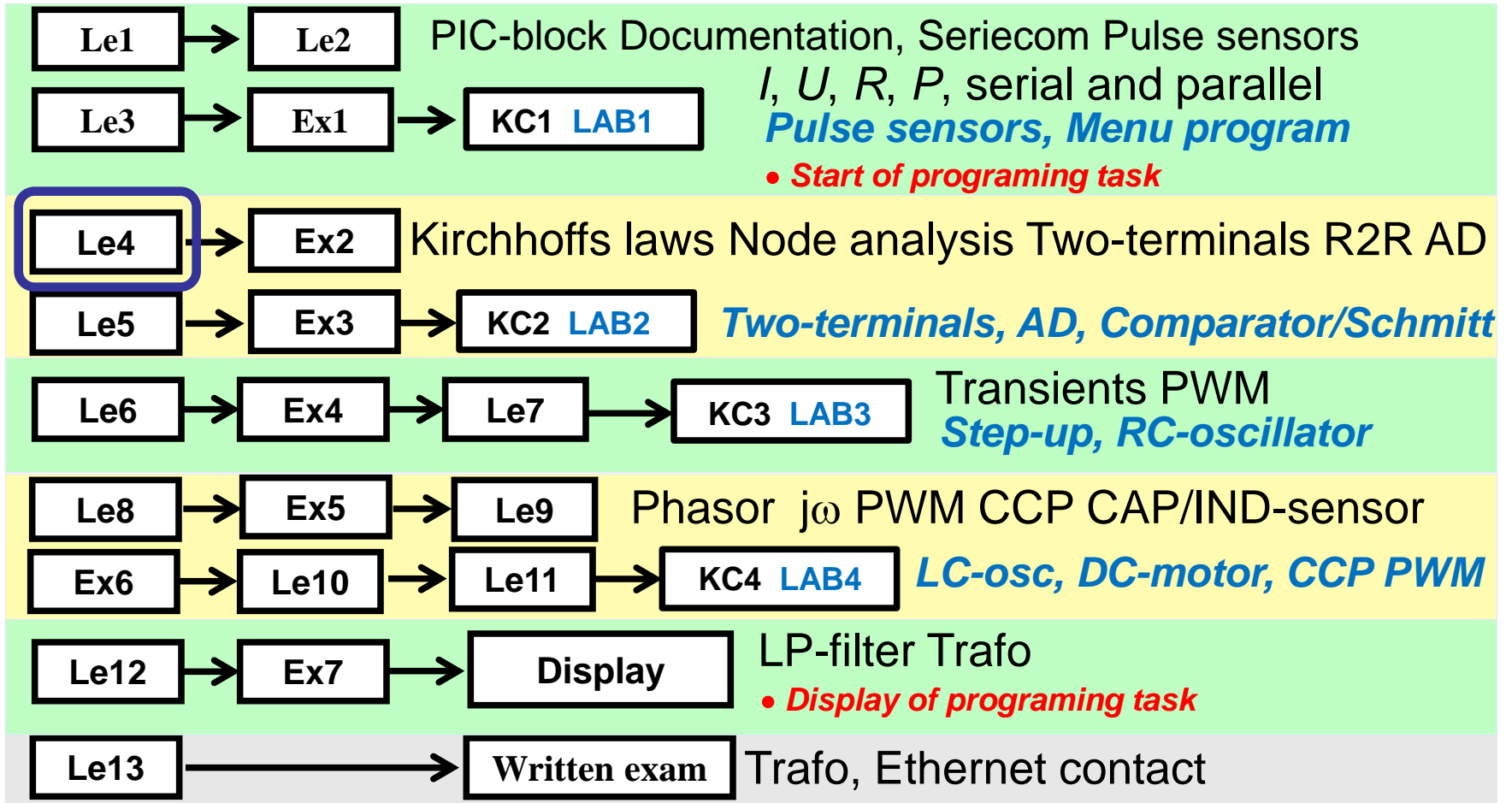


IE1206 Embedded Electronics



Kirchhoff



Gustav Robert Kirchhoff

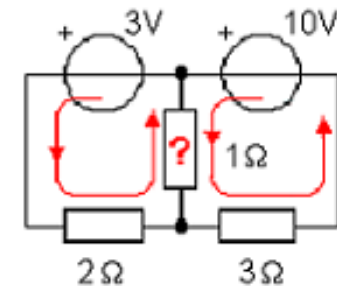
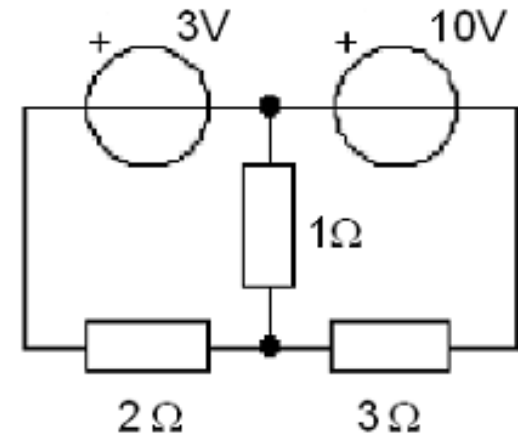
In 1847 formulated Kirchhoff two laws for electrical networks. These two laws Kirchhoff's current law and voltage law, together with the Ohm's law is the basis of all the electrical circuit theory.

Kirchhoffs laws

Ohm's Law is about one resistor one voltage drop and one current.

Usually you have more complex circuits with multiple voltage sources and many resistors. Voltage sources can be connected so that they work together or against each other. The resistors branches current between the voltage sources.

In the circuit in the figure two emfs 3V and 10V are driving current through a 1 Ω resistor, but from different directions. Which will win? The current from 3V emf must pass through a 2 Ω resistor and the current from 10V emf through a 3 Ω .

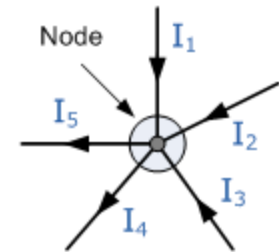


Do you dare to guess ...

Kirchhoffs current law

Ohm's law must be supplemented by a method that takes into account how the circuit is composed. Kirchhoff's two laws makes it possible to set up an equation system to solve the currents flowing in an electrical circuit and their directions. If one makes the circuit more complicated then one simultaneously creates the ability to set up more equations - Kirchhoff "promise "that the circuit is solvable!

$$\sum_{\text{Node}} I = 0 \quad I_1 + I_2 + I_3 - I_4 - I_5 = 0$$



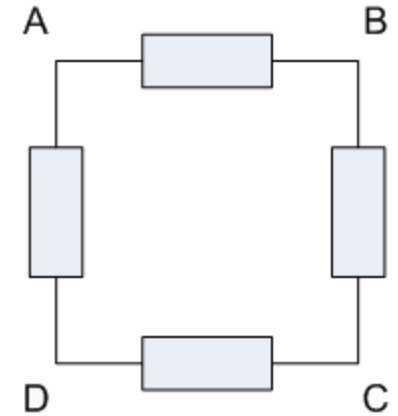
Kirchhoffs current law.

The electrical circuits consists of nodes. The sum of all currents to and from a node are zero. The current branches between the conductors connected to the node, but it passes the node without losses.

It is customary to count all currents that are entering a node with a plus sign, and all currents coming out of the node with the minus sign.

Kirchhoffs voltage law

$$\sum_{\text{ABCD}} U = 0 \quad U_{\text{AB}} + U_{\text{BC}} + U_{\text{CD}} + U_{\text{DA}} = 0$$



Kirchhoffs voltage law.

In electrical circuits are loops (or mesh). A loop starts and ends at the same point, eg. point **A**. Following a loop "full circle" and sums up all the voltage changes, at the the passage of emfs and the resistors voltage drops so the sum is zero.

One usually defines a positive voltage if one passes through an emf at the positive terminal, and a positive voltage if one passes through a resistor at the end where the current enters. Otherwise, the voltage is negative voltage.

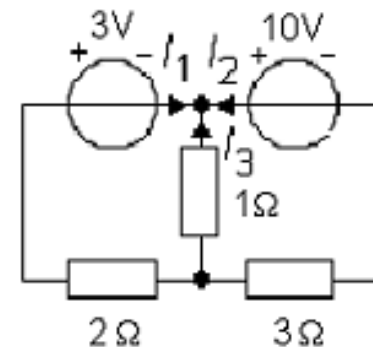
Vätternrundan (pronounced vet-turn-rund-an)

Motala-Hästholmen-Gränna-Jönköping-Fagerhult-Hjo-Karlsborg-Boviken-Hammarsundet-Medevi-Motala

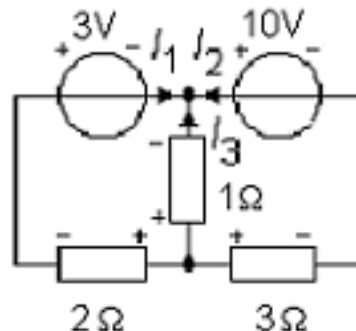
Work flow

You start by drawing out the currents I_1 , I_2 and I_3 . One may define the current direction in any way you want (if you have the wrong direction the calculations will ultimately provide a minus sign).

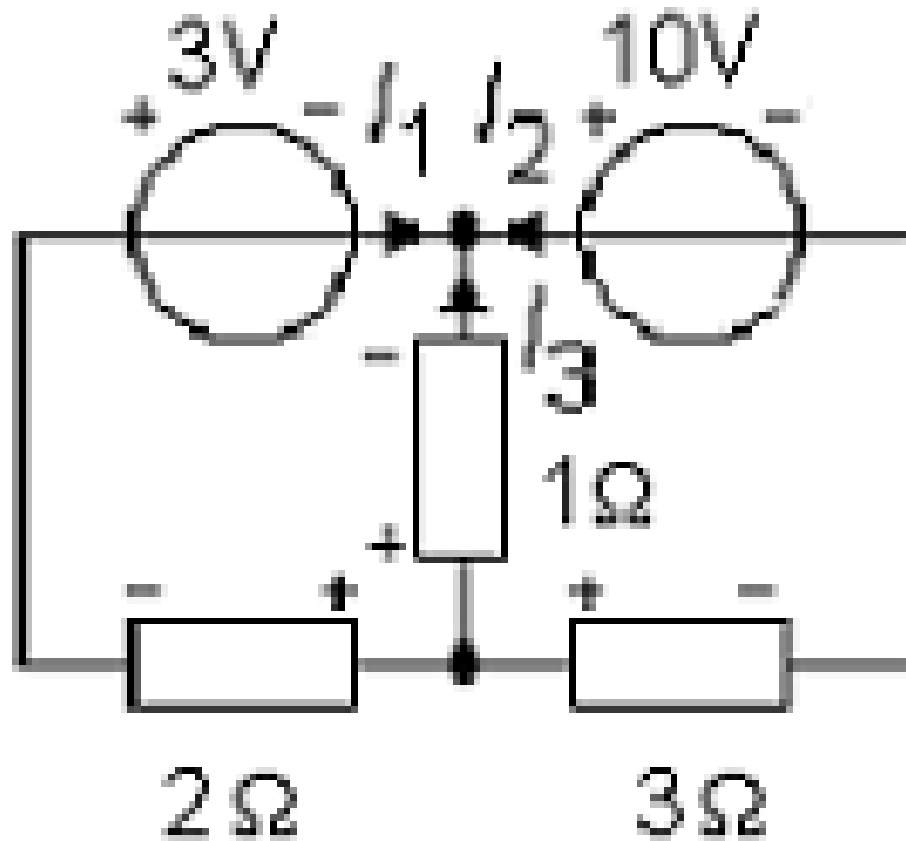
Here all currents are drawn in the direction toward the node - so, it can not be, so at least some currents will have a minus sign!



When the currents are defined, highlight the voltage drop over the resistors. Where the current enters the resistor the voltage drop has its plus sign.



Work flow



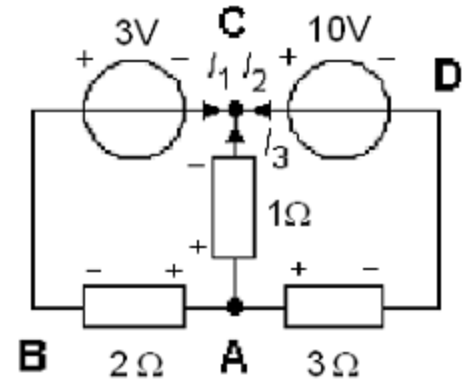
Equations

Kirchhoffs current law (sum of all currents in a node is 0, currents entering the node are taken with "+" sign, currents leaving the node are taken with "-" sign).

$$\sum_C I = 0 \quad I_1 + I_2 + I_3 = 0$$

Node A has the same currents as node C so it will not provide any new equation.

$$1 \cdot I_1 + 1 \cdot I_2 + 1 \cdot I_3 = 0$$



Kirchhoffs voltage law (sum of all voltages around a loop is 0).

Two loops:

$$\sum_{ABCA} U = 0 \quad -2 \cdot I_1 - 3 + 1 \cdot I_3 = 0 \quad \Leftrightarrow \quad -2 \cdot I_1 + 0 \cdot I_2 + 1 \cdot I_3 = 3$$

$$\sum_{ABCD} U = 0 \quad -2 \cdot I_1 - 3 - 10 + 3 \cdot I_2 = 0 \quad \Leftrightarrow \quad -2 \cdot I_1 + 3 \cdot I_2 + 0 \cdot I_3 = 13$$

Loop ACDA contains nothing more, we already have three equations – this loop is not needed.

Three equations and three unknowns = solvable!

"OHM's" law at matrix form

Three equations:

$$\begin{aligned}1 \cdot I_1 + 1 \cdot I_2 + 1 \cdot I_3 &= 0 \\-2 \cdot I_1 + 0 \cdot I_2 + 1 \cdot I_3 &= 3 \\-2 \cdot I_1 + 3 \cdot I_2 + 0 \cdot I_3 &= 13\end{aligned}$$

Matrix equation: $\mathbf{R} \cdot \mathbf{I} = \mathbf{U}$

$$\begin{pmatrix} 1 & 1 & 1 \\ -2 & 0 & 1 \\ -2 & 3 & 0 \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 13 \end{pmatrix}$$

Solution: $I_1 = -2$ $I_2 = 3$ $I_3 = -1$

(The currents I_1 and I_3 has the opposite direction of the assumed, just as we predicted, there must be currents having the directions out of the node)

We can see Kirchhoff's laws as a generalization of Ohm's law to apply to the entire circuits!

3x3 System - Mozilla Firefox

Arkiv Redigera Visa Historik Bokmärken Verktyg Hjälp

http://math.cowpi.com/systemsolver/3x3.htm

Math @ CowPi

System Solver 2 x 2 **3 x 3** 4 x 4 5 x 5

System of Three Equations & Three Unknowns

$1x + 1y + 1z = 0$

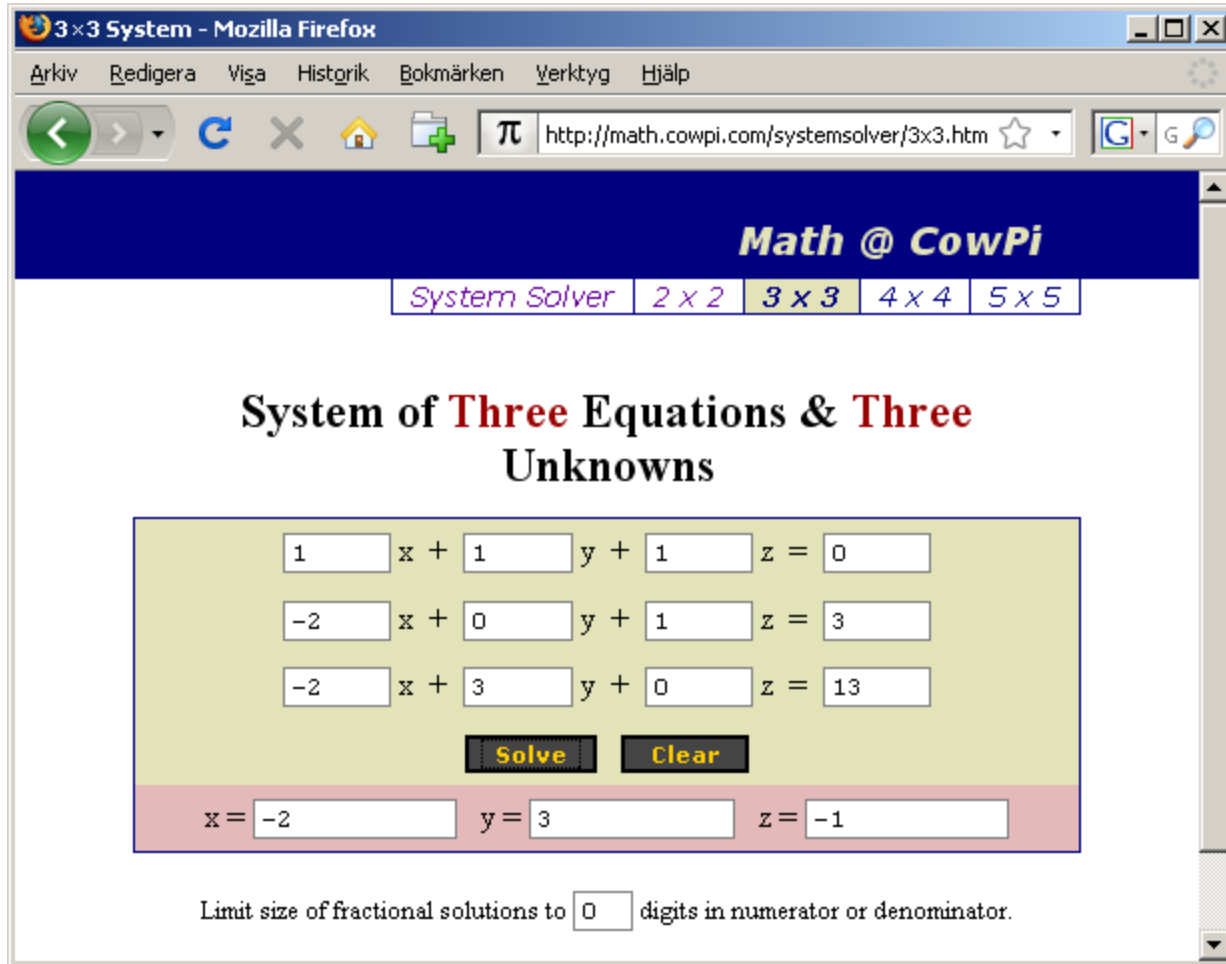
$-2x + 0y + 1z = 3$

$-2x + 3y + 0z = 13$

Solve **Clear**

x = -2 y = 3 z = -1

Limit size of fractional solutions to 0 digits in numerator or denominator.



CowPi Systemsolver

William Sandqvist william@kth.se

Kirchhoffs laws – try it yourself

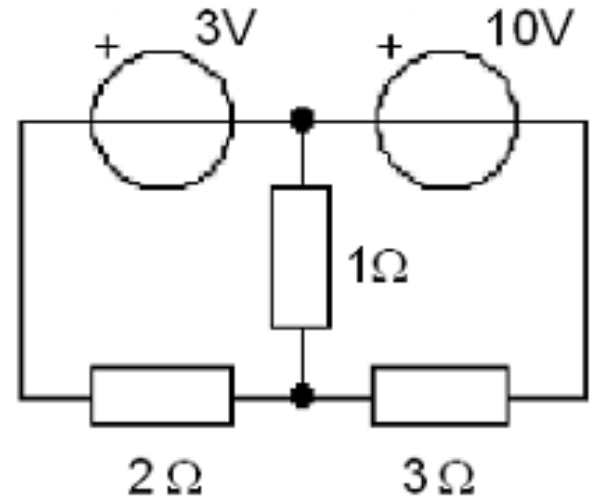
Define currents and use Kirchhoff's current law at the nodes.

$$\sum_{\text{Node}} I = 0$$

highlight the resistors voltage drop.

Use Kirchhoff's voltage law on loops.

$$\sum_{\text{loop}} U = 0$$



Solve the system of equations. Tips! In manual computation start with
expresse one of the currents in the others to reduce the number unknowns.

Independent equations

The method of Kirchhoff's laws are very flexible, you simply choose yourself how to set up the equations.

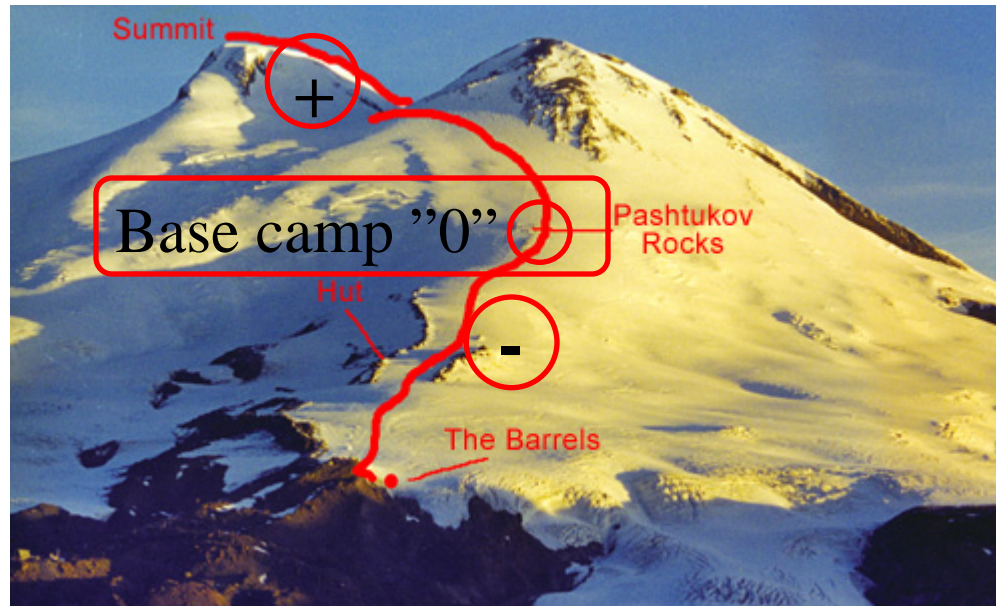
If the network is complex, it may be difficult to determine how many equations are needed, and to convince yourself that all equations are linearly independent.

There are therefore a number of systematic methodologies that ensure that you get the right number of independent equations. The scheme also means that you can take advantage of the special features that the equation systems have - This has led to some methods for rapid setup of the equation systems.

The professionals that makes extensive circuit calculations by hand earns on to learn these. (We wonder if there are so many of them nowadays ...?)

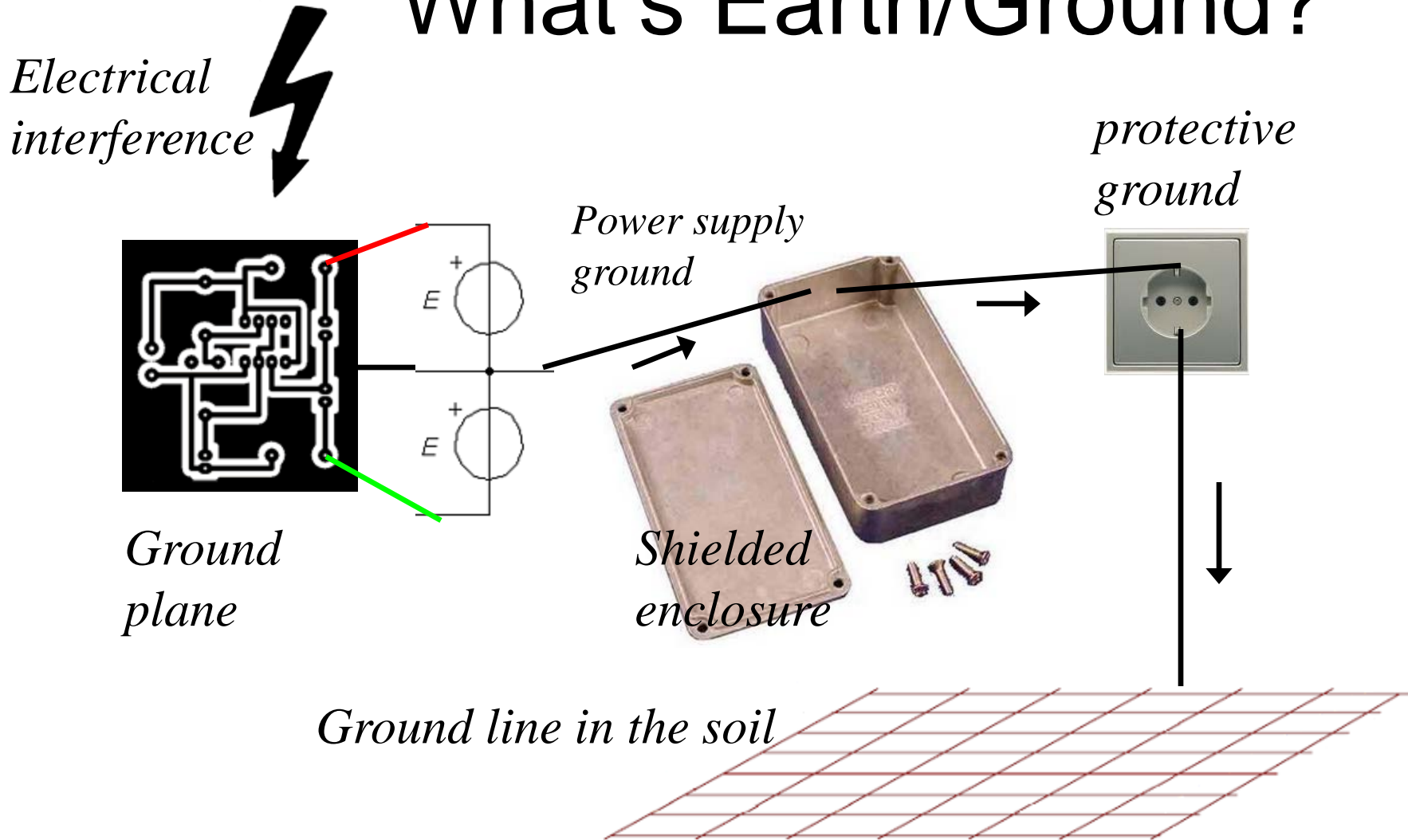
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Potential



The voltage relative to a reference point.
(Compare the height relative to the base camp for mountaineering expeditions.)

What's Earth/Ground?



What's ground on an aircraft?



What's ground on an aircraft?

The aircraft's casing is made of aluminum, it functions as a ground for all electrical equipment. No direct connection to earth is possible during a flight.

Static electricity is led away from the body to the surroundings with "static wicks" tips on the wings.



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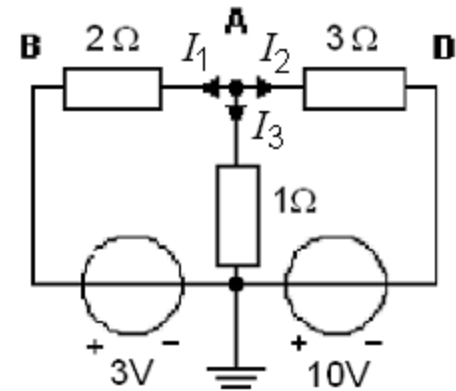
Node analysis

On more complex circuits need systematic methods that ensure that you set up the equations correctly are needed.

Node analysis is advantageous to use when there are few nodes with unknown potentials.

You can always decide that an arbitrarily selected node is the reference and assigned the potential 0V, earth. In this example, there is then only one node left with unknown potential and it is **A**.

Many times, eg in electronics, our electrical circuits has a "natural" ground that most components are connected to, and then it can be especially useful to use the node analysis method.



The previous example redrawn for node analysis.

*We have maintained current directions, but have drawn the currents to node **A** instead.*

Node analysis ...

Kirchhoff's current law:

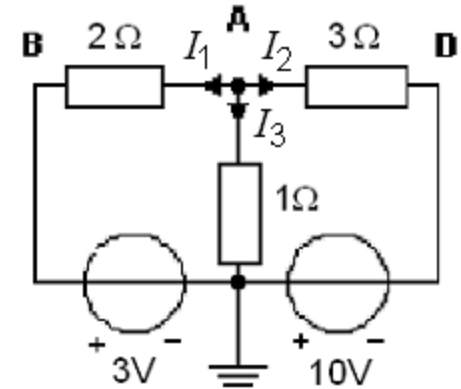
$$\sum_{\text{A}} I = 0 \quad -I_1 - I_2 - I_3 = 0 \quad \text{or} \quad I_1 + I_2 + I_3 = 0$$

OHM's law:

$$I_1 = \frac{U_{\text{AB}}}{2} = \frac{U_{\text{A}} - 3}{2}$$

$$I_2 = \frac{U_{\text{AD}}}{3} = \frac{U_{\text{A}} - (-10)}{3} = \frac{U_{\text{A}} + 10}{3}$$

$$I_3 = \frac{U_{\text{A0}}}{1} = \frac{U_{\text{A}} - 0}{1} = \frac{U_{\text{A}}}{1}$$



Node analysis ...

The expressions of branch currents can now be inserted into the current equation:

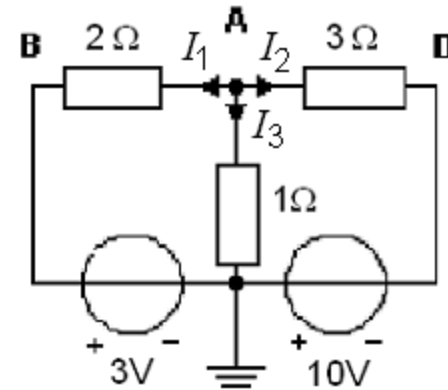
$$I_1 + I_2 + I_3 = 0$$

$$\frac{U_A - 3}{2} + \frac{U_A + 10}{3} + \frac{U_A}{1} = 0$$

Solve the unknown U_A :

$$\frac{3 \cdot (U_A - 3)}{6} + \frac{2 \cdot (U_A + 10)}{6} + \frac{6 \cdot U_A}{6} = 0$$

$$11 \cdot U_A + 11 = 0 \quad U_A = -\frac{11}{11} = -1 \text{ V}$$



At last, calculate currents I_1 I_2 I_3 :

$$I_1 = \frac{U_A - 3}{2} = \frac{-1 - 3}{2} = -2$$

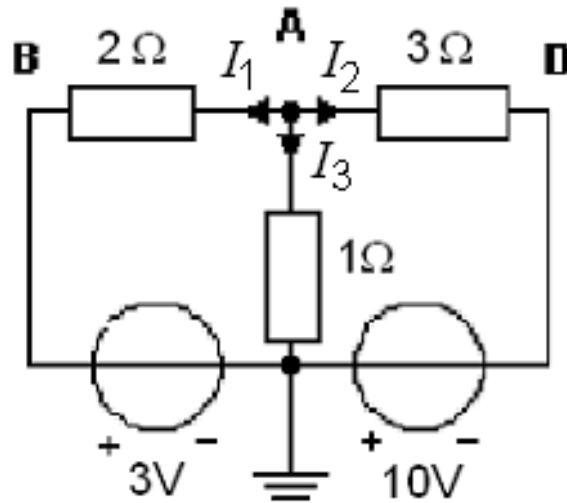
$$I_2 = \frac{U_A + 10}{3} = \frac{-1 + 10}{3} = 3$$

$$I_3 = \frac{U_A}{1} = \frac{-1}{1} = -1$$

Node analysis, work flow

- 1) Insert variables for all node potentials. An optional node is chosen as the reference (0), GND. Possibly the circuit has already been such a ground node.
- 2) Derive all branch currents with OHM's law.
- 3) Use Kirchhoffs current law on all nodes except one. This will generate the same amount of equations as the unknown node potentials.
- 4) Solve the equation system.

Node analysis – try it yourself



- 1) Insert variables for all node potentials. An optional node is chosen as the reference (0), GND. Possibly the circuit has already been such a ground node.
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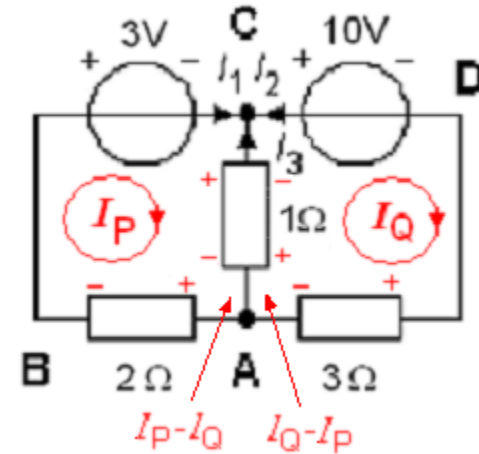
(Mesh analysis)

The most popular method is Mesh analysis. It can be used on any flat networks.

(A flat network can be drawn on a piece of paper without any wires or components crossing each other).

In figure **ABCA** and **ACDA** are *meshes* while the outer **ABCD** is a *loop*.

In all meshes we insert fictive "circulating" mesh currents, I_P and I_Q .



The real currents are:

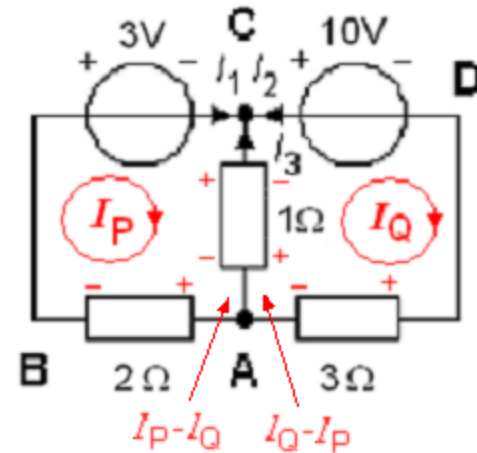
$$I_1 = I_P \quad I_2 = -I_Q \quad I_3 = I_Q - I_P$$

In the branch between the meshes the resulting current will be the difference between I_P and I_Q as these currents come from different directions.

Now we define the signs of the voltage drops of all resistors as the mesh-currents will give them.

(Maskanalys)

Kirchhoff's voltage law is set up for the meshes **P** and **Q**:



Common branch

$$\sum_{\text{P:ABCA}} U = 0 \quad -2 \cdot I_P - 3 - 1 \cdot (I_P - I_Q) = 0 \quad \Leftrightarrow \quad -3 \cdot I_P + 1 \cdot I_Q = 3$$

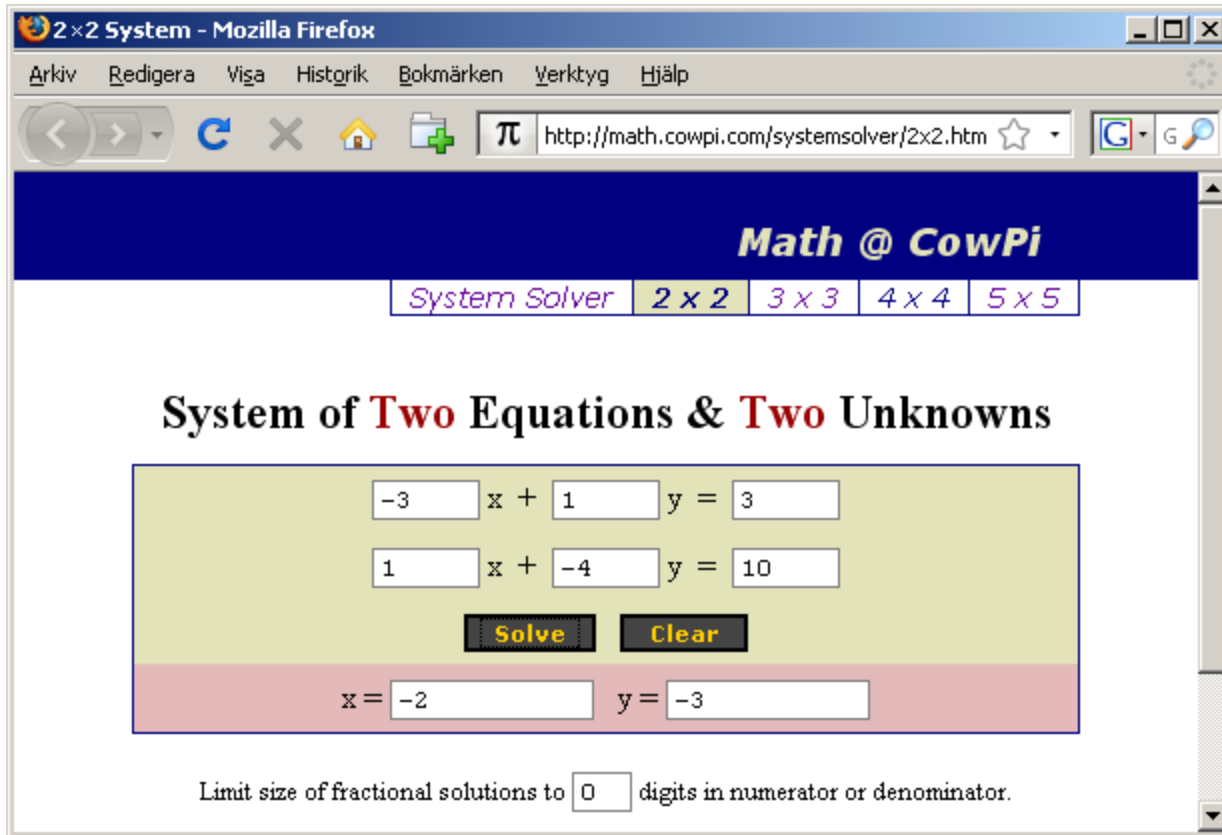
Common branch

$$\sum_{\text{Q:ACDA}} U = 0 \quad -1 \cdot (I_Q - I_P) - 10 - 3 \cdot I_Q = 0 \quad \Leftrightarrow \quad 1 \cdot I_P - 4 \cdot I_Q = 10$$

$$\begin{pmatrix} -3 & 1 \\ 1 & -4 \end{pmatrix} \cdot \begin{pmatrix} I_P \\ I_Q \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \end{pmatrix} \quad I_P = -2 \quad I_Q = -3$$

The real currents get

$$I_1 = I_P = -2 \quad I_2 = -I_Q = -(-3) = 3 \quad I_3 = I_Q - I_P = (-3) - (-2) = -1$$



The equation system now has two unknown variables not three!

[CowPi Systemsolver](http://math.cowpi.com/systemsolver/)

(Mesh analysis, work flow ...)

- 1) Insert variables for circulating currents in all meshes. The system of equations will be easier to solve if the direction of circulation is the same for all mesh currents (ie clockwise).
- 2) Set up Kirchhoff's voltage law for all meshes. This gives as many equations as the number of unknowns mesh currents.
- 3) Solve the equation system.

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Which method is most important to know and use?

- ***Most important.***

To set up the system of equations with Kirchhoff's laws is the principle close method. All currents and voltages defined are possible to directly "measure" and check, in a real circuit.

- **Important for electronics.**

Node analysis, working with potentials relative to circuit ground, fits like "hand in glove" on electronics. Often you only look at individual nodes, avoiding to set up systems of equations.

- **Important general knowledge.**

The mesh analysis is the classic, most widely used method. You must know that the method exists to avoid the risk of missing electronics general knowledge.

The advent of the mathematics program like Mathematica and simulation programs like Spice has fundamentally changed the nature of the calculations to do by hand!

- Equation system with Mathematica

$$\begin{pmatrix} 1 & 1 & 1 \\ -2 & 0 & 1 \\ -2 & 3 & 0 \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 13 \end{pmatrix}$$

Use **J** for I-vector because **I** is "protected" in Mathematica.

R = {{1,1,1},{-2,0,1},{-2,3,0}}

U = {{0},{3},{13}}

J = Inverse [R].U

(Shift + Enter starts the calculation)

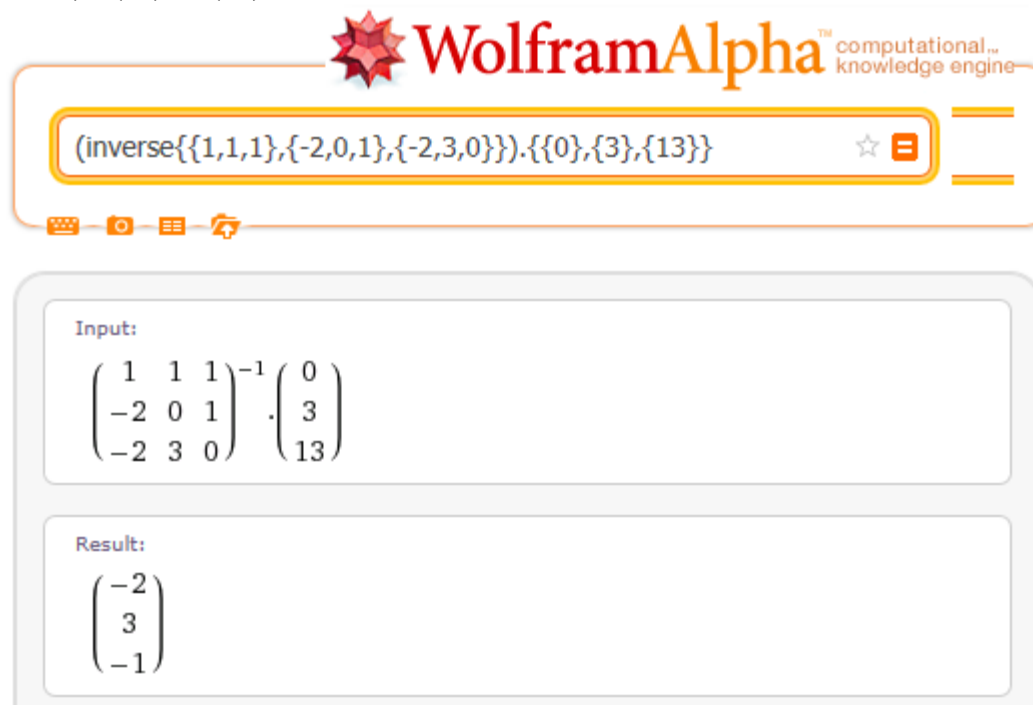
Mathematica answers:

{{-2},{3},{-1}}

• Wolfram Alpha online

$$\begin{pmatrix} 1 & 1 & 1 \\ -2 & 0 & 1 \\ -2 & 3 & 0 \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 13 \end{pmatrix}$$

Online on web:

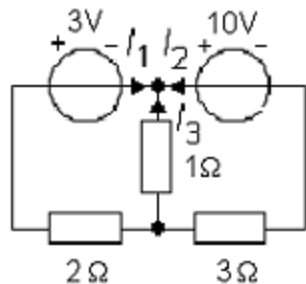


The screenshot shows the Wolfram Alpha interface. At the top is the Wolfram Alpha logo with the tagline "computational knowledge engine". Below the logo is a search bar containing the input: `(inverse{{1,1,1},{-2,0,1},{-2,3,0}}).{{0},{3},{13}}`. Below the search bar are several icons for input methods. The main content area is divided into two sections: "Input:" and "Result:". The "Input:" section displays the mathematical expression $\begin{pmatrix} 1 & 1 & 1 \\ -2 & 0 & 1 \\ -2 & 3 & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 0 \\ 3 \\ 13 \end{pmatrix}$. The "Result:" section displays the solution vector $\begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}$.

`inverse[{{1, 1, 1}, {-2, 0, 1}, {-2, 3, 0}}] . {{0}, {3}, {13}}`

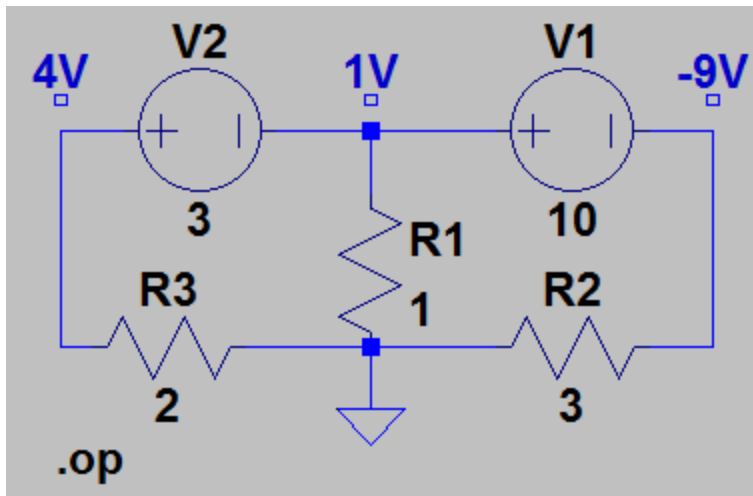
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• Simulation with LTSpice



Already fairly small circuits provides extensive equation systems that are difficult to solve by hand. It is therefore common to simulate the electrical circuits with computer programs.

SPICE developed at Berkeley in the mid-1970s (in FORTRAN programming language) it has laid the foundation for the simulation programs used today.

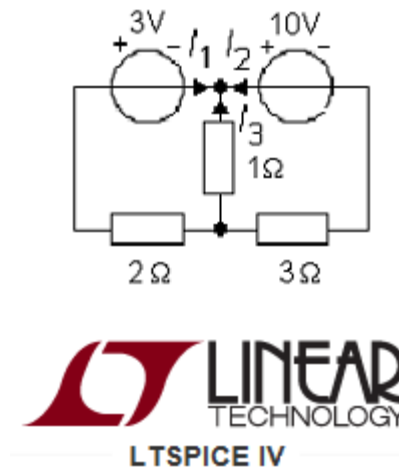
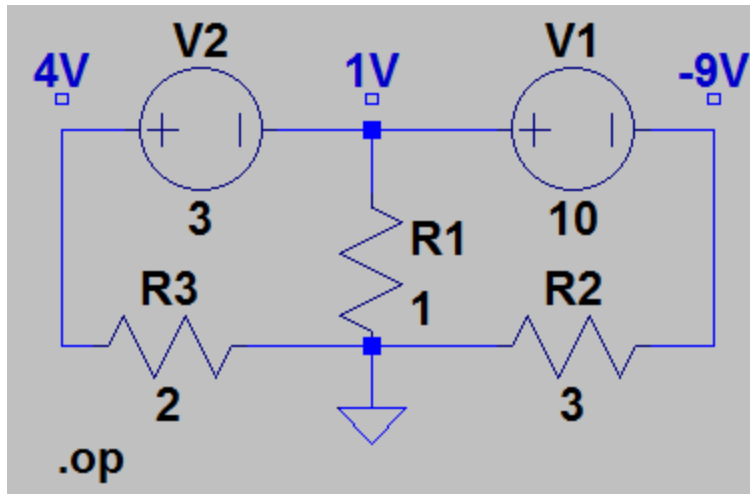


You can download the program for free and install on your own computer:

[LTspiceIV](#)



- Simulation with LTSpice



See [Tutorial](#) on course webb that shows how it's done!

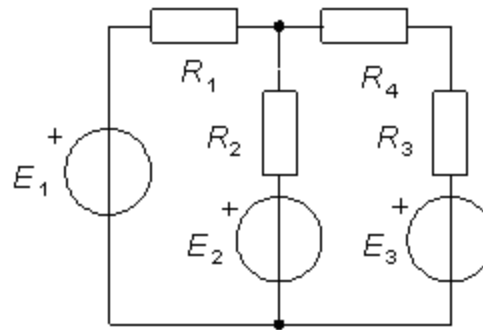
Web-task

Kirchhoffs laws

Use Kirchhoffs laws to first calculate the currents in this circuit.

Assume that

$$\begin{aligned} E_1 &= 50 \text{ V} & R_1 &= 0,18 \text{ } \Omega \\ E_2 &= 47 \text{ V} & R_2 &= 0,95 \text{ } \Omega \\ E_3 &= 46 \text{ V} & R_3 &= 0,9 \text{ } \Omega \\ & & R_4 &= 0,3 \text{ } \Omega \end{aligned}$$



Then, calculate the power in

- a) Resistor R_1 [W]
- b) Resistor R_4 [W]

Everyone of you has got an unique web-task to solve!

Write down your values in this table

a) P_{R1} [W]	b) P_{R4} [W]

Have fun! Good luck!

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