R_2 = 50\ \Omega$.

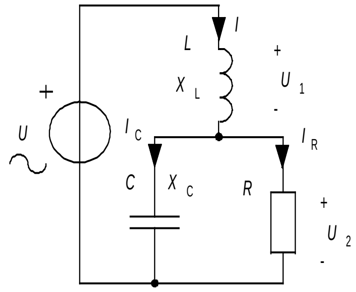
a) Calculate X_L.
b) Draw the phasor chart for this circuit. The phasor chart shall contain U U_{LR} U_{R2} I I_R I_L. Suggestion: use U_{LR} as the reference phase (horizontal, angle 0°). Phasor lengths must be at least a rough, proportionate. Mark angle φ in the phasor chart, the angle between I and U.



11.7

Draw the phasor chart for the circuit in the figure. At the actual frequency applies that $|X_C| = R$ and $|X_L| = R/2$.
The phasor chart should contain U U_1 U_2 I I_R I_C.
Mark angle φ in the phasor chart, the angle between I and U.

U_2 is a suitable reference phase.

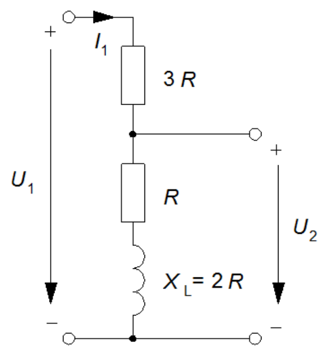


11.8

The figure shows a voltage divider. This is fed with an alternating voltage U_1 and the output voltage is U_2. At the actual frequency applies that $|X_L| = 2R$.

Draw the phasor with I_1, U_1 och U_2.

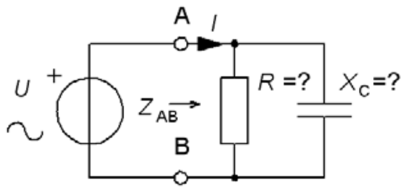
Use I_1 as reference phase (= horizontal). (*Strive to get the right proportions of phasors*)



Alternating voltage and current, jω-method

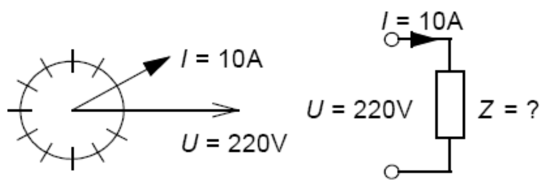
12.1

Set up the complex expression for current I expressed in U R C ω. Let U be reference phase which means a real numberl. Reply with an expression of the form a+jb.



12.2

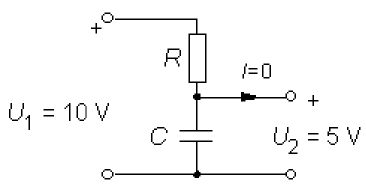
How can the impedance Z look like that has given rise to this phasors? Draw the circuit diagram diagram and calculate the components.
U = 220 V, f = 50 Hz.



12.3

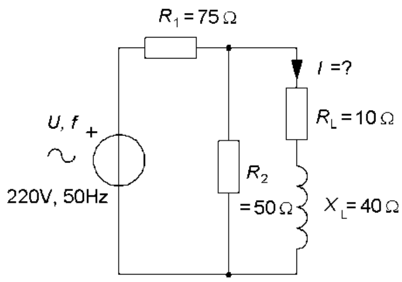
U_1 is a sinusoidal alternating voltage with the angular frequency ω.
Determine product $R{\cdot}C$.

(No current is drawn at U_2)



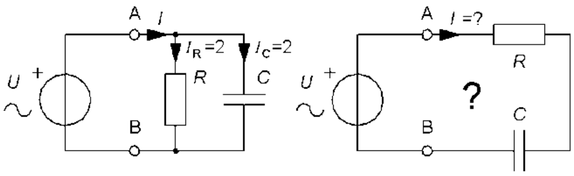
12.4

Determine the current I.
Use Thevenine equivalent.



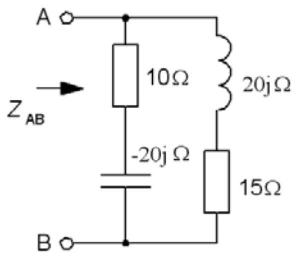
12.5

When a resistor R and a capacitor C are connected in parallel to a AC votage source U each of the two components gets the current 2A.
How big would the current be if the two were series connected to the voltage source?



12.6

Determine the complex impedance Z_{AB} for the circuit.

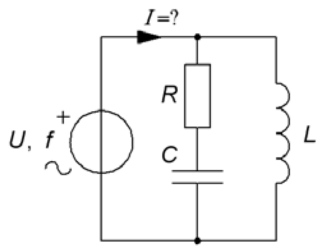


12.7

Derive an expression for complex current I (with U as reference phase).

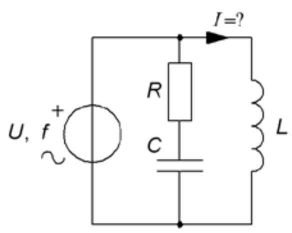
Note! One does not always havet o give the answer in the form a+jb. The same information, but with less effort, is if the answer is expressed as a ratio of complex numbers. Amount and arguments can if needed be taken from the numerator and the denominator directly.

$$\underline{I} = \frac{a + \mathrm{j}b}{c + \mathrm{j}d} \quad I = \frac{|a + \mathrm{j}b|}{|c + \mathrm{j}d|} \quad \arg(\underline{I}) = \arg(a + \mathrm{j}b) - \arg(c + \mathrm{j}d)$$



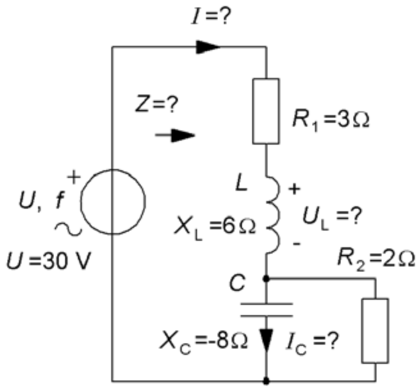
12.8

Derive an expression for complex current I (with U as reference phase).



12.9

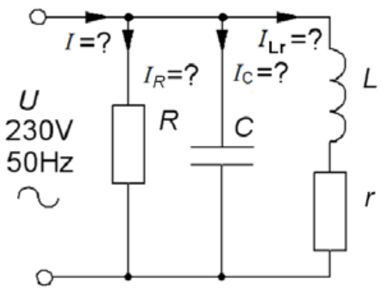
Calculate impedance *Z*.
Calculate current *I*.
Calculate I_C (use current branching formula).
Calculate U_L (use voltage division formula).



12.10

An AC circuit is connected to the AC mains with U = 230 V and f = 50 Hz. R = 46 Ω, $\omega L = R$, r = 32,5 Ω and C = 69 μF.

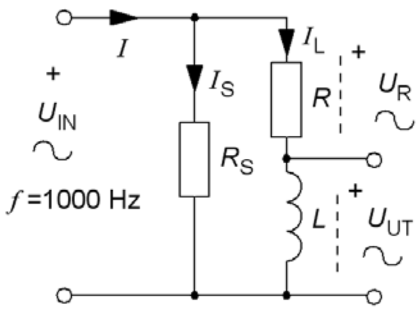
a) Calculate I_R
b) Calculate I_C
c) Calculate I_{Lr}
d) Calculate I



12.11

An AC voltage U_{IN} with the frequency f = 1000 Hz feeds a circuit with an inductance L = 10 mH in series with a resistor R = 50 Ω. In parallel with these are a resistor R_S = 100 Ω.
Given is voltage U_{UT} = 6,28 V.

a) Calculate I_L
b) Calculate U_R
c) Calculate U_{IN}
d) Calculate I

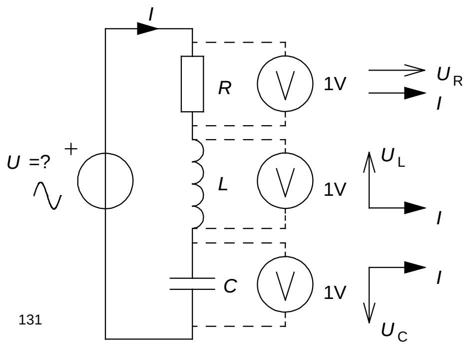


Resonance

13.1

In a circuit *R*, *L* and *C* are series connected. One measures the same voltage drop, 1 V, across all af the three components. How big is the supply voltage *U*?

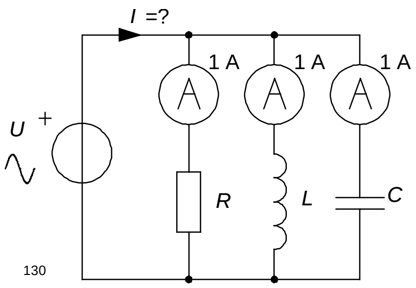
(*NOTE*! trick question)



13.2

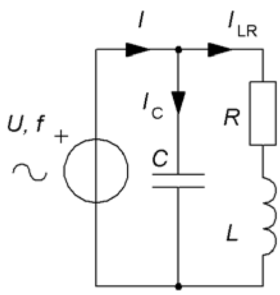
In a circuit *R*, *L* and *C* are in parallel. One measures the same current, 1 A, in the three parallel branches. How big is the current, *I*, that is taken from the supply voltage?

(*NOTE*! trick question)



13.3

At which frequency (expressed in *R L* and *C*) has the current *I* and voltage *U* the same phase?



13.4

A serial resonance circuit has the resonance frequency f_0 = 2000 Hz and the bandwith *BW* = 200 Hz.

a) Calculate the circuit Q-value.

b) One measures the coil resistance to R_S = 2 Ω. What value has X_L (at the resonance frequency)?

c) Calculate *L* and *C*.

d) Estimate the lower and upper limits of the bandwidth. Check that the estimate was reasonable.

13.5

A parallel resonant circuit is supplied from a current generator which supplies 80 mA at the resonant frequency f_0 =20 kHz.

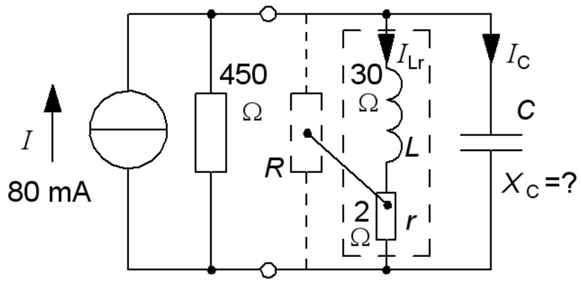
a) Check that the inductor has *Q* >10. Transform the serieresistance *r* to a parallel resistance *R*.

b) How large is the resulting impedance (source + resonant circuit) at the resonance frequency?

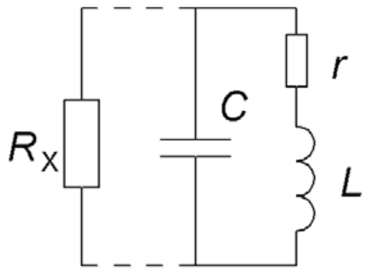
c) Calculate currents I_{Lr} and I_C.

d) What values has *L* and *C* ?

e) Calculate resulting Q-value and bandwith.

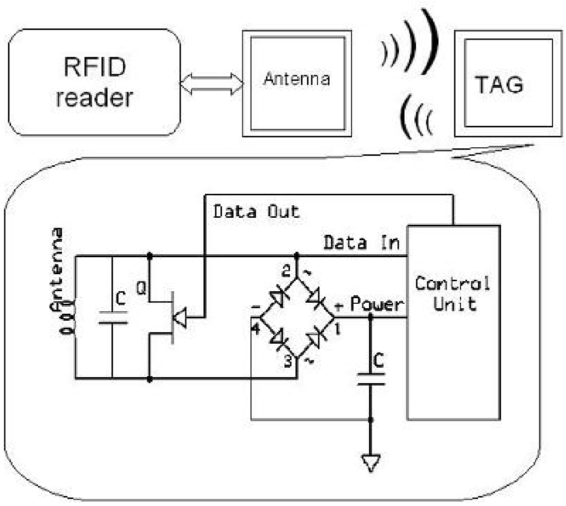


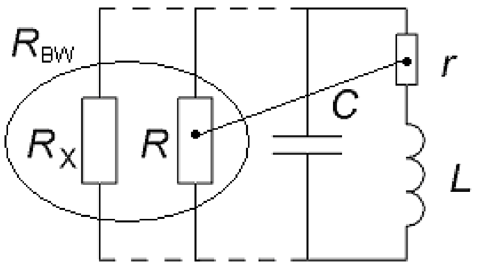
13.6



Stores theft buttons contains a resonant circuit with a high Q value, consisting of a small coil $L + r$ and a capacitor C. L = 5 μH r = 0,5 Ω and C = 25 pF.
a) Which resonant frequency has the circuit?
b) Which Q-value has the coil at resonance frequency?
c) one wishes to adjust the resulting Q-value of the resonance cirquit to exact Q = 500 by connecting a parallel resistor R_X. What value should R_X have?

13.7

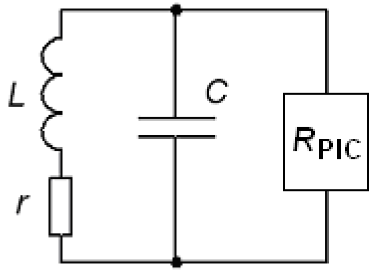




SL's access-card contains a so called RFID-tag. It communicates with the It communicates with a reader at the turnstyle at the frequency of 13.56 MHz and the data rate of 70 kHz. To "read" the data signal with this speed the resonance circuits included in the card and the reader needs to have a bandwidth at least twice the data rate ie 2·70 = 140 kHz.
(You only need to understand the resonant circuit $r\ L\ C$ in order to solve the task, not the RFID technology).
In figure is L = 2,5 μH and r = 1,5 Ω the embedded coil. C is the capacitance of the resonant circuit. The circuit parallel resistance R_X symbolises the processor connected to the resonant circuit, which consumes power from the resonant circuit.

a) Calculate the value of C that gives the wished resonance frequency?
b) What Q-value has the coil at resonance frequency?
c) Transform the coils serial resistans r to a parallel resistans R.
d) Suppose now that the coil only has parallel resistance.
What value should a parallel resistance R_{BW} have for the bandwith to be 140 kHz?
e) What value could the resistance R_X in parallel with R have for the bandwith to be 140 kHz?
($R_{BW} = R_X \| R$)

13.8



RFID-key-tags are used instead of door phone for access to apartment buildings. The communication is with radio frequency. The key tag containes a printed coil L r and a capacitor C, that forms a resonance circuit for the communication frequency. The resonance circuit is also loaded with a processor R_{PIC}.

L = 0,43 mH r = 4 Ω C = 3,77 nF R_{PIC} = 20 kΩ

a) What is the resonance frequency of the key tag? f = ? [kHz]

b) What Q-value has the printed coil at resonance frequency? Q_L = ? [times]

c) What resulting Q-value (with the processor included) will the resonance circuit have? Q_{res} = ? [times]

d) What is the resulting bandwidth of the resonance circuit? BW = ? [kHz]

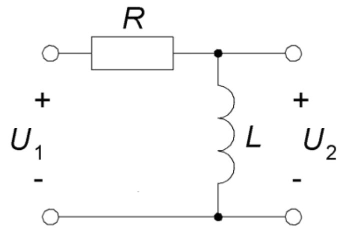
Filters

14.1

The figure shows a simple filter with L and R.

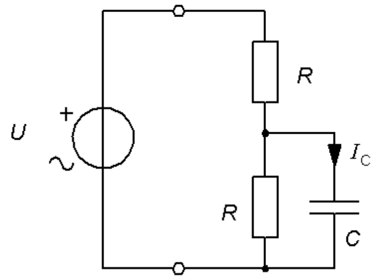
a) Derive the filter's transfer function.
b) Enter the filter magnitude function and phase function.
c) Give an expression for the filter cut-off frequency f_G.
d) What kind of filters is it LP HP BP BS ?

$$\frac{\underline{U}_2}{\underline{U}_1} = ? \quad \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = ? \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = ? \quad f_G = ? \quad LP\ HP\ BP\ BS\ ?$$



14.2

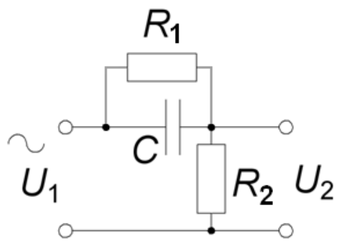
Set up an expression for $I_C(U, \omega, R, C)$.



14.3

The figure shows a simple filter with a capacitor C and two resistors R_1 and R_2.

a) Derive the filter transfer function, the ratio between the complex voltages $\frac{\underline{U}_2}{\underline{U}_1}$.



b) What values of the transfer function magnitude and phase angle going towards at the low frequencies?
What values of the transfer function magnitude and phase angle going towards at the very high frequencies?
What kind of filters is it, LP HP BP BS?

c) Set up an expression of the filter cutoff frequency (then the numerator real part and imaginary part are equal)?

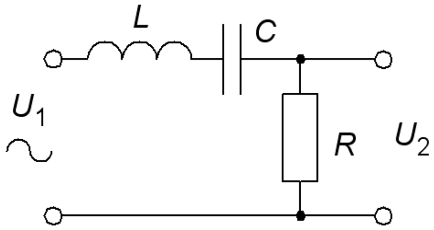
d) Suppose that R_1 = 1 kΩ and R_2 = 2 kΩ and C = 1 μF. Calculate cut off frequency.

14.4

The figure shows a simple filter with L C and R.

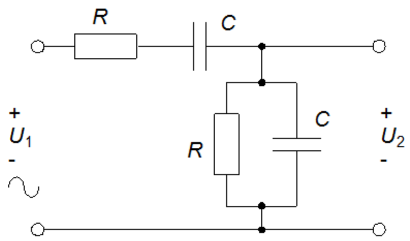
a) Derive the filter's transfer function.
b) Derive the filter magnitude function and phase function.
c) At what frequency will the numerator be purely real? Give an expression for this frequency f_x.
d) What is the value of the magnitude function at this frequency? What value does the phase function have at this frequency?
e) Examine the filter's magnitude and phase for very small frequencies ($f \approx 0$) and for very high frequencies ($f \approx \infty$).
What kind of filters is it, LP HP BP BS ?

$$\frac{\underline{U}_2}{\underline{U}_1} = ? \quad \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = ? \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = ? \quad f_X = ? \quad LP\ HP\ BP\ BS\ ?$$



14.5

Wien bridge often occur as feedback networks in amplifiers. (The two R , and the two C are the same).



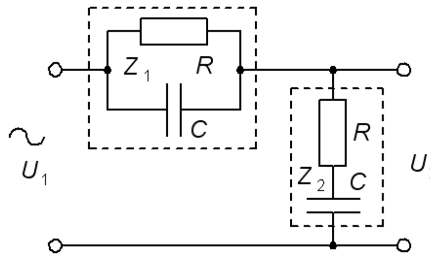
What value does $\frac{U_2}{U_1}$ approach at high, respective low frequencies?

For what value of f (expressed in R and C) does U_2 and U_1 has the same phase angle?

How big is the ratio $\frac{U_2}{U_1}$ at this frequency?

14.6

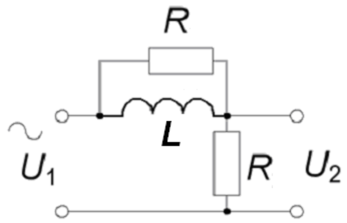
The figure shows the Wien bridge "backwards".



a) Derive the filter's transfer function.
b) (Sketch magnitude function and phase function.)
c) What amount and phase angle has the transfer function when $\omega = 1/RC$?

14.7

The figure shows a simple filter with two R and a L.

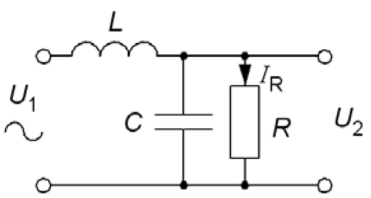


a) Derive the filter's complex transfer function $\underline{U}_2/\underline{U}_1$.
b) At what angular frequency ω_X will the magnitude function be $|\underline{U}_2|/|\underline{U}_1| = 1/\sqrt{2}$?
Give an expression for this frequency ω_X with $R\,L$.

c) What value is the transfer function magnitude going towards at the low frequencies, $\omega \approx 0$? What value has the phase function at very low frequencies?
d) What value is the transfer function magnitude going towards at very high frequencies, $\omega \approx \infty$? What value has the phase function at very high frequencies?

14.8

The figure shows a simple filter with L C and R.



a) Derive the filter's transfer function $\underline{U}_2/\underline{U}_1$.
b) At what angular frequency ω_x will denominator be purely imaginary?
Give an expression for this frequency ω_x with $R\,L$ and C.

c) What value has the transfer function magnitude at this angular frequency, ω_x?
d) What value has the phase function at this angular frequency, ω_x ?
e) Give an expression for the transfer function $\underline{I}_R/\underline{U}_1$. (Note! You alredy have $\underline{U}_2/\underline{U}_1$ from a)

$$a)\ \frac{\underline{U}_2(\omega)}{\underline{U}_1(\omega)} = ? \quad b)\ \omega_X(R,L,C) = ? \quad c)\ \left|\frac{\underline{U}_2(\omega_X)}{\underline{U}_1(\omega_X)}\right| = ? \quad d)\ \arg\left(\frac{\underline{U}_2(\omega_X)}{\underline{U}_1(\omega_X)}\right) = ? \quad e)\ \frac{\underline{I}_R(\omega)}{\underline{U}_1(\omega)} = ?$$

Transformer, inductive coupling

15.1

For a transformer in operation the following data was measured:

Primary			Secondary		
N_1	U_1	I_1	N_2	U_2	I_2
600	225 V	**?**	200	**?**	9 A

Calculate the two missing values.

15.2

For a transformer in operation the following data was measured:

Primary			Secondary		
N_1	U_1	I_1	N_2	U_2	I_2
?	230 V	2A	150	**?**	12 A

Calculate the two missing values.

15.3

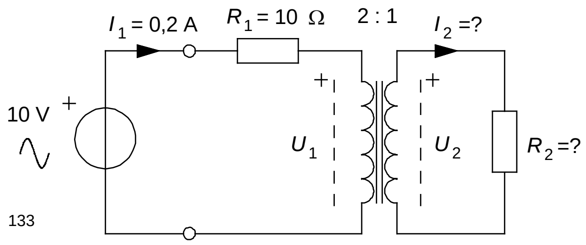
For a transformer in operation the following data was measured:

Primary			Secondary		
N_1	U_1	I_1	N_2	U_2	I_2
600	225 V	**?**	**?**	127 V	9 A

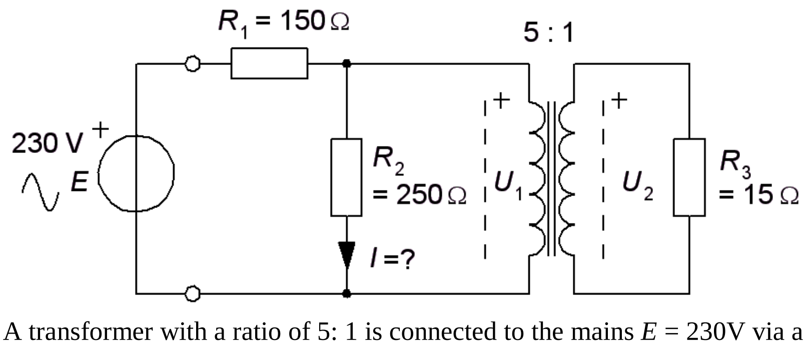
Calculate the two missing values.

15.4

U = 10 V, 50 Hz and I_1 = 0,2 A. Calculate I_2 and R_2.



15.5

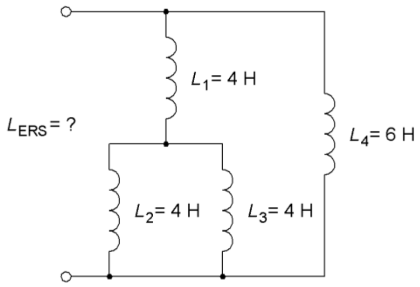


A transformer with a ratio of 5: 1 is connected to the mains E = 230V via a voltage divider R_1 = 150 Ω and R_2 = 250 Ω. The secundari side of the transformer has the load R_3 = 15 Ω.

Calculate the current I through resistor R_2.

15.6

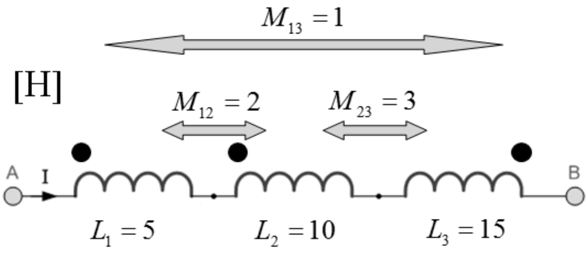
Four coils are connected as shown. They are totally independent of each other, they have no common flow.
How large is the resulting inductance L_{ERS} = ?



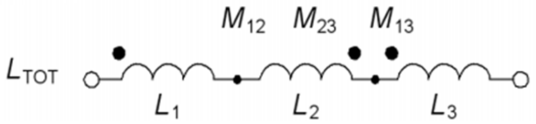
15.7

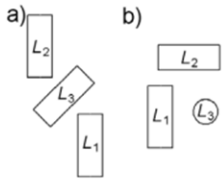
Calculate the total inductance of three series connected coils placed so that they access portions of each flow.
L_1 = 5 [H], L_2 = 10 [H], L_3 = 15 [H],
M_{12} = 2 [H], M_{23} = 3 [H], M_{13} = 1 [H].

L_{TOT} = ? [H].



15.8





Three inductors L_1 = 12, L_2 = 6, L_3 =5 [H] are series connected.

When inductors are close to each other the placement on the circuit board can be important. In the figure to the left a) will inductors to have a portion of the magnetic lines in common. They then have the mutual inductances M_{12} = 3, M_{23} = 1, M_{13} = 1 [H].

In the figure to the right b) the inductors are mounted three dimensional so that there are no shared power magnetic lines.

a) Calculate the total inductance for the arrangement in figure a). L_{TOT} = ?

b) Calculate the total inductance for the arrangement in figure b). L_{TOT} = ?

Solutions

Equivalent resistance

1.1

$$R_{\text{tot}} = 2 \cdot \left(\frac{1 \cdot (0{,}5 + 0{,}5)}{1 + 0{,}5 + 0{,}5} \right) = 1\Omega$$

1.2

$$R_{\text{tot}} = \frac{R_4 \left(R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3} \right)}{R_4 + \left(R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3} \right)} = \frac{30 \cdot \left(1 + \frac{21 \cdot 42}{21 + 42} \right)}{30 + \left(1 + \frac{21 \cdot 42}{21 + 42} \right)} = \frac{30(1 + 14)}{30 + 1 + 14} = \frac{30 \cdot 15}{45} = 10\ \Omega$$

1.3

$$R_{45} = 1 + 2 = 3 \quad R_{345} = \frac{6 \cdot 3}{6 + 3} = 2 \quad R_{2345} = 4 + 2 = 6 \quad R_{\text{ERS}} = R_{12345} = \frac{1 \cdot 6}{1 + 6} = 0{,}86\ \Omega$$

1.4

$$R_{\text{ERS}} = 0{,}5 + 1{,}6 + \frac{5{,}2\left(2{,}7 + \frac{7 \cdot 3}{7 + 3} \right)}{5{,}2 + 2{,}7 + \frac{7 \cdot 3}{7 + 3}} = 4{,}6\ \Omega$$

1.5

The three resistors $R_1 \ldots R_3$ are connected in parallel

[kΩ] : $R_{1,2,3} = \dfrac{1}{\frac{1}{28} + \frac{1}{84} + \frac{1}{56}} = 15{,}27$. The resistor R_4 is in parallel with a "short circuit wire" (R=0),

$\dfrac{0 \cdot R_4}{0 + R_4} = 0$. Totally we get $R_{\text{tot}} = R_{1,2,3} + 0 = 15{,}27\ \text{k}\Omega$.

1.6

The four resistors $R_2 \ldots R_5$ are in parallel.

$R_{2,3,4,5} = \dfrac{1}{\left(\frac{1}{12} + \frac{1}{12} + \frac{1}{24} + \frac{1}{24} \right)} = 4\ \Omega$ and thereafter connected in series with R_1. $R_{\text{tot}} = 4 + 2 = 6\ \Omega$.

1.7

$$R_{\text{TOT}} = 15 + \frac{1}{\frac{1}{15} + \frac{1}{15} + \frac{1}{15}} + \frac{1}{\frac{1}{15} + \frac{1}{15} + \frac{1}{15} + \frac{1}{15} + \frac{1}{15}} = 15 + 5 + 3 = \mathbf{23}\ \Omega$$

1.8

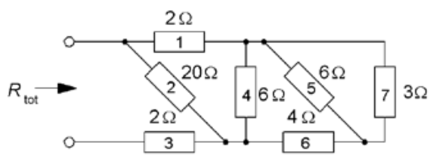
The circuit consits of two equal parallel branches. One of the branches have:

$R_{\text{ERS}} = \dfrac{20 \cdot 5}{20 + 5} + 2 = 6$. Then the total equivalent resistance is: $R_{\text{TOT}} = \dfrac{6 \cdot 6}{6 + 6} = \mathbf{3\ \Omega}$

1.9

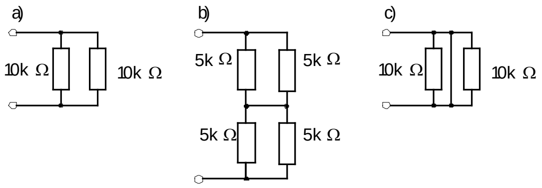
$$R_{tot} = \Big(\big(\big((R_7 \parallel R_5) + R_6 \big) \parallel R_4 \big) + R_1 \Big) \parallel R_2 + R_3$$

$$R_{5,7} = \frac{6 \cdot 3}{6 + 3} = 2 \quad R_{4,5,6,7} = \frac{(2 + 4) \cdot 6}{2 + 4 + 6} = 3$$



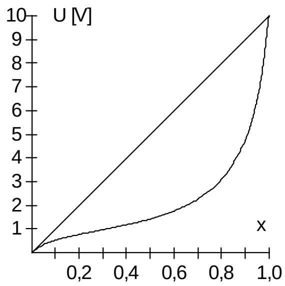
$$R_{1,2,4,5,6,7} = \frac{(3 + 2) \cdot 20}{3 + 2 + 20} = 4 \quad R_{tot} = 4 + 2 = 6\ \Omega$$

1.10



a) 10/2 = 5k **b**) 5/2+5/2 = 5k **c**) 0

1.11



Resistivity and resistors temperature dependence

2.1

A smart trainee go and get a Ω-meter and measures the resistance in the two series connected wires of the cable.
This measurement gives $2R = 2{,}3\ \Omega$.
Each wire then has the resistance $R = 1{,}15\ \Omega$.
The length l of the cable can be calculated:
$l = (R\times A) / \rho = 1{,}15\times2{,}5/0{,}018 = 159{,}7$ m
It had been troublesome to measure out the length of the cablewith measuring tape!

2.2

$$R_2 = \frac{U}{I} = \frac{20}{0{,}11} = 182\ \Omega \quad R_1 = 98\ \Omega \quad \alpha = 4{,}5\cdot 10^{-3} \quad t_1 = 98\ °\text{C}$$

$$R_2 = R_1 + R_1\cdot\alpha(t_2 - t_1) \quad \Leftrightarrow \quad t_2 = \frac{182-98}{98\cdot 4{,}5\cdot 10^{-3}} + 22 = 212{,}5\ °\text{C}$$

The radiation thermometer thus showed 60° improper!

2.3

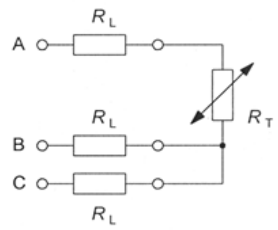
a) $\alpha_{NI} = 6{,}7\cdot 10^{-3}$

$$R_{\text{koka}} = R_{\text{rum}} + R_{\text{rum}}\cdot\alpha_{\text{NI}}(t_{\text{koka}} - t_{\text{rum}}) = 50 + 50\cdot 6{,}7\cdot 10^{-3}\cdot 75 = \mathbf{75{,}}1\ \Omega$$

b) $$I = \frac{E}{R+R_1} = \frac{12}{75{,}1+25} = \frac{12}{100{,}1} = 0{,}12\ \text{A} \quad \Leftrightarrow \quad P = I^2\cdot R = 0{,}12^2\cdot 75{,}1 = \mathbf{1{,}08\ W}$$

2.4

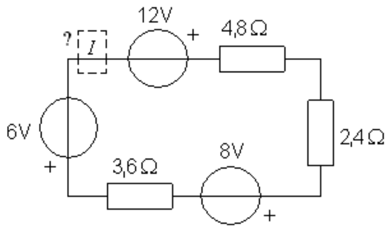
Describe the principle of so-called three wire connection.

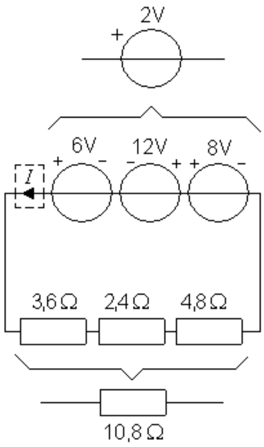


At three-wire measurement, measure first the resistance between A and B. Then you get with $2 \cdot R_L$ to much. By measuring the resistance between B and C you find out how much $2 \cdot R_L$ is and is thereby able to correct the first reading.

Serial – parallel circuits

3.1





$$8+6-12=2 \quad 3{,}6+2{,}4+4{,}8=10{,}8 \qquad I=\frac{2}{10{,}8}=0{,}19\text{ A}$$

3.2

a) $R_{RES} = 2\Omega$ b) $I = 4$ A, $U = 8$ V c) $I_1 = 1$ A, $I_2 = 2$ A, $I_3 = 1$ A, $U_1 = 6$ V

3.3

We calculate two equivalent resistances.

$$R_{1,2}=\frac{24\cdot 12}{24+12}=8\ \Omega \qquad R_{3,4,5}=\frac{1}{\frac{1}{9}+\frac{1}{18}+\frac{1}{6}}=3\Omega$$

U can be calculated with the voltage division formula:

$$U_{R2}=E\frac{R_{1,2}}{R_{1,2}+R_{3,4,5}}=12\frac{8}{8+3}=8{,}73\text{V}$$

Voltage over $R_{3,4,5}=E-U=12-8{,}73=3{,}27$ V and then $I=\frac{E-U}{R_5}=3{,}27/6=0{,}55$ A .

3.4

$$R_{tot}=4+\frac{\frac{4}{2}\cdot(0{,}5+1{,}5)}{\frac{4}{2}+(0{,}5+1{,}5)}=5 \qquad I_{tot}=\frac{E}{R_{tot}}=\frac{10}{5}=2\text{ A}$$

The current is divided between thre parallel branches: 4//4//2. Over these is the voltage =

$=E-I_{tot}\cdot 4=10-2\cdot 4=2$ V . We get $I=\frac{2}{4}=0{,}5$ A and $U=2\frac{0{,}5}{0{,}5+1{,}5}=0{,}5$ V

3.5

We calculates an equivalent resistance. $\dfrac{6\cdot(1+2)}{6+1+2}=2$

U_{R1} = 36 V. U_{R3} can be calculated with the voltage division rule: $36\dfrac{2}{4+2}=12$

then $I = I_{R3} = \dfrac{U_{R3}}{R_3}$ = 12 / 6 = 2 A

Voltage U over R_5 can be calculated by voltage division rule: $12\dfrac{2}{1+2}=8$

3.6

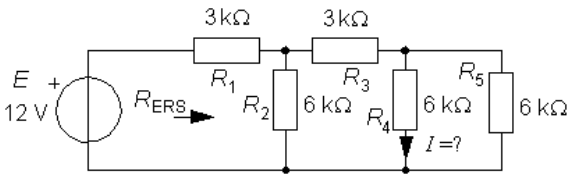
a) $R_{ERS} = R_1 + \left(R_2 \,\|\, (R_3 + (R_4 \,\|\, R_5))\right) =$

$R_{45} = \dfrac{6\cdot 6}{6+6} = 3 \quad R_{345} = 3+3 = 6$

$R_{2345} = \dfrac{6\cdot 6}{6+6} = 3 \quad R_{ERS} = R_{12345} = 3+3 = 6$

b) $I_{R1} = \dfrac{E}{R_{ERS}} = \dfrac{12}{6} = 2 \quad I_{R3} = \dfrac{I_{R1}}{2} = \dfrac{2}{2} = 1$

$I_{R4} = \dfrac{I_{R3}}{2} = \dfrac{1}{2} = 0{,}5\,[\text{mA}]$



Batteries

4.1

a) C_{20} = 60 Ah means that the discharge has been ongoing for 20 hours and that the battery capacity $I\times t$ was 60 Ah. The constant discharge current I that was used was then 60/20 = 3 A.

b) The capacity number is developed at the current 3 A. Then one can assume that the battery capacity is unchanged at the nearby current value of 1 A. We get $t = C/I$ = 60/1 = 60 h.

c) The high current 300 A is a completely different operating condition than that used by the manufacturer to develop the capacity number. From experience (here given) we know that the capacity gets lower at high currents. Therefore, it is expected that the capacity's reduced till 70%. $C' = 0{,}7\times C = 0{,}7\times 60 = 42$.

We get $t = C'/I$ = 42/300 = 0,14 h 0,14×60 = 8,4 min.

4.2

a) $I = \dfrac{E}{R_I + R} \quad \Leftrightarrow \quad 0{,}123 = \dfrac{1{,}4}{R_I + 10} \quad \Leftrightarrow \quad R_I = \dfrac{1{,}4}{0{,}123} - 10 = 1{,}38\ \Omega$

b) $I_{MAX} = \dfrac{E}{R_I} = \dfrac{1{,}4}{1{,}38} = 1{,}01$ A

4.3

a) $U = 6 \quad I = 1{,}75 \quad n\cdot E - n\cdot R_i \cdot I - U = 0 \quad n = \dfrac{U}{E - I\cdot R_i} = \dfrac{6}{1{,}1-1{,}75\cdot 0{,}2} = \mathbf{8\ st}$

b) At serial connection the voltage is increased but the capacity will be the same. C = 3000 mAh.

$C = I\cdot t \quad \Rightarrow \quad I = \dfrac{C}{t} = \dfrac{3}{1} = \mathbf{3\ A}$

c) $24 - 8\cdot 1{,}1 - R\cdot 3 - 8\cdot 0{,}2\cdot 3 = 0 \quad R = \dfrac{24-8\cdot 1{,}1 - 8\cdot 0{,}2\cdot 3}{3} = \mathbf{3{,}47\ \Omega}$

4.4

Three equal batteries can be merged into one with E = 10V and R_I = 6/3 = 2 Ω.
a) I = 10/(2 + 2) = 2,5 A. U = 2·2,5 = 5 V.
b) Two equal batteries can be merged to one with E = 10V and R_I = 6/2 = 3 Ω.
Kirchoff's current law gives:

- $I_1 - I_2 - I = 0$

Kirchoff's voltage law around two meshes gives:

- $10 - 3 \cdot I_1 + 10 - 6I_2 = 0 \quad \Leftrightarrow \quad -3I_1 - 6I_2 + 0I = -20$
- $6I_2 - 10 - 2I = 0 \quad \Leftrightarrow \quad 0I_1 + 6I_2 - 2I = 10$

On matrix form:

$$\begin{pmatrix} 1 & -1 & -1 \\ -3 & -6 & 0 \\ 0 & 6 & -2 \end{pmatrix} \bullet \begin{pmatrix} I_1 \\ I_2 \\ I \end{pmatrix} = \begin{pmatrix} 0 \\ -20 \\ 10 \end{pmatrix} \qquad I_1 = \mathbf{2{,}78}\text{ A} \quad I_2 = \mathbf{1{,}94}\text{ A} \quad I = \mathbf{0{,}83}\text{ A}$$

$U = I \cdot 2 = 0{,}83 \cdot 2 = \mathbf{1{,}67}$ V

Kirchoff's laws will be next in course!

Kirchoff's current law and voltage law

5.1

I_1 = 5 A, I_2 = 2,5 A, I_3 = 2,5 A och I_4 = 5 A.

5.2

$$E = R_{TOT} \cdot I = 2{,}62 \cdot 10 = 26{,}2 \text{ V}$$

$$I_4 = \frac{E}{4} = \frac{26{,}2}{4} = \mathbf{6{,}55\ A} \qquad I_1 = I - I_4 = 10 - 6{,}55 = \mathbf{3{,}45\ A}$$

$$I_3 = \frac{5{,}5}{2} = \mathbf{2{,}75\ A}$$

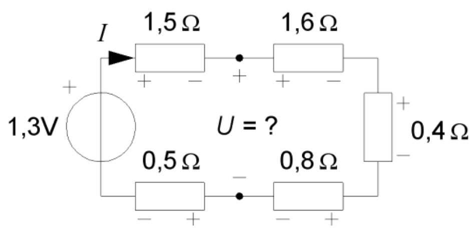
5.3

$$I = \frac{1{,}3}{1{,}5 + 1{,}6 + 0{,}4 + 0{,}8 + 0{,}5} = 0{,}27$$

$$U_{0,5} = 0{,}5 \cdot 0{,}27 = 0{,}14$$

$$U_{1,5} = 1{,}5 \cdot 0{,}27 = 0{,}41$$

U = – 0,14 + 1,3 – 0,41 = 0,76 V
or U = 0,27·(0,8+0,4+1,6) = 0,76 V

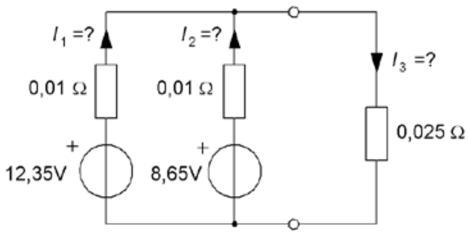


Kirchoffs lagar, ekvationssystem

6.1

Kirchoff's current law:

$I_1 + I_2 - I_3 = 0 \quad \Rightarrow \quad I_3 = I_1 + I_2$



Kirchoff's voltage law:

$12{,}35 - 0{,}01I_1 - 0{,}025I_3 = 0 \quad \Leftrightarrow \quad 12{,}35 - 0{,}01I_1 - 0{,}025(I_1 + I_2) = 0 \quad \Leftrightarrow \quad 0{,}035I_1 + 0{,}025I_2 = +12{,}35$

Kirchoff's voltage law:

$8{,}65 - 0{,}01 \cdot I_2 - 0{,}025I_3 = 0 \quad \Leftrightarrow \quad 8{,}65 - 0{,}01 \cdot I_2 - 0{,}025(I_1 + I_2) = 0 \quad \Leftrightarrow \quad 0{,}025I_1 + 0{,}035I_2 = 8{,}65$

two equations, two unknowns, thus solvable.

$$\begin{pmatrix} 0{,}035 & 0{,}025 \\ 0{,}025 & 0{,}035 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 12{,}35 \\ 8{,}65 \end{pmatrix} \quad I_1 = 360 \text{ A} \quad I_2 = -10 \text{ A} \quad I_3 = I_1 + I_2 = 350 \text{ A}$$

The bad battery degrades the starting current with 10 A!

6.2

Kirchoff's current law gives:

- $-I_1 + I_2 + I_3 = 0$

Kirchoff's voltage law around two meshes gives:

- $E_1 - I_1 \cdot R_1 - I_2 \cdot R_4 E_3 - I_1 \cdot R_3 = 0 \quad \Leftrightarrow \quad -3I_1 - 15I_2 + 0I_3 = -9$
- $-E_2 - I_3 \cdot R_2 + I_2 \cdot R_4 = 0 \quad \Leftrightarrow \quad 0I_1 + 15I_2 - 2I_3 = 21$

On matrix form:

$$\begin{pmatrix} -1 & 1 & 1 \\ -3 & -15 & 0 \\ 0 & 15 & -2 \end{pmatrix} \bullet \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -9 \\ 21 \end{pmatrix} \quad \text{solution:} \qquad I_1 = \mathbf{-2} \text{ A} \quad I_2 = \mathbf{1} \text{ A} \quad I_3 = \mathbf{-3} \text{ A}$$

6.3

a) Over 18Ω-resistor is E_1 18 V.
(I_3 = -18/18 = -1 A in opposite direction than stated in figure)

b) $E_2 - R_1 I_2 - E_1 = 0 \quad \Rightarrow I_2 = -\dfrac{E_1 - E_2}{R_1} = -\dfrac{16 - 12}{6} = -1 \text{ A}$.

I_2 has opposite direction than stated in figure.

c) $I_1 + I_2 + I_3 = 0$ $\qquad\qquad$ $I_1 = 1 + 1 = 2$ A

6.4

Kirchoff's current law gives:

- $I_1 - I_2 - I = 0$

Kirchoff' voltage law around two meshes gives:

- $-I_2 \cdot R_3 + E_2 - I_1 \cdot R_2 = 0 \quad \Leftrightarrow \quad I_1 + 3I_2 + 0 \cdot I = 12$
- $I_2 \cdot R_3 - I \cdot R_1 - E_1 - I \cdot R = 0 \quad \Leftrightarrow \quad 0I_1 + 3I_2 - 6I = 6$

On matrix form:

$$\begin{pmatrix} 1 & -1 & -1 \\ 1 & 3 & 0 \\ 0 & 3 & -6 \end{pmatrix} \bullet \begin{pmatrix} I_1 \\ I_2 \\ I \end{pmatrix} = \begin{pmatrix} 0 \\ 12 \\ 6 \end{pmatrix} \quad \text{solution:}$$

$I_1 = 3{,}33$ A $\quad I_2 = 2{,}89$ A $\quad I = \mathbf{0{,}44\ A}$

$\Rightarrow \quad U = I \cdot R = 4 \cdot 0{,}44 = \mathbf{1{,}78\ V}$

6.5

Kirchoff's current law:

$I_1 + I_2 + I_3 = 0$

Kirchoff's voltage law (left):

$-25 - 2 \cdot I_1 + 3 \cdot I_2 + 60 = 0$

reorder:

$-2 \cdot I_1 + 3 \cdot I_2 + 0 \cdot I_3 = -35$

Kirchoff's voltage law (right):

$-60 - 3 \cdot I_2 + 6 + 5 \cdot I_3 - 20 = 0$

reorder:

$0 \cdot I_1 - 3 \cdot I_2 + 5 \cdot I_3 = 74$

Equation system on matrix form:

$$\begin{pmatrix} 1 & 1 & 1 \\ -2 & 3 & 0 \\ 0 & -3 & 5 \end{pmatrix} \bullet \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -35 \\ 74 \end{pmatrix}$$

a) $\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 1{,}87 \\ -10{,}4 \\ 8{,}55 \end{pmatrix}$

b) Voltage at voltmeter $U = -E_3 - R_3 \cdot I_3 = -6 - 5 \cdot 8{,}55 = -48{,}75$ V

6.6

Kirchoff's current law:

- $I_1 + I_2 + I_3 = 0$

Kirchoff's voltage law around two meshes gives:

- $-I_1 \cdot R_1 + I_3 \cdot R_2 - E_2 = 0 \quad \Leftrightarrow \quad -4I_1 + 0I_2 + 6I_3 = 15$
- $E_2 - I_3 \cdot R_2 + E_1 + I_2 \cdot R_3 - E_3 = 0 \quad \Leftrightarrow \quad 0I_1 + 8I_2 - 6I_3 = -8$

On matrix form:

$$\begin{pmatrix} 1 & 1 & 1 \\ -4 & 0 & 6 \\ 0 & 8 & -6 \end{pmatrix} \bullet \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 15 \\ -8 \end{pmatrix}$$

solution: $I_1 = \mathbf{-1{,}56}\text{A} \quad I_2 = \mathbf{0{,}1}\text{A} \quad I_3 = \mathbf{1{,}46}\text{A}$

Potential, node analysis, dependent sources

7.1

$U_{R1} = 12 \cdot \frac{100}{100 + 110 + 120} = 3{,}64 \text{ V} \quad U_{R2} = 12 \cdot \frac{110}{100 + 110 + 120} = 4 \text{ V} \quad U_{R3} = 12 \cdot \frac{120}{100 + 110 + 120} = 4{,}37 \text{ V}$

Socket	a)	b)	c)	d)
Voltmeter [V]	-4,37	0	+4	4+3,64 = 7,64

7.2

$$-I_1 - I_2 + 1 = 0 \quad I_1 + I_2 = 1$$

$$I_2 = \frac{U}{R_2} = \frac{U}{12}$$

$$I_1 = \frac{U-E}{R_1} = \frac{U-24}{6}$$

$$1 = \frac{U}{12} + \frac{U-24}{6} = \frac{2\cdot U - 48 + U}{12} \quad \Leftrightarrow \quad 12 = 3\cdot U - 48$$

$$U = 20 \text{ V}$$

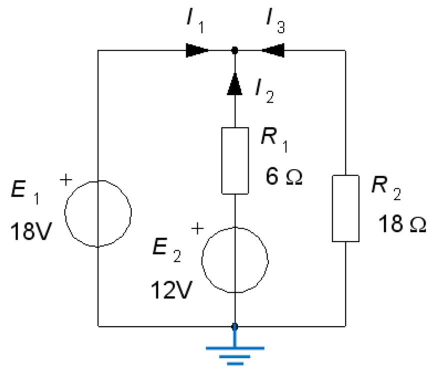
$$I_2 = \frac{20}{12} = 1{,}67$$

$$I_1 = \frac{20-24}{6} = -0{,}67$$

$$I_1 + I_2 = 1 \quad -0{,}67 + 2{,}67 = 1$$

7.3 Node analysis

$I_1 + I_2 + I_3 = 0 \quad I_1 = -I_2 - I_3$
$E_1 = 18$ V
$I_3 = -\ (E_1 -\ 0)/R_2 = -18/18$
$= -1$ A
$I_2 = -(E_1 - E_2)/R_1 = -(18 - 12)/6 = = -1$A
$I_1 = -\ I_2 -\ I_3\ = -\ (-1) -\ (-1) =\ 2$ A



7.4 Dependent source

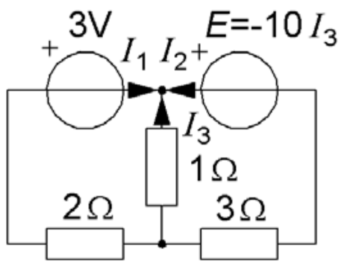
Kirchoff's current law: $I_1 + I_2 + I_3 = 0$

Kirchoff's voltage law (not dependent emf):

$$-2I_1 - 3 + 1I_3 = 0 \quad \Leftrightarrow \quad -2I_1 + 0I_2 + 1I_3 = 3$$

Kirchoff's voltage law (with the dependent emf):

$$-1I_3 - (-10I_3) + 3I_2 = 0 \quad \Leftrightarrow \quad 0I_1 + 3I_2 + 9I_3 = 0$$

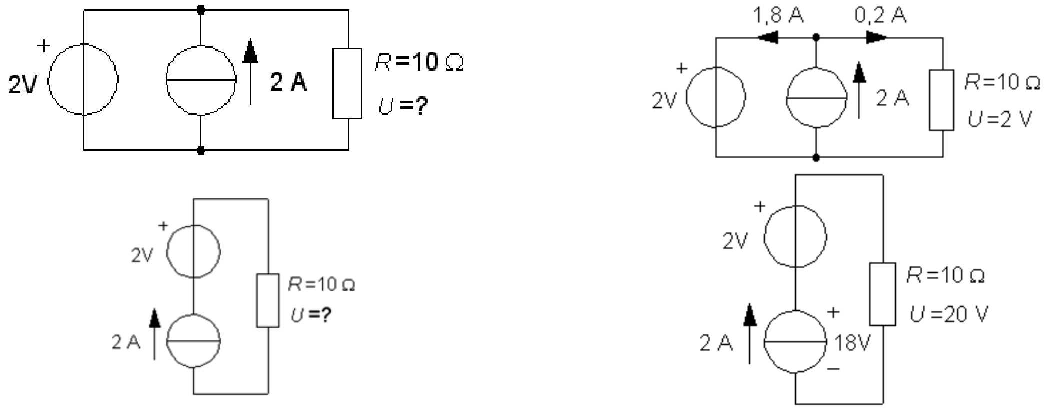


$$\begin{pmatrix} 1 & 1 & 1 \\ -2 & 0 & 1 \\ 0 & 3 & 9 \end{pmatrix} \bullet \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \qquad I_1 = -2 \quad I_2 = 3 \quad I_3 = -1$$

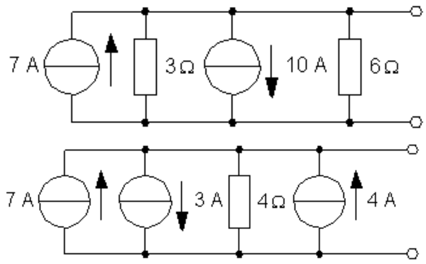
(numerical values are the same as in the course throughout lecture example …)

Thevenin and Norton equivalents, superposition

8.1

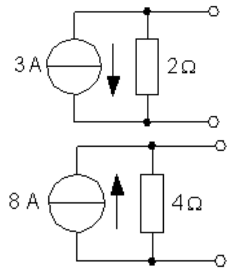


8.2



$$7-10=-3$$

$$\frac{3\cdot 6}{3+6}=2$$



8.3

$E_K = 1$ V, $R_I = 1$ Ω

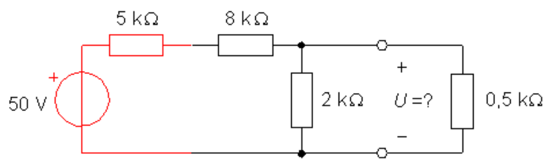
8.4

a) $$U_{R16}=100\cdot\frac{\dfrac{16\cdot(4+12)}{16+4+12}}{4+\dfrac{16\cdot(4+12)}{16+4+12}}=100\frac{8}{12}=66{,}67 \qquad U_{R12}=66{,}67\cdot\frac{12}{4+12}=50\text{ V}$$

b) c) $$R_I=12\,\|\,(4+4\,\|\,16)=\frac{12\left(4+\dfrac{4\cdot16}{4+16}\right)}{12+4+\dfrac{4\cdot16}{4+16}}=4{,}5\text{ k}\Omega \quad\Rightarrow\quad I_K=\frac{50}{4{,}5}=11{,}1\text{ mA}$$

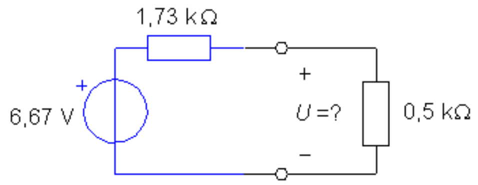
d) $$R_X=R_I \quad\Rightarrow\quad P=\frac{E_0^2}{4\cdot R_I}=\frac{50^2}{4\cdot4{,}5\cdot10^3}=0{,}114\text{ W}$$

8.5



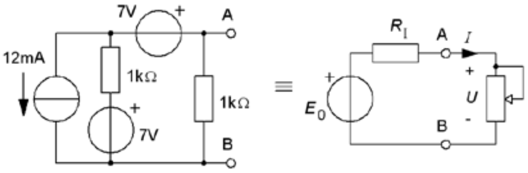
$$\frac{2\cdot(5+8)}{2+5+8}=1{,}73$$

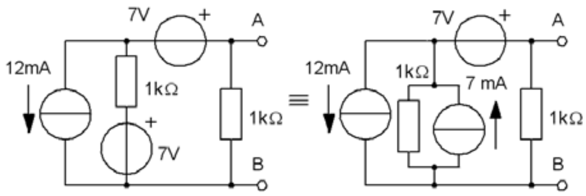
$$50\cdot\frac{2}{2+5+8}=6{,}67$$



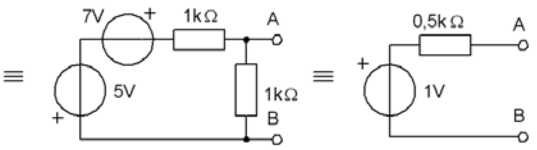
$$U = 6{,}67 \cdot \frac{0{,}5}{0{,}5 + 1{,}73} = 1{,}49\ \mathrm{V}$$

8.6

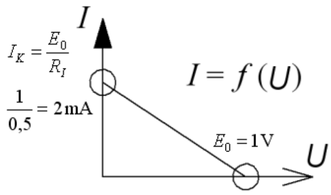




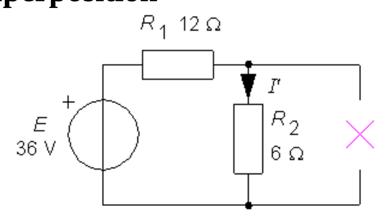
a) 7V and 1k transforms to a current source, 7 mA and 1k. Then two currentsources can be merged into one 5 mA and 1k. Transform to voltage source 5V and 1k. Total 2 V with a voltage divider. Last 1V and 0,5 k.

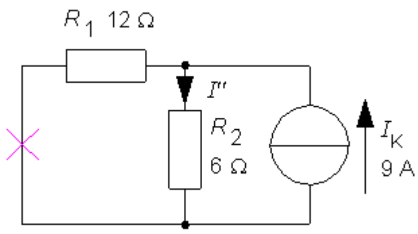


b) The two port $I = f(U)$ is a line. Dry run $U = E_0 = 1\mathrm{V}$ and short circuit $I_K = E_0/R_I = 2$ mA are two points on the line.



8.7 superposition





$$I' = \frac{E}{R_1 + R_2} = \frac{36}{12+6} = 2$$

$$I'' = I_K \cdot \frac{R_1}{R_1 + R_2} = 9 \cdot \frac{12}{12+6} = 6$$

A turned down current source is a circuit break!
I’ one can calculate with OHM’s law.

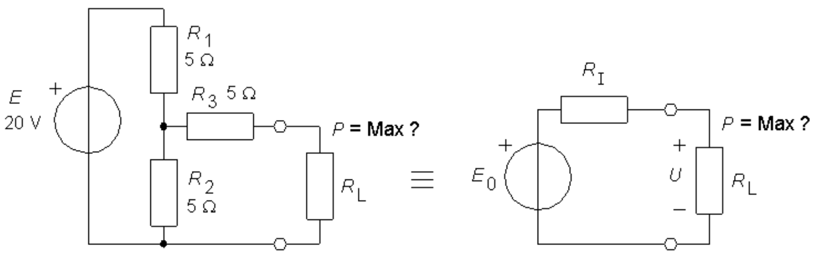
A turned down voltage source is a circuit short cut.
I” one can calculate with current branching.

I = *I’* + *I”* = 2 + 6 = 8 A

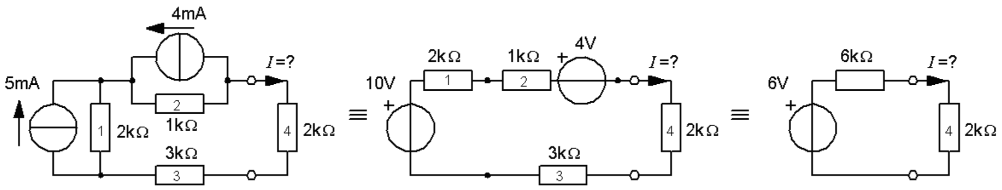
8.8

$$R_I = R_3 + \frac{R_1 \cdot R_2}{R_1 + R_2} = 5 + \frac{5 \cdot 5}{5+5} = 7{,}5 \qquad E_0 = E \cdot \frac{R_2}{R_1 + R_2} = 20 \cdot \frac{5}{5+5} = 10$$

$$P_{MAX} = \frac{E_0^2}{4 \cdot R_I} = \frac{10^2}{4 \cdot 7{,}5^2} = 3{,}33\ \mathrm{W}$$



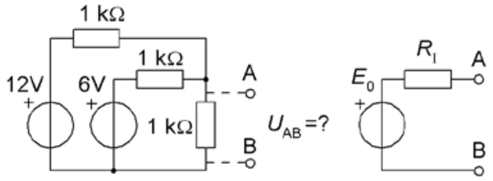
8.9



5mA||2kΩ ⇔ 10V+2kΩ, 4mA||1kΩ ⇔ 4V+1kΩ ⇒ 6V+6kΩ

$$I = \frac{E_0}{R_I + R_L} = \frac{6}{6+2} = 0{,}75\ \text{mA}$$

8.10



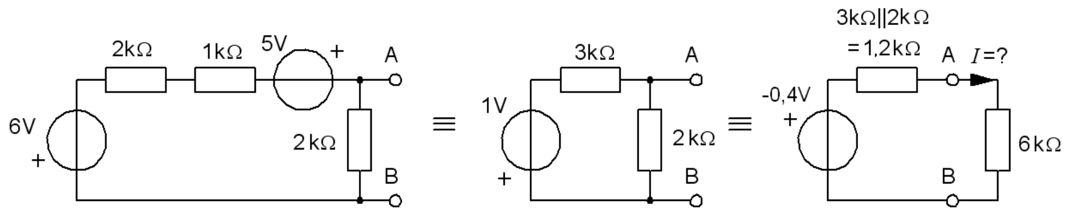
$$R_I = \frac{1}{\frac{1}{1\text{k}\Omega} + \frac{1}{1\text{k}\Omega} + \frac{1}{1\text{k}\Omega}} = \frac{1}{3}\ \text{k}\Omega$$

Suppose A and B short cut. The third 1 kΩ resistor is then without current.

$$I_K = \frac{12\text{V}}{1\text{k}\Omega} + \frac{6\text{V}}{1\text{k}\Omega} = 18\ \text{mA} \quad I_K = \frac{E_0}{R_I} \quad \Rightarrow \quad E_0 = I_K \cdot R_I = 18 \cdot \frac{1}{3} = 6\text{V}$$

The voltage U_{AB} has the same value as E_0.

8.11



The current source and 1 kΩ resistor can be transformed into a voltage source. The whole circuit will be a 1V voltage source with a voltage divider.

$$E_0 = 1\frac{2}{3+2} = 0{,}4\ \text{V} \quad R_I = \frac{3 \cdot 2}{3+2} = 1{,}2\ \text{k}\Omega$$

The unloaded voltage will be 0,4V, and the internal resistance 3kΩ||2kΩ = 1,2kΩ. Note that the voltage source 0,4V is counter defined in the original figure.

At last the the current (with electronics prefixes: mA kΩ V) I = -0,4/(1,2+2) = -0,125 [mA]

Capacitance, magnetism, inductance

9.1

$W = \frac{1}{2} \cdot C \cdot U^2$ a) $W = \frac{1}{2} \cdot C \cdot U^2 = \frac{1}{2} \cdot 1000 \cdot 10^{-6} \cdot 100^2 = 5 \text{ J, Ws}$

$Q = C \cdot U$ b) $Q = C \cdot U = 1000 \cdot 10^{-6} \cdot 100 = 0{,}1 \text{ C, As}$

$I = \frac{Q}{t}$ c) $I = \frac{Q}{t} = \frac{0{,}1}{1/2000} = 200 \text{ A}$

$P = \frac{W}{t}$ d) $P = \frac{W}{t} = \frac{5}{1/2000} = 10 \text{ kW}$

d) $U = \frac{Q}{C} = \frac{I_{\text{Ladda}} \cdot t_{\text{Ladda}}}{C} \Rightarrow t_{\text{Ladda}} = \frac{C \cdot U}{I_{\text{Ladda}}} = \frac{1000 \cdot 10^{-6} \cdot 100}{10 \cdot 10^{-3}} = 10 \text{ s}$

9.2

$\Delta Q = C \cdot \Delta U = 1 \cdot (5 - 2{,}5) = 2{,}5 \text{ As}$ $t = \frac{\Delta Q}{I} = \frac{2.5}{10 \cdot 10^{-3}} = 250 \text{ s} = 4 \text{ min}$

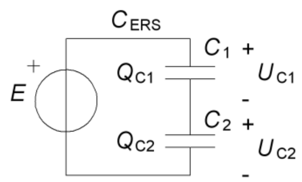
9.3

Capacitance values are added, parallel connection is the same thing as if capacitor areas are added. The capacitor with the lowest withstanding voltage determines the equivalent capacitor rated voltage. It is in that capacitor the impact will occur.

$C_{\text{ERS}} = C_1 + C_2 = 4 + 2 = 6$ μF 50V

9.4

No current or charge can pass trough a capacitor. Two serias connected capacitors must always have the same charge! $Q_{C1} = Q_{C2}$.

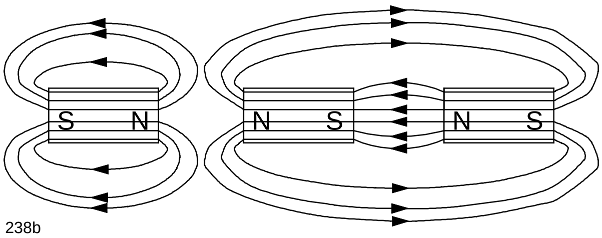


$Q_{C1} = Q_{C2} = Q = C_{\text{ERS}} \cdot E = C_1 \cdot U_{\text{C1}} = C_2 \cdot U_{\text{C2}}$

$C_{\text{ERS}} = \frac{6 \cdot 12}{6 + 12} = 4 \ \mu\text{F} \quad Q = 4 \cdot 10^{-6} \cdot 10 = 40 \ \mu\text{C}$

$U_{\text{C1}} = \frac{Q}{C_1} = \frac{40 \cdot 10^{-6}}{6 \cdot 10^{-6}} = 6{,}66 \text{ V} \qquad U_{\text{C2}} = E - U_{\text{C1}} = 10 - 6{,}66 = 3{,}33 \text{ V}$

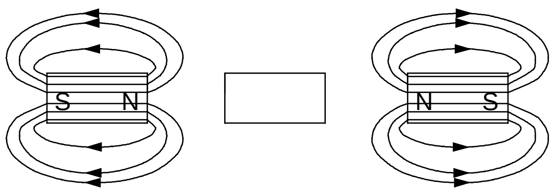
9.5



238b

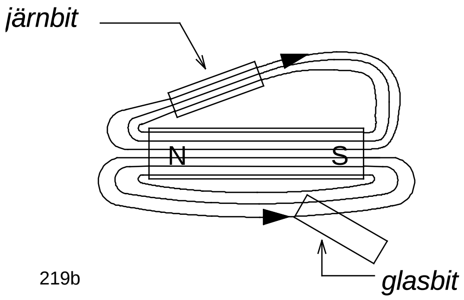
9.6

The metal part has the permabilitety $\mu_r = 1$, which is the same as for air. It does nor influence the magnets in any way.
The magnets distance from each other is large, so the magnetic fields is the same as from completely alone magnets.

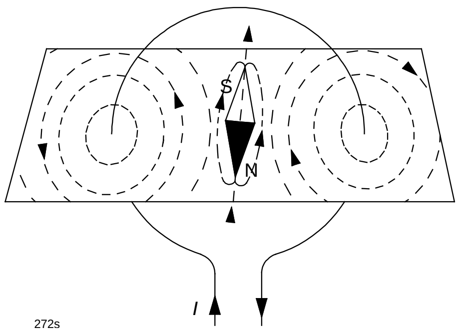


238c

9.7



9.8

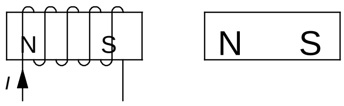


9.9

Rigt hand rule:

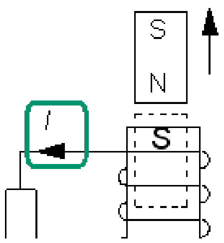
"If you keep on the coil with your right hand so that the fingers pointing in the direction of the current, the thumb is pointing toward the north end."

This force is attractive because the electromagnet and permanent magnet face different poles to each other.



9.10

The current is to counteract the movement. So will it be if the magnet leaves the coil at the "south side" (= attraction between the coil and magnet). Right hand rule then gives the current direction out from the winding.



9.11

$$L = 1 = 100^2 \cdot K \quad \Rightarrow \quad K = 10^{-4}$$

$$0{,}5 = N_X^2 \cdot 10^{-4} \quad \Rightarrow \quad N_X = \sqrt{5000} = 71 \quad 71 - 100 = 29$$

Unwind 29 turns (100-71) for the inductance to decrease by half.

Transients with RC and L/R

10.1

The circuit has the time constant $\tau = R \cdot C = 500 \cdot 500 \cdot 10^{-6} = 0{,}25\,\text{s}$. Capacitor is at first moment uncharged, when connected it will charge to + 10 V. For the voltages Kirchoff' voltage law applies $E + U_C + U_R = 0$.
At time $t = 0$: $10 + 0 + U_R = 0$. At time $t = \infty$: $10 + 10 + U_R = 0$.
We get for U_R:
$u_\infty = 0 \quad u_0 = 10$.

a) $x(t) = x_\infty - (x_\infty - x_0)\,\mathrm{e}^{-\frac{t}{\tau}} \quad \Rightarrow \quad u_R(t) = 0 - (0-10)\,\mathrm{e}^{-\frac{t}{0{,}25}} = 10\,\mathrm{e}^{-4t}$

$$2 = 10\,\mathrm{e}^{-4t} \quad \Leftrightarrow \quad 0{,}2 = \mathrm{e}^{-4t} \quad \Leftrightarrow \quad \ln 0{,}2 = \ln \mathrm{e}^{-4t} = -4t \quad \Rightarrow \quad t = -\frac{\ln 0{,}2}{4} = 0{,}4\,\text{s}$$

b) When the voltage over C is 2 V it will be 8 V over R.

$$8 = 10\,\mathrm{e}^{-4t} \quad \Leftrightarrow \quad 0{,}8 = \mathrm{e}^{-4t} \quad \Leftrightarrow \quad \ln 0{,}8 = \ln \mathrm{e}^{-4t} = -4t \quad \Rightarrow \quad t = -\frac{\ln 0{,}8}{4} = 0{,}06\,\text{s}$$

10.2

When the three components has the same voltage it will be $\frac{1}{3}10 = 3{,}33\,\text{V}$.

The two resistors can be merged to one R' = 2kΩ. We let $U_C(t)$ be $x(t)$ in the formula for exponetial process.
Startvalue $U_C(t)$ = 0 (from the beginning the capacitor is empty)
Endvalue $U_C(t=\infty) = 10$ V (the capacitor is charged to the full voltage after a long time)
$\tau = R \cdot C = 2 \cdot 10^3 \cdot 1000 \cdot 10^{-6} =$ 2s.

$$x(t) = x_\infty - (x_\infty - x_0)\,\mathrm{e}^{-\frac{t}{\tau}}$$

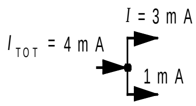
$$U_C(t) = 10 - (10-0)\,\mathrm{e}^{-\frac{t}{\tau}} \quad \Rightarrow \quad U_C(t) = 10 - 10\,\mathrm{e}^{-\frac{t}{2}}$$

$$0{,}667 = \mathrm{e}^{-0{,}5 \cdot t} \quad \Leftrightarrow \quad \ln(0{,}667) = -0{,}5 \cdot t \quad \Rightarrow \quad t = \frac{0{,}405}{0{,}5} = \mathbf{0{,}81\,s}$$

10.3

The two resistors can be merged to one $R = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{5 \cdot 15}{5+15} = 3750\,\Omega$.

a) Timeconstant is: $\tau = R \cdot C = 3750 \cdot 10 \cdot 10^{-6} =$ **0,038 s**.
b) When current through R_1 is 3 mA it will be 1 mA through R_2. Both resisors are in parallel and has the same voltage over them. Cureents will then be inversely proportional to the resistances. The total current is I_{TOT} = 4 mA. Use $x = I_{TOT}$ in the formula for exponetial process. From the beginning the capacitor is empty and then $I_{TOT} = \frac{E}{R} = 5{,}9 \cdot 10^{-3}$. After a long time the capacitor will be full and then I_{TOT} = 0.



$$x(t) = x_\infty - (x_\infty - x_0)\,\mathrm{e}^{-\frac{t}{\tau}}$$

$$I_{TOT}(t) = 0 - (0 - \frac{E}{R})\,\mathrm{e}^{-\frac{t}{\tau}} \quad \Rightarrow \quad I_{TOT}(t) = 59 \cdot 10^{-3}\,\mathrm{e}^{-\frac{t}{0{,}038}}$$

$$4 \cdot 10^{-3} = 5{,}9 \cdot 10^{-3}\,\mathrm{e}^{-26{,}3 \cdot t} \quad \Leftrightarrow \quad \ln(\frac{4}{5{,}9}) = -26{,}3 \cdot t \quad \Rightarrow \quad t = \frac{2{,}69}{26{,}3} = \mathbf{0{,}014\,s}$$

10.4

a) Series connected capacitors:

$$C = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{25 \cdot 15}{25 + 15} \cdot 10^{-6} = 9{,}38 \cdot 10^{-6} = 9{,}38\ \mu\text{F}$$

$$\tau = R \cdot C = 330 \cdot 10^3 \cdot 9{,}38 \cdot 10^{-6} = \mathbf{3{,}1\ s}$$

b) $x(t) = x_\infty - (x_\infty - x_0)\,\mathrm{e}^{-\frac{t}{\tau}}$

After long time ($t = \infty$) there will be E = 15 V over the series connected capacitors. The charge Q is the same in both capacitors (no charge can pass through a capacitor).

$$E = \frac{Q}{C} \quad \Rightarrow \quad Q = E \cdot C = 15 \cdot 9{,}38 \cdot 10^{-6} = 141\ \mu\text{C} \qquad\qquad u_{C2}(t = \infty) = \frac{Q}{C_2} = \frac{143 \cdot 10^{-6}}{15 \cdot 10^{-6}} = 9{,}38\ \text{V}$$

At beginning the capacitors are empty. $u_{C2}(t = 0) = 0$. We get:

$$u_{C2}(t) = u_{C2_\infty} - (u_{C2_\infty} - u_{C2_0})\,\mathrm{e}^{-\frac{t}{3{,}1}} \quad \Leftrightarrow \quad u_{C2}(t) = 9{,}38\left(1 - \mathrm{e}^{-0{,}32 \cdot t}\right)$$

$u_{C2}(t) = 2$ vid t = ?

$$2 = 9{,}38\left(1 - \mathrm{e}^{-0{,}32 \cdot t}\right) \quad \Leftrightarrow \quad \left(\frac{2}{9{,}38} - 1\right) = -\mathrm{e}^{-0{,}32 \cdot t} \quad \Leftrightarrow \quad \ln(0{,}79) = \ln(e^{-0{,}32 \cdot t}) \quad \Leftrightarrow \quad -0{,}24 = -0{,}32 \cdot t \quad \Rightarrow \quad t = \mathbf{0{,}75\ s}$$

10.5

The capacitor is first charged to 5 V, at the change of the circuit it will be further charged towards 15 V.

We get $u_\infty = 15 \quad u_0 = 5$. The time constant of the circuit is $\tau = R \cdot C = 2000 \cdot 1000 \cdot 10^{-6} = 2\ \text{s}$.

$$x(t) = x_\infty - (x_\infty - x_0)\mathrm{e}^{-\frac{t}{\tau}} \quad \Rightarrow \quad u(t) = 15 - (15 - 5)\,\mathrm{e}^{-\frac{t}{2}} = 15 - 10\mathrm{e}^{-\frac{t}{2}}$$

$$10 = 15 - 10\,\mathrm{e}^{-\frac{t}{2}} \quad \Leftrightarrow \quad -5 = -10\,\mathrm{e}^{-\frac{t}{2}} \quad \Leftrightarrow \quad \ln\frac{5}{10} = \ln\,\mathrm{e}^{-\frac{t}{2}} = -\frac{t}{2} \quad \Rightarrow \quad t = -2\ln\frac{5}{10} = 1{,}39\ \text{s}$$

When the capacitor is fully charged the current will cease. This will happen after about 10 s (5 time constants).

10.6

$u(t) = 2 \qquad u_\infty = 0 \qquad u_0 = 12 \qquad \tau = R \cdot C = 110 \cdot 10000 \cdot 10^{-6} = 1{,}1$ s

$$x(t) = x_\infty - (x_\infty - x_0)\,\mathrm{e}^{-\frac{t}{\tau}} \quad \Rightarrow \quad 2 = 0 - (0 - 12)\mathrm{e}^{-\frac{t}{1{,}1}}$$

$$\frac{2}{12} = \mathrm{e}^{-\frac{t}{1{,}1}} \quad \Leftrightarrow \quad 1{,}1 \cdot \ln\left(\frac{1}{6}\right) = -t \quad \Rightarrow \quad t = 1{,}97 \approx \mathbf{2\ s}$$

An exponentially process can be considered as terminated after $5 \cdot \tau = 5 \cdot 1{,}1 =$ **5,5 s**.

10.7

x_0 = entity start value $\quad i(0) = 0$

x_∞ = entity end value $\quad i(\infty) = \frac{E}{R} = \frac{12}{12} = 1 \quad$ b) $\frac{12}{24} = 0{,}5$

τ = time constant $= \frac{L}{R} = \frac{0{,}8}{12} = 0{,}06 \qquad$ b) $\frac{0{,}8}{24} = 0{,}03$

$$x(t) = x_\infty - (x_\infty - x_0)\,\mathrm{e}^{-\frac{t}{\tau}}$$

a) $i(t) = 1 - (1 - 0)\,\mathrm{e}^{-\frac{t}{0{,}06}} = 1 - \mathrm{e}^{-\frac{t}{0{,}06}} \quad \Rightarrow \quad i(t = 0{,}1) = 1 - \mathrm{e}^{-\frac{0{,}1}{0{,}06}} = \mathbf{0{,}81}$ A

b) $i(t) = 0{,}5 - (0{,}5 - 0)\,\mathrm{e}^{-\frac{t}{0{,}03}} = 0{,}5 \cdot (1 - \mathrm{e}^{-\frac{t}{0{,}03}}) \quad \Rightarrow \quad i(t = 0{,}1) = 0{,}5 \cdot (1 - \mathrm{e}^{-\frac{0{,}1}{0{,}03}}) = \mathbf{0{,}48}$ A

10.8

a) The inductor has a current inertia so at the beginning the current stays at 0.

b) After a long time the current through the inductor is constant (no change), $\frac{di}{dt} = 0$, and the emf over the inductor is then $e = L\frac{di}{dt} = 0$. The inductor zero emf short circuits the parallell 100 Ω resistor. Current is limited by the serias resistor vith the value 100 Ω. $I = \frac{10}{100} = 0{,}1\ \text{A}$.

c) When the switch breaks the circuit the current will subside to zero with a time constant of

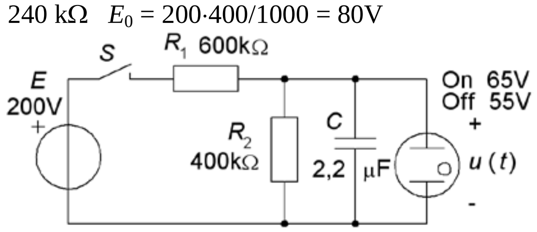
$$\tau = \frac{L}{R} = \frac{1}{100} = 0{,}01\ \text{s}$$

$$x(t) = x_\infty - (x_\infty - x_0)\,\mathrm{e}^{-\frac{t}{\tau}} \quad \Rightarrow \quad i_\mathrm{L}(t) = 0 - (0 - 0{,}1)\,\mathrm{e}^{-\frac{t}{0{,}01}} = 0{,}1\mathrm{e}^{-\frac{t}{0{,}01}} .$$

10.9

The circuit has the Thevenin equivalent: $R_\mathrm{I} = 600||400 = 240\ \text{k}\Omega$ $E_0 = 200\cdot 400/1000 = 80\text{V}$

a)



$$\tau = R_I \cdot C = 240 \cdot 10^3 \cdot 2{,}2 \cdot 10^{-6} = 0{,}528$$

$$t = \tau \cdot \ln \frac{all}{the\ rest} = 0{,}528 \cdot \ln \frac{80-0}{80-65} = 0{,}88\ s$$

b)

$$\tau = 0{,}528$$

$$t = \tau \cdot \ln \frac{all}{the\ rest} = 0{,}528 \cdot \ln \frac{80-55}{80-65} = 0{,}27\ s$$

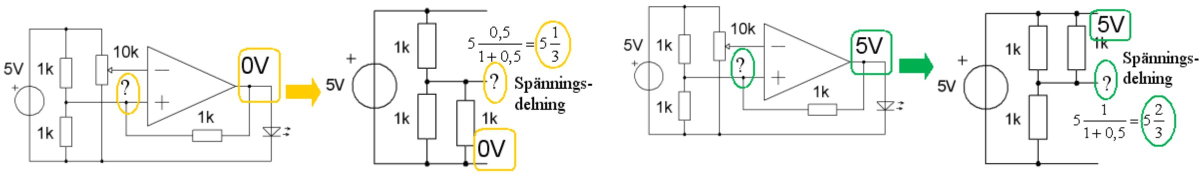
c)

If R_2 is gone E will not be voltage divided. E = 200. Time constant will change.

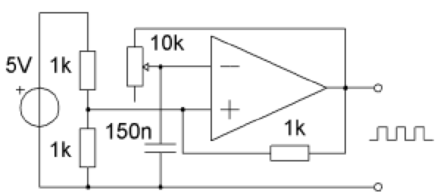
$$\tau = R_1 \cdot C = 600 \cdot 10^3 \cdot 2{,}2 \cdot 10^{-6} = 1{,}32$$

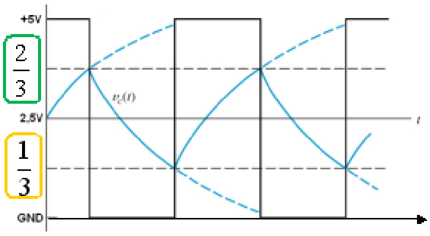
$$t = \tau \cdot \ln \frac{all}{the\ rest} = 1{,}32 \cdot \ln \frac{200-55}{200-65} = 0{,}094\ s$$

10.10



$U_{TH-} = 5\frac{1}{3} \approx 1{,}67\ \text{V}$ $\qquad$ $U_{TH+} = 5\frac{2}{3} \approx 2{,}33\ \text{V}$





$$\tau = R \cdot C = 5 \cdot 10^{3} \cdot 150 \cdot 10^{-9} = 0{,}75 \cdot 10^{-3}$$

$$t_1 = \tau \cdot \ln \frac{all}{the\ rest} = 0{,}75 \cdot 10^{-3} \cdot \ln \frac{5 - \frac{1}{3} \cdot 5}{\frac{1}{3} \cdot 5} = 0{,}75 \cdot 10^{-3} \cdot \ln 2 = 5{,}2\ \text{ms}$$

$$t_2 = t_1 \quad T = 2 \cdot t_1 = 2 \cdot 5{,}2 \cdot 10^{-3} = 10{,}4\ \text{ms} \quad f = \frac{1}{T} = \frac{1}{10{,}4 \cdot 10^{-3}} = 962\ \text{Hz}$$

We can shorten away the supply voltage 5V. The frequency is thus independent of the supply voltage!

10.11

$R(\vartheta) = 100 \cdot (1 + 3{,}85 \cdot 10^{-3} \cdot \vartheta)\,[\,\Omega\,]$

$R_0 = 176\ \Omega$

$R(\ t = 10\ \text{min}\ ) = 139\ \Omega$

$R_\infty = R(\vartheta = 25°\,) = 100 \cdot (1 + 3{,}85 \cdot 10^{-3} \cdot 25) = 109{,}6\,[\,\Omega\,]$

$$x(t) = x_\infty - (x_\infty - x_0)\,\mathrm{e}^{-\frac{t}{\tau}} \quad \Rightarrow \quad R(t = 10\ \text{min}) = 139 = 109{,}6 - (109{,}6 - 176)\,\mathrm{e}^{-\frac{10}{\tau}}$$

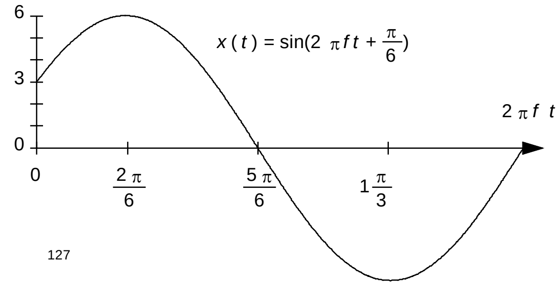
$$0{,}443 = \mathrm{e}^{-\frac{10}{\tau}} \quad \Leftrightarrow \quad \ln(0{,}443) = -\frac{10}{\tau} \quad \Rightarrow \quad \tau = \frac{10}{0{,}815} = \mathbf{12{,}3\ minuter}$$

AC voltage and current, phasor

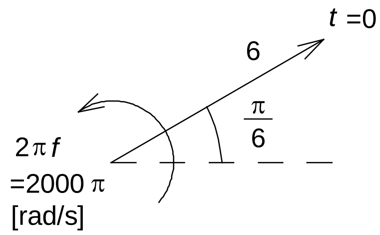
11.1

a) $x(t) = 6\sin(2000\pi \cdot t + \pi/6)$

b)



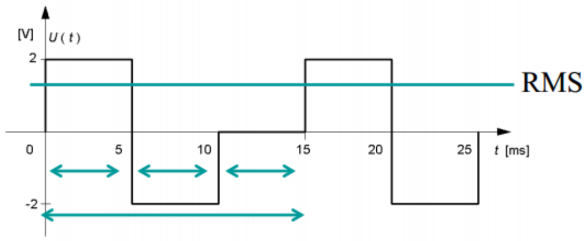
c)



11.2

$$U = \sqrt{\frac{\int_0^T u(t)^2\,\mathrm{d}t}{T}} = \sqrt{\frac{(2^2 + (-2)^2 + 0)\cdot 5\cdot 10^{-3}}{15\cdot 10^{-3}}} =$$

$$= \sqrt{\frac{8\cdot 5\cdot 10^{-3}}{15\cdot 10^{-3}}} = 1{,}63\ \mathrm{V}$$



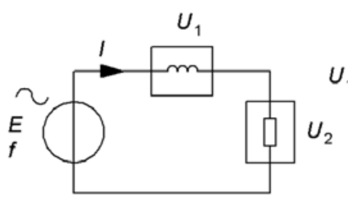
11.3

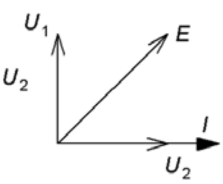
$$U = \sqrt{\frac{\int_0^T u(t)^2\,\mathrm{d}t}{T}} = \hat{U}\sqrt{\frac{\int_0^T \sin(\omega t)^2\,\mathrm{d}t}{T}} = \hat{U}\sqrt{\frac{\int_0^T \sin(\frac{2\pi t}{T})^2\,\mathrm{d}t}{T}} = \hat{U}\sqrt{\frac{1}{2}} = \frac{\hat{U}}{\sqrt{2}}$$

Phasor chart

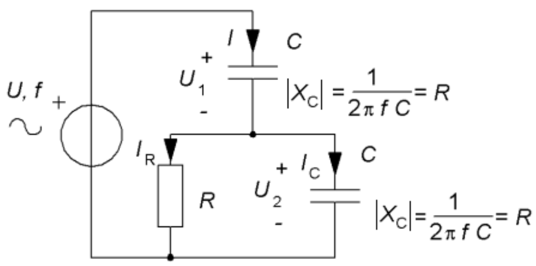
11.4

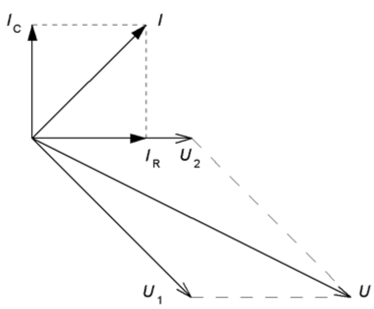
I dentify what components that have given the voltage phasors U_1 and U_2.
U_2 has the same phase as the current so it is a resistor that has given the voltage phasor. U_1 is earlier in phase than I so here we have a component with a current inertia, an inductance.



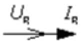


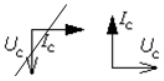
11.5



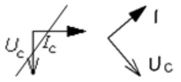


1) U_2 reference phase (=horisontal)

2) $\bar{I}_R \parallel \bar{U}_2$ 

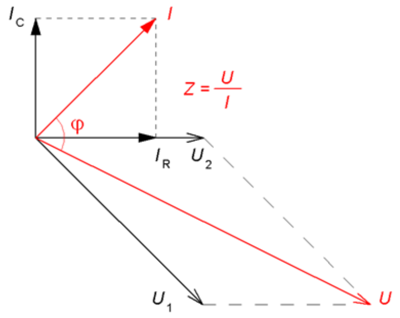
3) $\bar{I}_C \perp \bar{U}_2 \quad I_C = I_R = \dfrac{U_2}{R}$ 

4) $\bar{I} = \bar{I}_R + \bar{I}_C \quad I = \sqrt{2} \cdot I_C$

5) $\bar{U}_1 \perp \bar{I}$ 

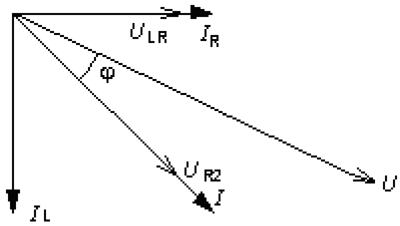
$U_1 = I \cdot R = I_C \cdot \sqrt{2} \cdot R \quad \Rightarrow \quad U_1 = \sqrt{2} \cdot U_2$

6) $\bar{U} = \bar{U}_1 + \bar{U}_2$



11.6

$X_L = 2 \cdot \pi \cdot f \cdot L = 2 \cdot \pi \cdot 50 \cdot 0{,}318 = 100\ \Omega$. We choose U_{LR} as reference phase. Current I_R has the same direction as U_{LR}. Current I_L is 90° after U_{LR} and has a equally long phasor as I_R because R_1 and L has the same resistance to the alternating current ($X_L = 100\ \Omega$, $R_1 = 100\ \Omega$). The two currents I_L and I_R can be added as vectors to a vector I, $\bar{I} = \bar{I}_R + \bar{I}_L$. I is $\sqrt{2}$ times longer than I_L and I_R (Pythagorean Theorem). Current I passes through the lower resistor R_2. The voltaged rop over U_{R2} will have the same direction as I and is $\sqrt{2}$ times bigger than U_{LR} (because the resistors are equal and the current is so many times larger). The voltage U can finally determined as the vector sum of U_{LR} och U_{R2} ;
$\bar{U} = \bar{U}_{LR} + \bar{U}_{R2}$.

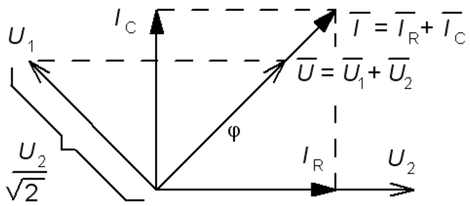


The angle φ is the angle between voltage U over the whole circuit and the current I flowing in to it.

11.7

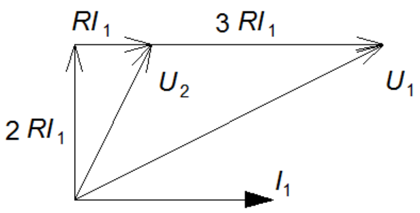
Start with U_2 as reference phase. The current I_R has the same direction as U_2. ($U_2 = I_R \cdot R$)
Current I_C is 90° before U_2 and is equally big as I_R
(because $X_C = R$)
Currents I_C and I_R are summed to I. $\bar{I} = \bar{I}_R + \bar{I}_C \quad I = \sqrt{2} \cdot I_R$
(Pythagorean Theorem)

U_1 is 90° before I. $U_1 = I \cdot X_L = \sqrt{2} \cdot I_R \cdot \dfrac{R}{2} = \dfrac{I_R \cdot R}{\sqrt{2}}$



The voltages U_1 and U_2 are summed to the voltage U. $\bar{U} = \bar{U}_1 + \bar{U}_2$.
(One can se that U will get equally big as U_1 !)

11.8



Alternating voltage and current, jω-method

12.1

$$\underline{I} = \underline{I}_{\mathrm{R}} + \underline{I}_{\mathrm{C}} = \frac{U}{R} + \frac{U}{\dfrac{1}{\mathrm{j}\omega C}} = \frac{U}{R} + \mathrm{j}\omega C \cdot U$$

12.2

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{220}{10\cdot\cos(30°) + 10\,\mathrm{j}\cdot\sin(30°)} = \frac{220}{8{,}6+5\,\mathrm{j}}\cdot\frac{(8{,}6-5\,\mathrm{j})}{(8{,}6-5\,\mathrm{j})} = \frac{1892-1100\,\mathrm{j}}{99} = 19{,}1 - 11{,}1\,\mathrm{j}$$

this impedance could be a resistor R = 19,1 Ω in serias with a capacitor with the reaktance X_{C} = -11,1 Ω.

$$X_C = -\frac{1}{\omega C} = -11{,}1 \quad\Rightarrow\quad C = -\frac{1}{2\pi\cdot 50\cdot(-11{,}1)} = 287\ \mu\mathrm{F}$$

12.3

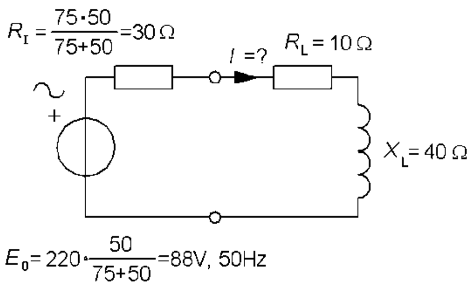
$$\underline{U}_2 = \underline{U}_1 \cdot \frac{\dfrac{1}{\mathrm{j}\omega C}}{R + \dfrac{1}{\mathrm{j}\omega C}} \cdot \frac{(\mathrm{j}\omega C)}{(\mathrm{j}\omega C)} = \underline{U}_1 \cdot \frac{1}{1+\mathrm{j}\omega RC} \quad\Rightarrow\quad \frac{U_1}{U_2} = \sqrt{1+R^2\omega^2C^2} = \frac{10}{5} = 2$$

$$1 + R^2\omega^2C^2 = 4 \quad\Leftrightarrow\quad R\omega C = \sqrt{3} \quad\Leftrightarrow\quad RC = \frac{\sqrt{3}}{\omega}$$

12.4

$$\underline{I} = \frac{U}{\underline{Z}} \quad\Rightarrow\quad I = \frac{88}{|(30+10)+\mathrm{j}40|} =$$

$$= \frac{88}{\sqrt{(30+10)+40^2}} = 1{,}56\ \mathrm{A}$$



12.5

Parallel connection:

$$\underline{I} = \underline{I}_{\mathrm{R}} + \underline{I}_{\mathrm{C}} = \frac{U}{R} + U\cdot\mathrm{j}\omega C$$

$$I_R = \frac{U}{R} \quad I_C = U\omega C$$

$$\underline{I} = 2 + 2\,\mathrm{j}$$

Serial connection:

$$\underline{I} = \frac{U}{R + \dfrac{1}{\mathrm{j}\omega C}} \quad\Rightarrow\quad I = \frac{U}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}}$$

$$R = \frac{1}{\omega C} = \frac{E}{2} \quad\Rightarrow\quad I = \frac{U}{U\cdot\sqrt{\frac{1}{4}+\frac{1}{4}}} = \sqrt{2}\ \mathrm{A}$$

12.6

$$\underline{Z}_{AB} = \frac{(15+\mathrm{j}20)\cdot(10-\mathrm{j}20)}{15+\mathrm{j}20+10-\mathrm{j}20} = \frac{550-\mathrm{j}100}{25} = 22 - \mathrm{j}4\,[\Omega]$$

12.7

$$\underline{Z} = \frac{\left(R+\dfrac{1}{\mathrm{j}\omega C}\right)\cdot\mathrm{j}\omega L}{R+\mathrm{j}\omega L+\dfrac{1}{\mathrm{j}\omega C}} = \frac{\dfrac{L}{C}+\mathrm{j}\omega LR}{R+\mathrm{j}\left(\omega L-\dfrac{1}{\omega C}\right)} \qquad \underline{I} = \frac{U}{\underline{Z}} = U\,\frac{R+\mathrm{j}\left(\omega L-\dfrac{1}{\omega C}\right)}{\dfrac{L}{C}+\mathrm{j}\omega LR}$$

12.8

Voltage U is directly over the parallel branch with the inductance L.

$$\underline{I} = \frac{U}{\mathrm{j}\omega L} = -\mathrm{j}\frac{U}{\omega L}$$

12.9

$$\underline{Z}_{\mathrm{R2C}} = \frac{2\cdot(-8\mathrm{j})}{2-8\mathrm{j}}\cdot\frac{(2+8\mathrm{j})}{(2+8\mathrm{j})} = 1{,}88-0{,}47\,\mathrm{j}$$

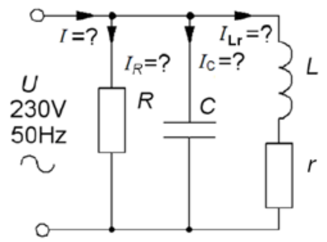
$$\underline{Z} = 3+6\,\mathrm{j}+1{,}88-0{,}47\,\mathrm{j} = 4{,}88+5{,}53\mathrm{j} \quad \mathrm{Z} = \sqrt{4{,}88^2+5{,}53^2} = 7{,}38\,\Omega$$

$$\underline{I} = \frac{U}{\underline{Z}} = \frac{30}{4{,}88+5{,}53\mathrm{j}}\cdot\frac{(4{,}88-5{,}53\mathrm{j})}{(4{,}88-5{,}53\mathrm{j})} = \frac{146{,}5-165{,}9\,\mathrm{j}}{54{,}41} = 2{,}7-3\mathrm{j} \quad I = \sqrt{2{,}7^2+3^2} = 4\ \mathrm{A}$$

$$\underline{I}_{\mathrm{C}} = \frac{2(2{,}7-3\mathrm{j})}{2-8\mathrm{j}}\cdot\frac{(2+8\mathrm{j})}{(2+8\mathrm{j})} = 0{,}86+0{,}46\mathrm{j} \quad I_{\mathrm{C}} = \sqrt{0{,}86^2+0{,}46^2} = 0{,}98\ \mathrm{A}$$

$$\underline{U}_{\mathrm{L}} = 30\frac{6\,\mathrm{j}}{3+6\,\mathrm{j}+(1{,}88\text{-}0{,}47\mathrm{j})} = 30\frac{6\,\mathrm{j}}{4{,}88+5{,}53\mathrm{j}}\cdot\frac{(4{,}88-5{,}53\mathrm{j})}{(4{,}88-5{,}53\mathrm{j})} = 18{,}3+16{,}2\,\mathrm{j} \quad U_{\mathrm{L}} = 24{,}4\ \mathrm{V}$$

12.10



a) $\underline{U}$ andh $\underline{I}_R$ are in phase and will be our reference phase with,

$$\arg(\underline{U}) = 0$$

$$U = R\cdot I_R \quad \Rightarrow \quad I_R = \frac{U}{R} = \frac{230}{46} = 5\mathrm{A}$$

b)

$$\underline{U} = \frac{1}{j\omega C}\underline{I}_C \quad \Rightarrow \quad U = \frac{1}{\omega C}I_C \quad \Rightarrow \quad I_C = U\cdot\omega C = 230\cdot 2\cdot\pi\cdot 50\cdot 69\cdot 10^{-6} \approx 5\mathrm{A}$$

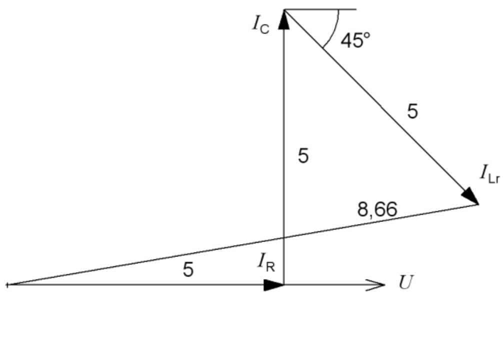
$$\arg(\underline{I}_C) = \arg(\underline{U}) + \arg(j\omega C) = 0° + 90° = 90°$$

c)

$$\underline{U} = \underline{Z}_{Lr}\cdot\underline{I}_{Lr} = (r + j\omega L)\cdot\underline{I}_L \quad \Rightarrow \quad I_{Lr} = \frac{U}{\sqrt{r^2+(\omega L)^2}} = \frac{230}{\sqrt{32{,}5^2+(32{,}5)^2}} \approx 5\mathrm{A}$$

$$\arg(\underline{I}_{Lr}) = \arg(\underline{U}) - \arg(\underline{Z}_{Lr}) = 0° - \arctan\frac{32{,}5^2}{32{,}5^2} = -45°$$

d)



$$\underline{I} = \underline{I}_C + \underline{I}_R + \underline{I}_{Lr}$$

$$I = \sqrt{(I_C - I_{Lr}\cdot\sin 45°)^2 + (I_R + I_L\cdot\cos 45°)^2} =$$

$$= \sqrt{(5-5\cdot 0{,}71)^2 + (5+5\cdot 0{,}71)^2} =$$

$$= \sqrt{1{,}46^2 + 8{,}54^2} = \sqrt{75} \approx 8{,}66\mathrm{A}$$

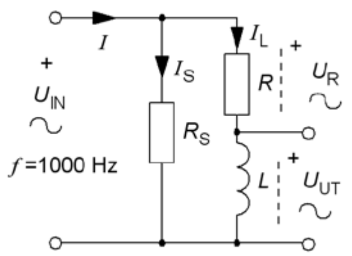
12.11

a) $\underline{U}_{UT}$ are chosen as reference phase, $\arg(\underline{U}_{UT}) = 0$

$\underline{U}_{UT} = j\omega L \cdot \underline{I}_1 \quad \underline{U}_{UT} = U_{UT} = 6{,}28$

$$\underline{I}_L = \frac{\underline{U}_{UT}}{j\omega L} = \frac{6{,}28}{j \cdot 2\pi \cdot 1000 \cdot 10 \cdot 10^{-3}} - 0{,}1j$$

$I_L = 0{,}1\ A$



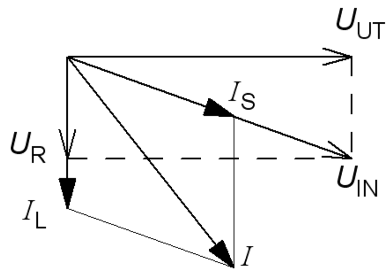
b) $\underline{U}_R = R \cdot \underline{I}_L = -50 \cdot 0{,}1j = -5j \quad U_R = 5\ \mathrm{V}$

c) $\underline{U}_{IN} = \underline{U}_R + \underline{U}_{UT} = 6{,}28 - 5j \quad U_{IN} = \sqrt{6{,}28^2 + 5^2} = 8{,}0\ \mathrm{V}$

$$\underline{I}_S = \frac{\underline{U}_{IN}}{R_S} = \frac{6{,}28 - 5j}{100} = 0{,}063 - 0{,}05j$$

d) $\underline{I} = \underline{I}_L + \underline{I}_S = -0{,}1j + 0{,}063 - 0{,}05j = 0{,}062 - 0{,}15j$

$I = \sqrt{0{,}063^2 + 0{,}15^2} = 0{,}16\ \mathrm{A}$



Resonance

13.1

Because $|U_C| = |U_L|$ there is resonance. The voltage drops over L and C will cancel each other and left over will be 1 V over R.
U = 1 V.

13.2

Because $|I_C| = |I_L|$ there is resonance. I = 1 A, the current in L and C is a circulating current, $I_C = -I_L$.

13.3

$$\underline{I} = \underline{I}_{\mathrm{C}} + \underline{I}_{\mathrm{LR}} = \frac{U}{\frac{1}{\mathrm{j}\omega C}} + \frac{U}{R + \mathrm{j}\omega L} \cdot \frac{(R - \mathrm{j}\omega L)}{(R - \mathrm{j}\omega L)} = U \cdot \left(\mathrm{j}\omega C + \frac{R - \mathrm{j}\omega L}{R^2 + (\omega L)^2} \right) =$$

$$= U \cdot \left(\frac{R}{R^2 + (\omega L)^2} + \mathrm{j}\left(\omega C - \frac{\omega L}{R^2 + (\omega L)^2}\right) \right)$$

We have here chosen U as reference phase, real. Curerent I must then also be reall to be in phase with voltage. This provides the condition that $\mathrm{Im}\left[\underline{I}\right] = 0$.

$$\omega C = \frac{\omega L}{R^2 + (\omega L)^2} \quad \Rightarrow \quad \omega^2 = \frac{1}{LC} - \frac{R^2}{L^2} \quad \omega = 2\pi f \quad \Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$$

This frequency is the **resonance frequency**.

13.4

a) Q-value. $BW = \dfrac{f_0}{Q} \quad \Rightarrow \quad Q = \dfrac{f_0}{BW} = \dfrac{2000}{200} = 10$.

b) $R_\mathrm{S} = 2\ \Omega \quad Q = \dfrac{X_\mathrm{L}}{R_\mathrm{S}} \quad \Rightarrow \quad X_\mathrm{L} = Q \cdot R_\mathrm{S} = 10 \cdot 2 = 20\ \Omega$

c) $X_\mathrm{L} = 2\pi f_0 L \quad \Rightarrow \quad L = \dfrac{X_\mathrm{L}}{2\pi f_0} = \dfrac{20}{2\pi \cdot 2000} = 1{,}59\ \mathrm{mH} \qquad X_\mathrm{L} = X_\mathrm{C}$

$$X_\mathrm{C} = \frac{1}{2\pi f_0 C} \quad \Rightarrow \quad C = \frac{1}{2\pi f_0 X_\mathrm{C}} = \frac{1}{2\pi \cdot 2000 \cdot 20} = 3{,}98\ \mu\mathrm{F}$$

d) $f_1 \approx f_0 - \dfrac{BW}{2} = 1900 \quad f_1 \approx f_0 + \dfrac{BW}{2} = 2100$

$$f_0 = \sqrt{f_1 \cdot f_2} = \sqrt{1900 \cdot 2100} = 1997 \approx 2000\ \text{OK!}$$

13.5

a) The coil Q-value, parallel resistance. $Q = \dfrac{X_\mathrm{L}}{R_\mathrm{S}} = \dfrac{30}{2} = 15 \quad R = Q^2 \cdot r = 15^2 \cdot 2 = 450\ \Omega$

b) $Z_{ERS} = 450 \,||\, 450 = 225\ \Omega$

c) $I \cdot Z_\mathrm{ERS} = 80 \cdot 10^{-3} \cdot 225 = 18\ \mathrm{V} \quad \underline{I}_\mathrm{C} = \dfrac{18}{-\mathrm{j}30} \quad \Rightarrow \quad I_\mathrm{C} = \quad 0{,}6\ \mathrm{A} \angle +90°$

$$\underline{I}_\mathrm{Lr} = \frac{18}{2 + \mathrm{j}30} \quad \Rightarrow \quad I_\mathrm{L} \approx \quad 0{,}6\ \mathrm{A} \angle -86°$$

d) $L = \dfrac{X_\mathrm{L}}{2\pi f_0} = \dfrac{30}{2\pi \cdot 20 \cdot 10^3} = 0{,}24\ \mathrm{mH} \quad C = \dfrac{1}{2\pi f_0 \cdot |X_C|} = \dfrac{1}{2\pi \cdot 20 \cdot 10^3 \cdot 30} = 265\ \mathrm{nF}$

e) $Q_\mathrm{TOT} = \dfrac{225}{30} = 7{,}5 \quad BW = \dfrac{f_0}{Q} = \dfrac{20 \cdot 10^3}{7{,}5} = 2{,}67\ \mathrm{kHz}$

13.6

a) $f_0 = \dfrac{1}{2\pi\sqrt{L \cdot C}} = \dfrac{1}{2\pi\sqrt{5 \cdot 10^{-6} \cdot 25 \cdot 10^{-12}}} = 14{,}2\ \mathrm{MHz}$

b) The coil Q-value

$$Q = \frac{2\pi f_0 \cdot L}{r} = \frac{2\pi \cdot 14{,}2 \cdot 10^6 \cdot 5 \cdot 10^{-6}}{0{,}5} = 894$$

c) Total parallel resistance for $Q = 500$ will be:

$$R_{Q500} = 500 \cdot 2\pi f_0 \cdot L = 500 \cdot 2\pi \cdot 14{,}2 \cdot 10^6 \cdot 5 \cdot 10^{-6} = 224\ \mathrm{k\Omega}$$

The coil resistance transformed to parallel resistace will ber:

$$R = Q^2 \cdot r = 894^2 \cdot 0{,}5 = 400\ \mathrm{k\Omega}$$

Chose R_X so that:

$$R_{Q500} = R_X \,||\, R \quad \Rightarrow \quad R_X = \frac{R \cdot R_{Q500}}{R - R_{Q500}} = \frac{400 \cdot 224}{400 - 224} = 507\ \mathrm{k\Omega}$$

13.7

$$f_0 = \frac{1}{2\pi\sqrt{L \cdot C}} \quad \Rightarrow \quad C = \frac{1}{4\pi^2 L f_0^2} = \frac{1}{4\pi^2 \cdot 2{,}5 \cdot 10^{-6} \cdot (13{,}56 \cdot 10^6)^2} = 55 \text{ pF}$$

$$Q = \frac{\omega L}{r} = \frac{2\pi f_0 \cdot L}{r} = \frac{2\pi \cdot 13{,}56 \cdot 10^6 \cdot 2{,}5 \cdot 10^{-6}}{1{,}5} = 142$$

$$R = Q^2 \cdot r = 142^2 \cdot 1{,}5 = 30{,}25 \text{ k}\Omega$$

$$Q_{\text{BW}} = \frac{f_0}{\Delta f} = \frac{13{,}56 \cdot 10^6}{140 \cdot 10^3} = 96{,}86 \quad Q_{\text{BW}} = \frac{R_{\text{BW}}}{2\pi \cdot f_0 \cdot L} \quad \Rightarrow$$

$$R_{\text{BW}} = Q_{\text{BW}} \cdot 2\pi \cdot f_0 \cdot L = 96{,}86 \cdot 2\pi \cdot 13{,}56 \cdot 10^6 \cdot 2{,}5 \cdot 10^{-6} = 20{,}63 \text{ k}\Omega$$

$$R_{\text{BW}} = R_X \parallel R \quad \Rightarrow \quad R_X = \frac{R \cdot R_{\text{BW}}}{R - R_{\text{BW}}} = \frac{30{,}25 \cdot 20{,}63}{30{,}25 - 20{,}63} \cdot 10^3 = 64 \text{ k}\Omega$$

If the electronics are power efficient and does not consume more current than a 64 k resistor the resonance circuit will have a bandwidth of 140 kHz. Unnecessarily large bandwidth increases sensitivity to disturbances.

13.8

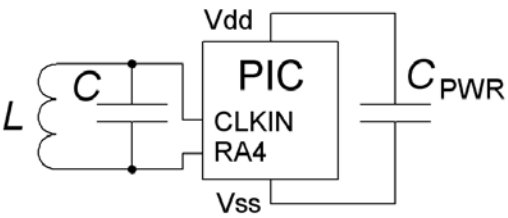
$$a) \quad f_0 = \frac{1}{2\pi\sqrt{L \cdot C}} = \frac{1}{2\pi\sqrt{0{,}43 \cdot 10^{-3} \cdot 3{,}77 \cdot 10^{-9}}} = 125 \text{ kHz}$$

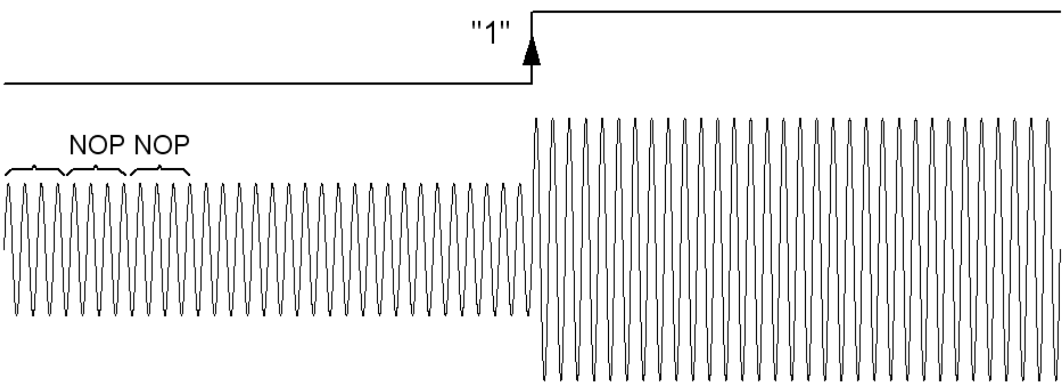
$$b) \quad Q_L = \frac{2\pi f_0 \cdot L}{r} = \frac{2\pi \cdot 125 \cdot 10^3 \cdot 4{,}3 \cdot 10^{-3}}{4} = 84{,}4$$

$$c) \quad R_{PIC} = 20 \cdot 10^3 \quad r_{PIC} = \frac{R_{PIC}}{Q_L^2} = \frac{20 \cdot 10^3}{84{,}4^2} = 2{,}8\,\Omega \quad Q_{res} = \frac{2\pi f_0 \cdot L}{r_{PIC} + r_L} = \frac{338}{4 + 2{,}8} = 50$$

$$d) \quad Q_{res} = \frac{f_0}{BW} \quad \Rightarrow \quad BW = \frac{f_0}{Q_{res}} = \frac{125 \cdot 10^3}{50} = 2{,}5 \text{ kHz}$$

Interested in the function? PIC processor has "protection diodes" inside the chip – these will now act as rectifier diodes and rectifi the 125-kHz signal to provide power to the CPU. A capacitor C_{PWR} keeps the supply voltage stable (the chip internal capacitance will be enough). The processor uses the 125-kHz-signal as its clock (`CLKIN`). One instruction takes 4 clockcycles. A program that sends a bit ”1” can look like this:





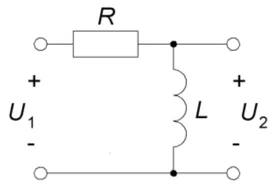

```
TRISA.5 = 1; /* CLKIN is input */
PORTA.4 = 0;
TRISA.4 = 0;  /* RA4 pin at GND,  RF-signal is damped */
nop(); nop(); nop(); nop(); nop(); nop(); nop();
TRISA.4 = 1; /* RA4 pin is threestate, RF-signal is undamped */
nop(); nop(); nop(); nop(); nop(); nop(); nop();
```

Filter

14.1

$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{j\omega L}{R + j\omega L} = \frac{j\omega \frac{L}{R}}{1 + j\omega \frac{L}{R}} \qquad \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = \frac{|\, j\omega L\,|}{|\, R + j\omega L\,|} = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$$

$$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = \arg\left(\frac{j\omega L}{R + j\omega L}\right) = \arg(j\omega L) - \arg(R + j\omega L) = 90° - \arctan\left(\frac{\omega L}{R}\right) =$$



$$= \arctan\left(\frac{R}{\omega L}\right) \qquad R^2 = (\omega L)^2 \quad \omega = 2\pi \cdot f_G \quad \Rightarrow \quad f_G = \frac{R}{2\pi L} \qquad HP$$

14.2

$$R \,||\, C = \frac{R \cdot \frac{1}{\mathrm{j}\omega C}}{R + \frac{1}{\mathrm{j}\omega C}} \cdot \frac{\mathrm{j}\omega C}{\mathrm{j}\omega C} = \frac{R}{1 + \mathrm{j}\omega RC} \qquad \underline{I}_C = \frac{\underline{U}_\mathrm{C}}{\frac{1}{\mathrm{j}\omega C}} = \underline{U}_\mathrm{C} \cdot \mathrm{j}\omega C$$

$$\underline{U}_\mathrm{C} = U \frac{\frac{R}{1 + \mathrm{j}\omega RC}}{R + \frac{R}{1 + \mathrm{j}\omega RC}} \cdot \frac{\frac{1 + \mathrm{j}\omega RC}{R}}{\frac{1 + \mathrm{j}\omega RC}{R}} = U \frac{1}{1 + \mathrm{j}\omega RC + 1} \quad \Rightarrow \quad \underline{I}_C = U \frac{\mathrm{j}\omega C}{2 + \mathrm{j}\omega RC}$$

14.3

$$R_1 \,||\, C = \frac{R_1 \cdot \frac{1}{\mathrm{j}\omega C}}{R_1 + \frac{1}{\mathrm{j}\omega C}} \cdot \frac{\mathrm{j}\omega C}{\mathrm{j}\omega C} = \frac{R_1}{\mathrm{j}\omega R_1 C + 1}$$

$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{R_2}{R_2 + \frac{R_1}{\mathrm{j}\omega R_1 C + 1}} \cdot \frac{\mathrm{j}\omega R_1 C + 1}{\mathrm{j}\omega R_1 C + 1} = \frac{R_2(\mathrm{j}\omega R_1 C + 1)}{R_2(\mathrm{j}\omega R_1 C + 1) + R_1} = \frac{\mathrm{j}\omega R_1 R_2 C + R_2}{\mathrm{j}\omega R_1 R_2 C + (R_1 + R_2)}$$

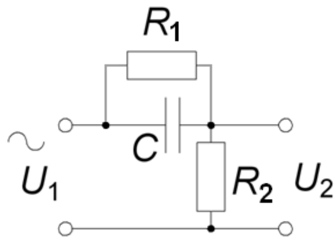
$$\omega \approx 0 \Rightarrow \quad \frac{\underline{U}_2}{\underline{U}_1} \approx \frac{R_2}{R_1 + R_2} \quad \frac{U_2}{U_1} = \frac{1}{3} \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) \approx 0°$$

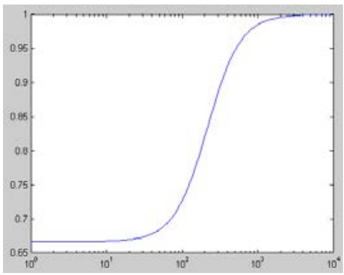
$$\omega \to \infty \Rightarrow \quad \frac{U_2}{U_1} = 1 \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) \approx 0°$$

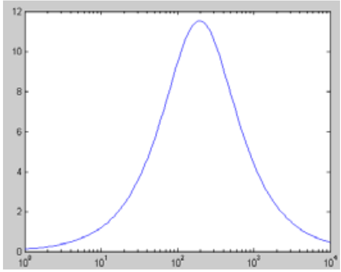
The crossover frequency is when the numerators real part is equal to its imaginary part:

$$\omega R_1 R_2 C = (R_1 + R_2) \quad \Rightarrow \quad \omega = \frac{(R_1 + R_2)}{R_1 R_2 \cdot C} \quad \Rightarrow \quad f = \frac{(R_1 + R_2)}{2\pi \cdot R_1 R_2 \cdot C}$$

$$f = \frac{(1 \cdot 10^3 + 2 \cdot 10^3)}{2\pi \cdot 1 \cdot 10^3 \cdot 2 \cdot 10^3 \cdot 1 \cdot 10^{-6}} = 239 \text{ Hz}$$







The filter is a high pass filter, HP, that passes high frequencies but attenuates 1/3 at low frequencies. The phase shift is greatest at the "cutoff frequency" 240Hz when it amounts to ca. 12°.

14.4

$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{R}{j\omega L + \frac{1}{j\omega L} + R} = \frac{R}{R + j(\omega L - \frac{1}{\omega C})} \qquad \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = \frac{R}{|R + j(\omega L - \frac{1}{\omega C})|} = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

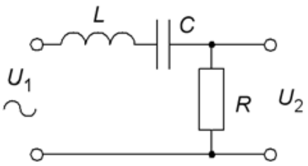
$$\arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = \arg\left(\frac{R}{R + j(\omega L - \frac{1}{\omega C})}\right) = \arg(R) - \arg\left(R + j(\omega L - \frac{1}{\omega C})\right) = 0° - \arctan\left(\frac{(\omega L - \frac{1}{\omega C})}{R}\right)$$

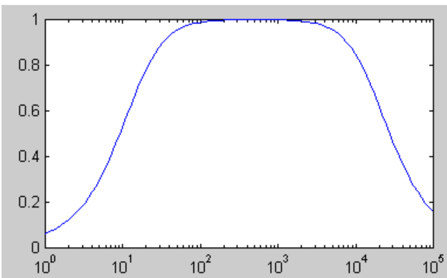
$$(\omega L - \frac{1}{\omega C}) = 0 \quad \Rightarrow \quad \omega_x = \frac{1}{\sqrt{L \cdot C}} \qquad f_x = \frac{1}{2\pi\sqrt{L \cdot C}}$$

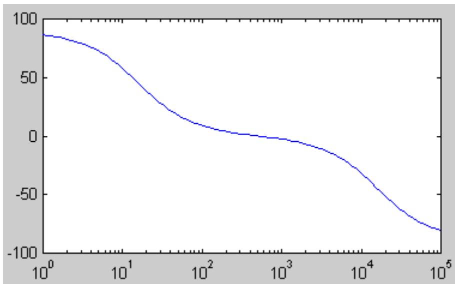
$$f = f_x \quad \Rightarrow \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = 1 \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = -\arctan(0) = 0°$$

$$f = 0 \quad \Rightarrow \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = 0 \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = \arg\left(\frac{1}{-\infty j}\right) = 90°$$

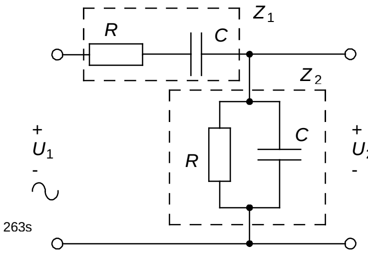
$$f = \infty \quad \Rightarrow \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = 0 \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = \arg\left(\frac{1}{+\infty j}\right) = -90° \qquad \Rightarrow \quad BP$$







14.5



define impedanses $\underline{Z}_1 = R + \frac{1}{j\omega C}$ och $\underline{Z}_2 = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC}$

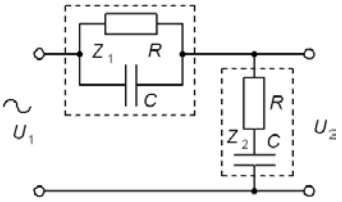
$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} = \frac{\frac{R}{1 + j\omega RC}}{R + \frac{1}{j\omega C} + \frac{R}{1 + j\omega RC}} = \frac{R}{3R + j\left(R^2 \cdot \omega C - \frac{1}{\omega C}\right)}$$

$\omega \to 0 \quad \Rightarrow \quad \dfrac{1}{\omega C} \to \infty$. När $\omega \to \infty \quad \Rightarrow \quad \omega C \to \infty$.

$\left(R^2 \cdot \omega C - \dfrac{1}{\omega C} \right) \to \infty \quad \Rightarrow \quad \dfrac{\underline{U}_2}{\underline{U}_1} \to 0$

When $R^2 \cdot \omega C - \dfrac{1}{\omega C} = 0$ at $\omega = \dfrac{1}{RC}$ then $\dfrac{\underline{U}_2}{\underline{U}_1} = \dfrac{1}{3}$.

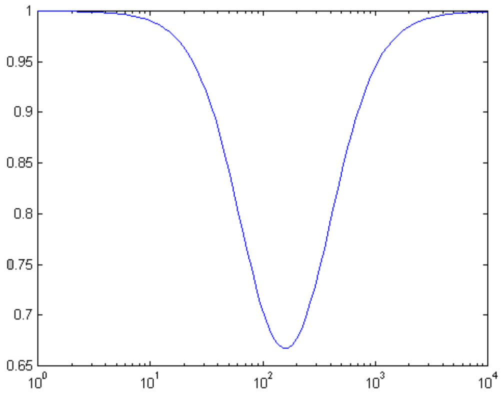
14.6

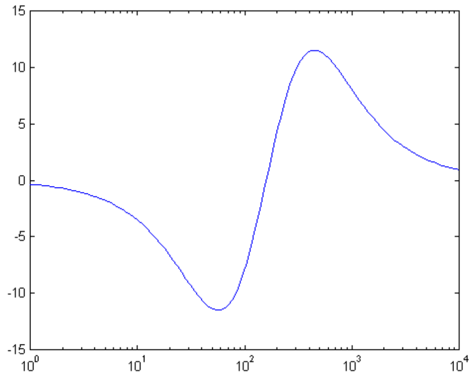


define impedanses $\underline{Z}_1 = \dfrac{R \cdot \dfrac{1}{\mathrm{j}\omega C}}{R + \dfrac{1}{\mathrm{j}\omega C}} = \dfrac{R}{1 + \mathrm{j}\omega RC}$ och $\underline{Z}_2 = R + \dfrac{1}{\mathrm{j}\omega C}$

$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{R + \dfrac{1}{\mathrm{j}\omega C}}{R + \dfrac{1}{\mathrm{j}\omega C} + \dfrac{R}{1 + \mathrm{j}\omega RC}} \cdot \frac{\mathrm{j}\omega C(1 + \mathrm{j}\omega RC)}{\mathrm{j}\omega C(1 + \mathrm{j}\omega RC)} = \ldots = \frac{(1 - \omega^2 R^2 C^2) + 2\mathrm{j}\omega RC}{(1 - \omega^2 R^2 C^2) + 3\mathrm{j}\omega RC}$$

$$\omega = \frac{1}{RC} \quad \Rightarrow \quad \omega RC = 1 \quad \omega^2 R^2 C^2 = 1 \quad \Rightarrow \quad \frac{\underline{U}_2}{\underline{U}_1} = \frac{2}{3} \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1} \right) = 0$$





14.7

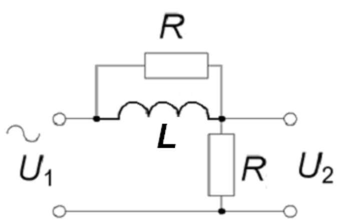
$$a) \qquad R \,||\, L = \frac{R \cdot j\omega L}{R + j\omega L} \qquad \frac{\underline{U}_2}{\underline{U}_1} = \frac{R}{R + \dfrac{R \cdot j\omega L}{R + j\omega L}} = \frac{1}{1 + \dfrac{1 \cdot j\omega L}{R + j\omega L}} = \frac{\dfrac{R + j\omega L}{R + j\omega L}}{\dfrac{R + j\omega L + j\omega L}{R + j\omega L}} = \frac{R + j\omega L}{R + j2\omega L}$$

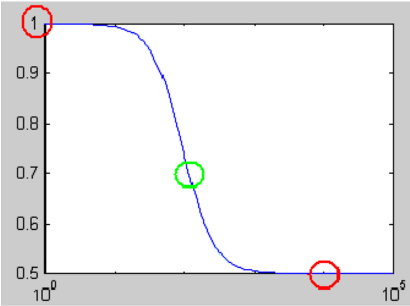
$$b) \qquad \left| \frac{\underline{U}_2}{\underline{U}_1} \right| = \left| \frac{R + j\omega L}{R + j2\omega L} \right| = \frac{1}{\sqrt{2}} \qquad \frac{\sqrt{R^2 + (\omega L)^2}}{\sqrt{R^2 + (2\omega L)^2}} = \frac{1}{\sqrt{2}} \qquad 2R^2 + 2(\omega L)^2) = R^2 + 4(\omega L)^2$$

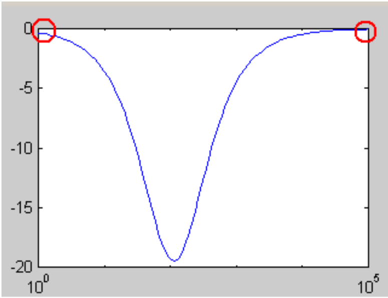
$$R^2 = 2(\omega L)^2 \quad \Rightarrow \quad \omega_X = \frac{R}{L\sqrt{2}}$$

$$c) \qquad \frac{R + j\omega L}{R + j2\omega L} \qquad \omega \to 0 \qquad \frac{R + 0}{R + 0} = 1 \quad \Rightarrow \quad \left| \frac{\underline{U}_2}{\underline{U}_1} \right| = 1 \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1} \right) = 0°$$

$$d) \quad \frac{R + j\omega L}{R + j2\omega L} \Rightarrow \frac{\frac{R}{\omega} + jL}{\frac{R}{\omega} + j2L} \quad \omega \to \infty \quad \frac{0 + jL}{0 + j2L} = \frac{1}{2} \Rightarrow \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = 0{,}5 \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = 0°$$







14.8

$$a) \frac{\underline{U}_2(\omega)}{\underline{U}_1(\omega)} = ? \quad b) \omega_X(R, L, C) = ? \quad c) \left|\frac{\underline{U}_2(\omega_X)}{\underline{U}_1(\omega_X)}\right| = ? \quad d) \arg\left(\frac{\underline{U}_2(\omega_X)}{\underline{U}_1(\omega_X)}\right) = ? \quad e) \frac{\underline{I}_R(\omega)}{\underline{U}_1(\omega)} = ?$$

$$a)\, b) \qquad R \| C = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \cdot \frac{j\omega C}{j\omega C} = \frac{R}{1 + j\omega RC}$$

$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{\frac{R}{1 + j\omega RC}}{j\omega L + \frac{R}{1 + j\omega RC}} \cdot \frac{1 + j\omega RC}{1 + j\omega RC} = \frac{R}{j\omega L(1 + j\omega RC) + R} =$$

$$= \frac{R}{(R - \omega^2 RLC) + j\omega L} \qquad RE\left[\frac{\underline{U}_2}{\underline{U}_1}\right] = 0 \Rightarrow \omega^2 RLC = R \quad \omega = \frac{1}{\sqrt{LC}}$$

$$c) \quad \frac{\underline{U}_2}{\underline{U}_1} = \frac{R}{(R - \omega^2 RLC) + j\omega L} = \left\{\omega = \frac{1}{\sqrt{LC}}\right\} = \frac{R}{0 + j\sqrt{\frac{L}{C}}} \qquad \frac{U_2}{U_1} = \frac{R}{\sqrt{\frac{L}{C}}} = R\sqrt{\frac{C}{L}}$$

$$d) \quad \arg\left[\frac{\underline{U}_2}{\underline{U}_1}\right] = \arg\left[\frac{R}{j\sqrt{\frac{L}{C}}}\right] = -90°$$

$$e) \quad \frac{\underline{I}_R}{\underline{U}_1} = ? \quad \underline{I}_R = \frac{\underline{U}_2}{R} \Rightarrow \frac{\underline{I}_R}{\underline{U}_1} = \frac{\underline{U}_2}{\underline{U}_1} \cdot \frac{1}{R} = \frac{1}{(R - \omega^2 RLC) + j\omega L}$$

Transformer, inductive coupling

15.1

Transformer has the voltage ratio $n = N_1/N_2 = 600/200 = 3$.

We get $U_2 = \frac{1}{n}U_1 = \frac{225}{3} = 75$ V and $I_1 = \frac{1}{n}I_2 = \frac{9}{3} = 3$ A.

15.2

$\frac{N_2}{N_1} = \frac{1}{6} \quad \Rightarrow \quad N_1 = 6 \cdot N_2 = 6 \cdot 150 = 900 \qquad U_2 = U_1/n = 230/6 = 38{,}3$ V

15.3

Transformer has voltage ratio

$$\frac{U_1}{U_2} = \frac{N_1}{N_2} = \frac{225}{127} = 1{,}77 \quad \Rightarrow \quad N_2 = \frac{U_2}{U_1}N_1 = \frac{600 \cdot 127}{225} = 339.$$

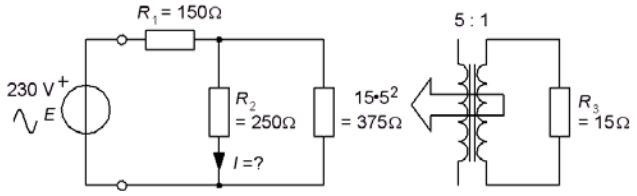
We get $I_1 = \frac{N_2}{N_1}I_2 = \frac{339}{600}9 = 5{,}08$ A .

15.4

$U_1 = 10 - 0{,}2 \cdot 10 = 8\,[\mathrm{V}] \qquad U_2 = \frac{1}{2}U_1 = 0{,}5 \cdot 8 = 4\,[\mathrm{V}] \qquad I_2 = \frac{2}{1}I_1 = 2 \cdot 0{,}2 = \mathbf{0{,}4}\,[\mathrm{A}]$

$R_2 = \frac{U_2}{I_2} = \frac{4}{0{,}4} = \mathbf{10}\,[\Omega]$

15.5



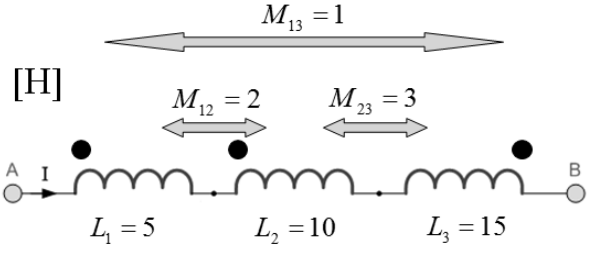
$R_3: \quad R_{1\leftarrow 2} = \left(\frac{N_1}{N_2}\right)^2 \cdot R_2 \quad \Rightarrow \quad 5^2 \cdot 15 = 375$

$250 \,\|\, 375 = 150 \qquad I = \frac{E/2}{250} = \frac{115}{250} = 0{,}46$ A

15.6

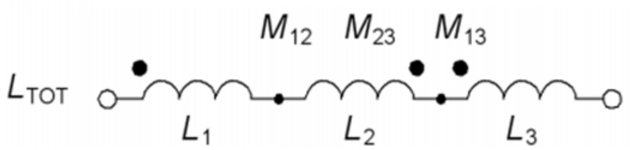
$$L_{\mathrm{ERS}} = \frac{\left(4 + \frac{4 \cdot 4}{4+4}\right) \cdot 6}{4 + \frac{4 \cdot 4}{4+4} + 6} = 3\ \mathrm{H}$$

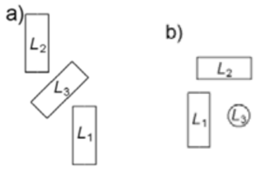
15.7



$L_{\mathrm{TOT}} =$
$L_1 + M_{12} - M_{13} +$
$L_2 + M_{12} - M_{23} +$
$L_3 - M_{23} - M_{13} =$
$= 5 + 2 - 1 + 10 + 2 - 3 + 15 - 3 - 1 = 26$ [H]

15.8





a) $L_{TOT} = L_1 - M_{12} + M_{13} +$

$L_2 - M_{12} - M_{23} +$

$L_3 - M_{23} + M_{13} =$

$= 12 - 3 + 1 + 6 - 3 - 2 + 5 - 1 + 1 = 16\,[\mathrm{H}]$

b) $L_{TOT} = L_1 + L_2 + L_3 = 12 + 6 + 5 = 23\,[\mathrm{H}]$