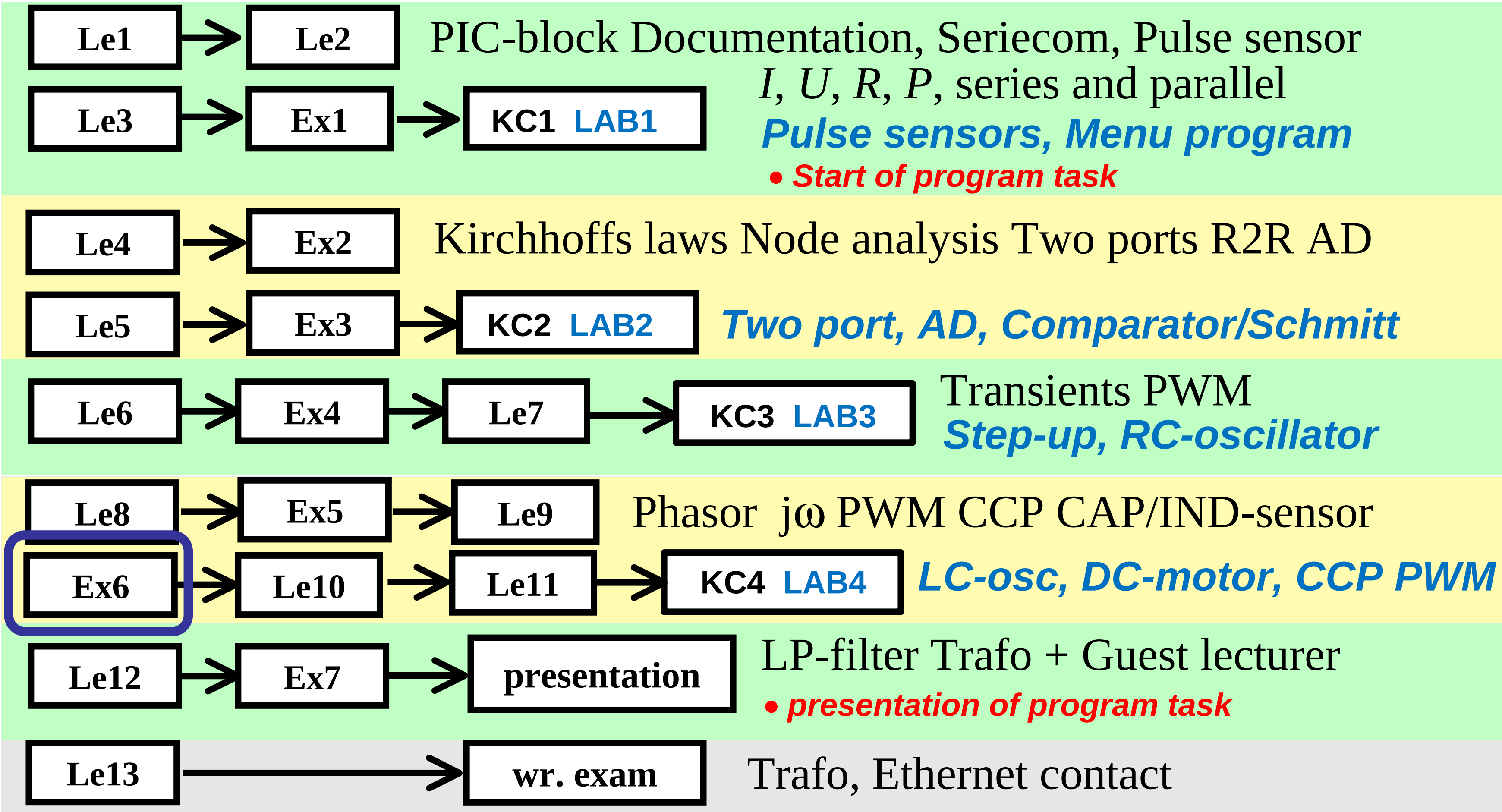
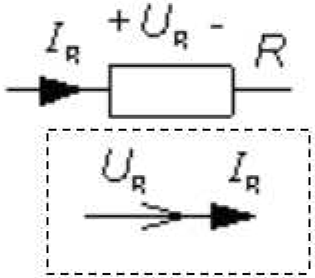
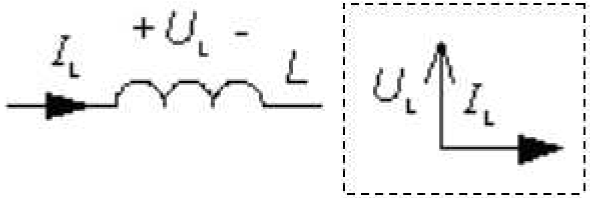
IE1206 Embedded Electronics

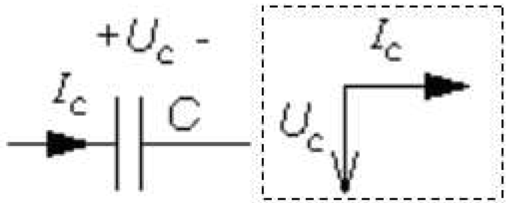


William Sandqvist william@kth.se

Phasor - vector







$$\omega = 2\pi\ f$$

$$\left|X_L\right| = \omega \cdot L$$

$$\left|X_C\right| = \frac{1}{\omega \cdot C}$$

$$\boxed{Z = \frac{U}{I}}$$

Complex phasors, jω-method

- Complex OHM's law for R L and C.

$$\underline{U}_{\mathrm{R}} = \underline{I}_{\mathrm{R}} \cdot R$$

$$\underline{U}_{\mathrm{L}} = \underline{I}_{\mathrm{L}} \cdot \mathrm{j} X_{\mathrm{L}} = \underline{I}_{\mathrm{L}} \cdot \mathrm{j}\omega L \qquad \omega = 2\pi \cdot f$$

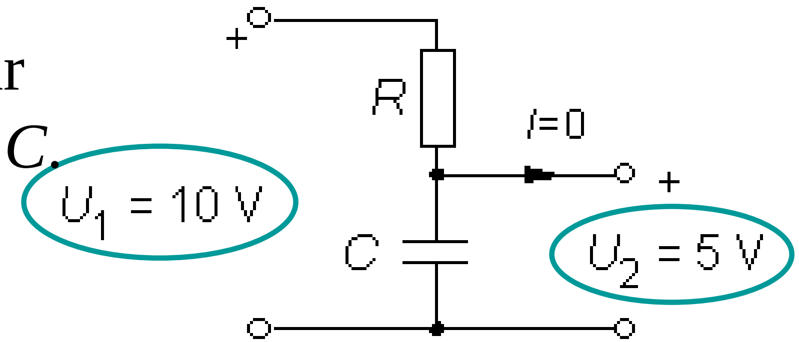
$$\underline{U}_{\mathrm{C}} = \underline{I}_{\mathrm{C}} \cdot \mathrm{j} X_{\mathrm{C}} = \underline{I}_{\mathrm{C}} \cdot \frac{1}{\mathrm{j}\omega C}$$

- Complex OHM's law for Z.

$$\boxed{\underline{U} = \underline{I} \cdot \underline{Z}} \qquad Z = \frac{U}{I} \qquad \varphi = \arg(\underline{Z}) = \arctan\left(\frac{\mathrm{Im}[\underline{Z}]}{\mathrm{Re}[\underline{Z}]}\right)$$

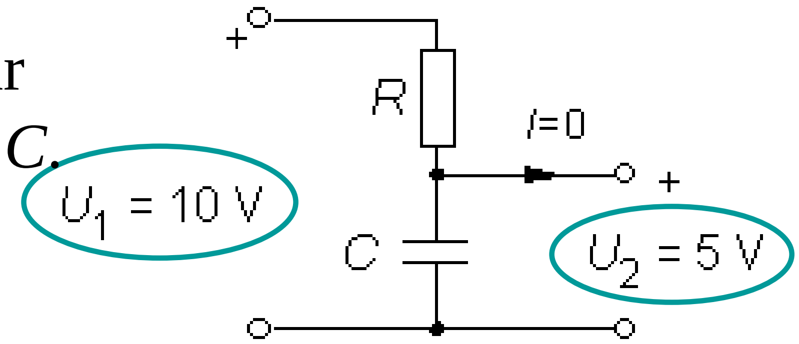
ω for half the voltage? (12.3)

U_1 is a sine voltage with the angular frequency ω. Decide the product $R \cdot C$. (No current is consumed at U_2).



ω for half the voltage? (12.3)

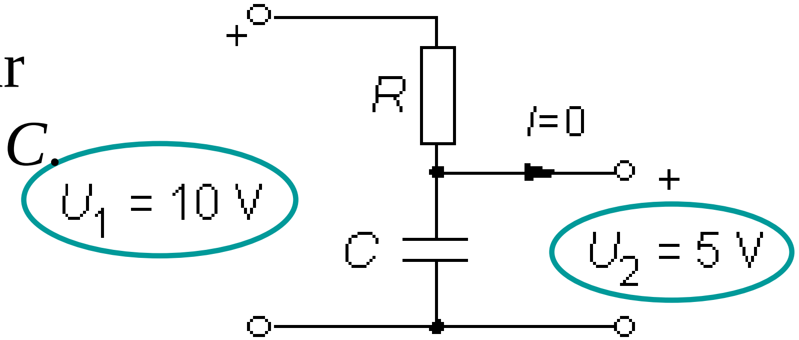
U_1 is a sine voltage with the angular frequency ω. Decide the product $R \cdot C$. (No current is consumed at U_2).



$$\underline{U}_2 = \underline{U}_1 \cdot \frac{\dfrac{1}{\mathrm{j}\omega C}}{R + \dfrac{1}{\mathrm{j}\omega C}}$$

ω for half the voltage? (12.3)

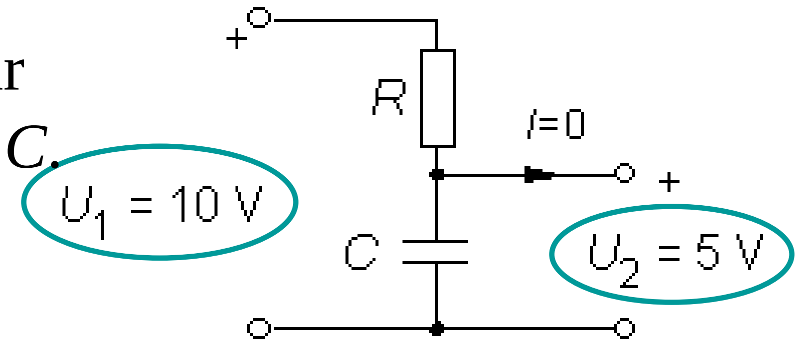
U_1 is a sine voltage with the angular frequency ω. Decide the product $R \cdot C$. (No current is consumed at U_2).



$$\underline{U}_2 = \underline{U}_1 \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \cdot \frac{(j\omega C)}{(j\omega C)} =$$

ω for half the voltage? (12.3)

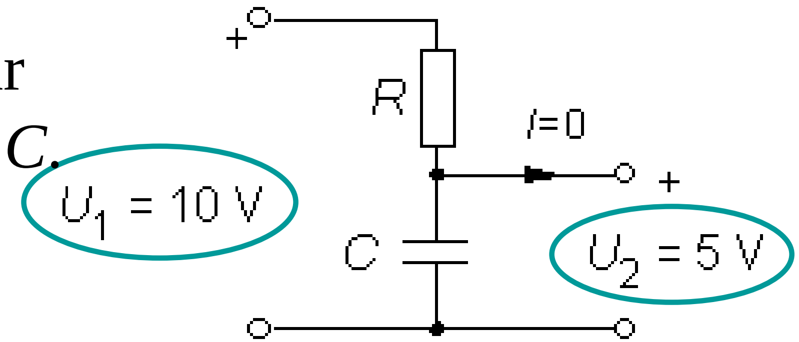
U_1 is a sine voltage with the angular frequency ω. Decide the product $R \cdot C$. (No current is consumed at U_2).



$$\underline{U}_2 = \underline{U}_1 \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \cdot \frac{(j\omega C)}{(j\omega C)} = \underline{U}_1 \cdot \frac{1}{1 + j\omega RC}$$

ω for half the voltage? (12.3)

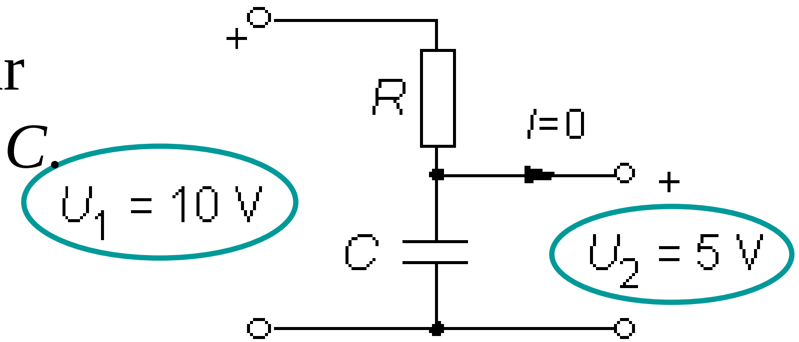
U_1 is a sine voltage with the angular frequency ω. Decide the product $R \cdot C$. (No current is consumed at U_2).



$$\underline{U}_2 = \underline{U}_1 \cdot \frac{\frac{1}{\mathrm{j}\omega C}}{R + \frac{1}{\mathrm{j}\omega C}} \cdot \frac{(\mathrm{j}\omega C)}{(\mathrm{j}\omega C)} = \underline{U}_1 \cdot \frac{1}{1 + \mathrm{j}\omega RC} \quad \Rightarrow \quad \frac{U_1}{U_2} = \sqrt{1 + R^2\omega^2C^2} = \frac{10}{5} = 2$$

ω for half the voltage? (12.3)

U_1 is a sine voltage with the angular frequency ω. Decide the product $R \cdot C$. (No current is consumed at U_2).

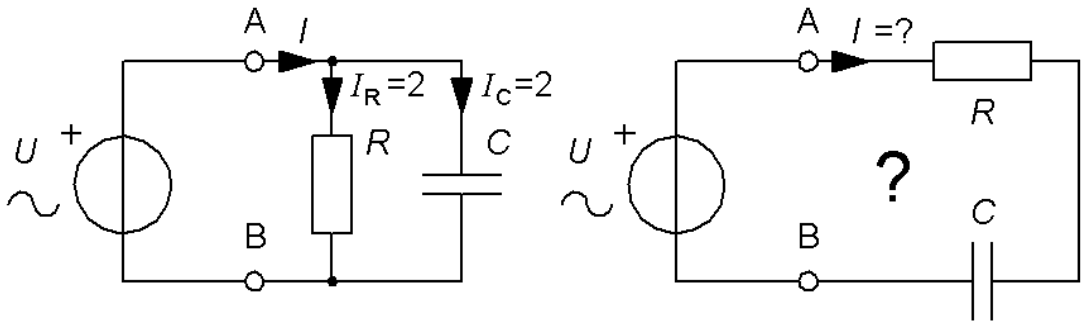


$$\underline{U}_2 = \underline{U}_1 \cdot \frac{\frac{1}{\mathrm{j}\omega C}}{R + \frac{1}{\mathrm{j}\omega C}} \cdot \frac{(\mathrm{j}\omega C)}{(\mathrm{j}\omega C)} = \underline{U}_1 \cdot \frac{1}{1 + \mathrm{j}\omega RC} \quad \Rightarrow \quad \frac{U_1}{U_2} = \sqrt{1 + R^2\omega^2C^2} = \frac{10}{5} = 2$$

$$1 + R^2\omega^2C^2 = 4 \quad \Leftrightarrow \quad R\omega C = \sqrt{3} \quad \Leftrightarrow \quad RC = \frac{\sqrt{3}}{\omega}$$

William Sandqvist william@kth.se

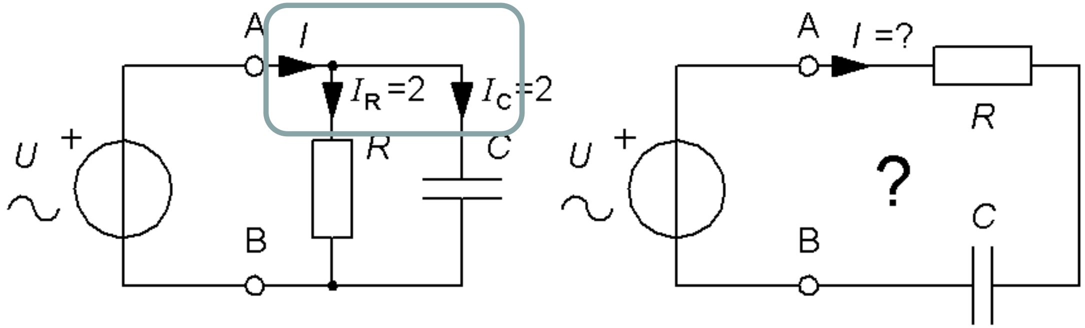
Compare serial with parallel (12.5)



When a resistor *R* and a capacitor *C* is connected in parallel to a voltage source *U* each of them get the current 2A.

How big would the current in the resistor be if the two were series connected to the voltage source?

Compare serial with parallel (12.5)

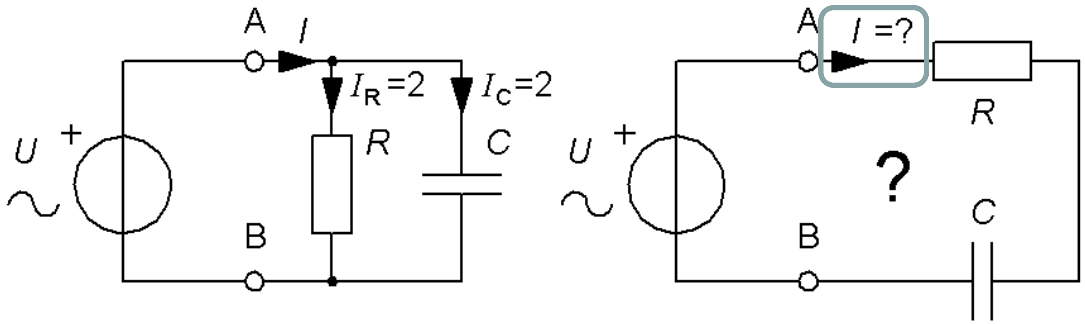


Parallel connection:

$$\underline{I} = \underline{I}_{\mathrm{R}} + \underline{I}_{\mathrm{C}} = \frac{U}{R} + \mathrm{j}U\omega C \quad \underline{I} = 2 + 2\mathrm{j}$$

$$I_R = \frac{U}{R} = 2 \quad I_C = U\omega C = 2 \quad \Rightarrow \quad R = \frac{1}{\omega C} = \frac{U}{2}$$

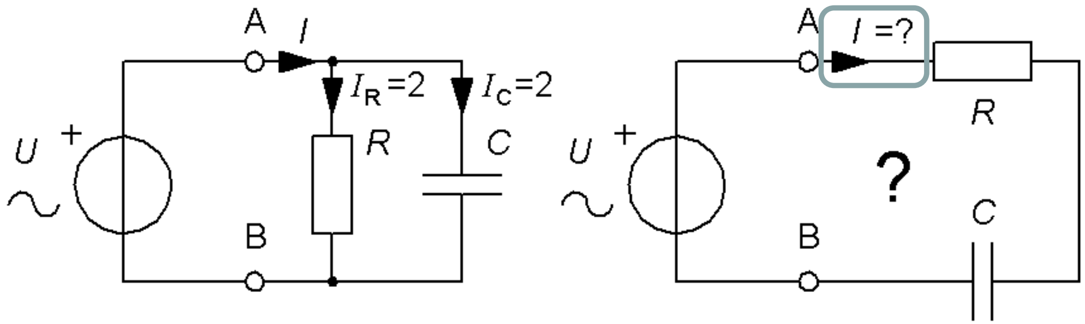
Compare serial with parallel (12.5)



Series connected:

$$\underline{I} = \frac{U}{R + \frac{1}{\mathrm{j}\omega C}} \quad \Rightarrow \quad I = \frac{U}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

Compare serial with parallel (12.5)



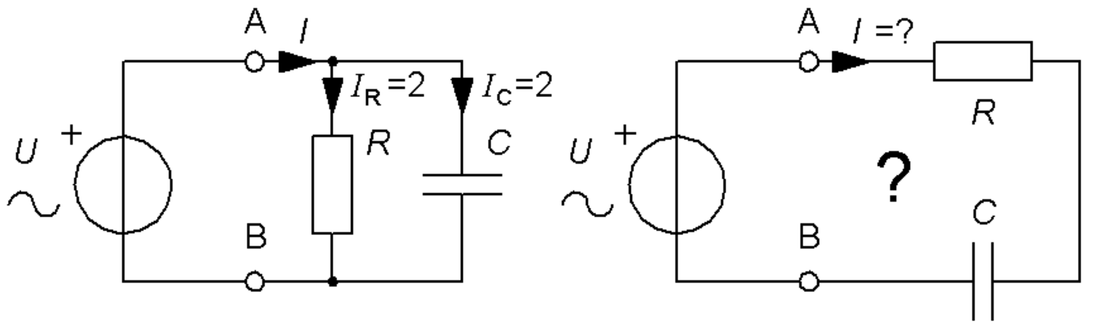
Series connected:

As before …

$$\underline{I} = \frac{U}{R + \frac{1}{j\omega C}} \Rightarrow I = \frac{U}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \quad R = \frac{1}{\omega C} = \frac{U}{2}$$

$$\Rightarrow \quad I = \frac{U}{\sqrt{\frac{U^2}{2^2} + \left(\frac{U}{2}\right)^2}} = \frac{U}{U \cdot \sqrt{\frac{1}{4} + \frac{1}{4}}} = \sqrt{2} \approx 1{,}414 \text{ A}$$

Compare serial with parallel (12.5)



Series connection:

As before …

$$\underline{I} = \frac{U}{R + \frac{1}{\mathrm{j}\omega C}} \Rightarrow I = \frac{U}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \quad R = \frac{1}{\omega C} = \frac{U}{2}$$

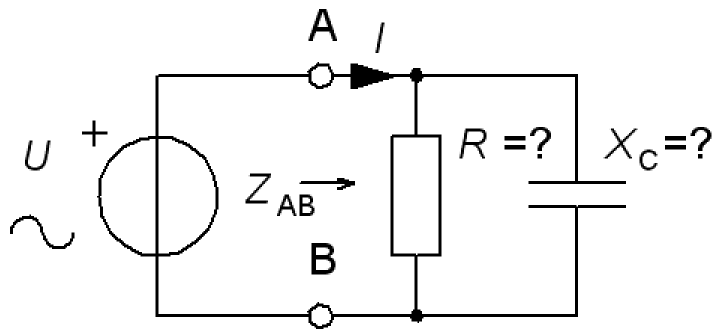
$$\Rightarrow I = \frac{U}{\sqrt{\frac{U^2}{2^2} + \left(\frac{U}{2}\right)^2}} = \frac{U}{U \cdot \sqrt{\frac{1}{4} + \frac{1}{4}}} = \sqrt{2}\ \mathrm{A}$$

Parallel 2A
Series 1,4A

William Sandqvist william@kth.se

Try yourself ... (12.1)

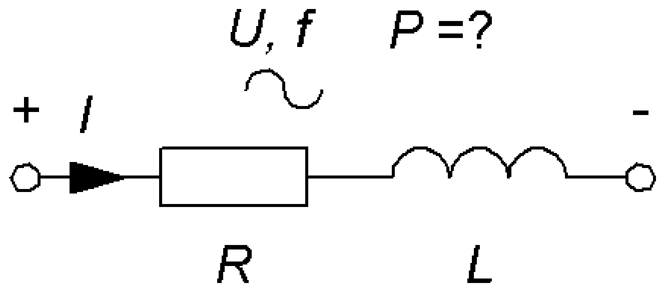
Set up the complex expression for current *I* expressed with *U R C* ω. Let *U* be reference phase, real. Answer with a expression of the form *a*+j*b*.



$$\underline{I} = \underline{I}_{\mathrm{R}} + \underline{I}_{\mathrm{C}} = \frac{U}{R} + \frac{U}{\dfrac{1}{\mathrm{j}\omega C}} = \frac{U}{R} + \mathrm{j}\omega C \cdot U$$

Active power in impedance

Set up an expression of the active power *P* for this impedance. There will only be power in the resistors.

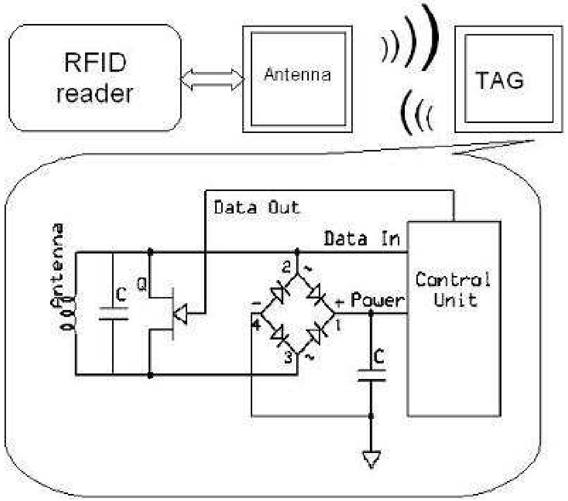


U reference phase, real.

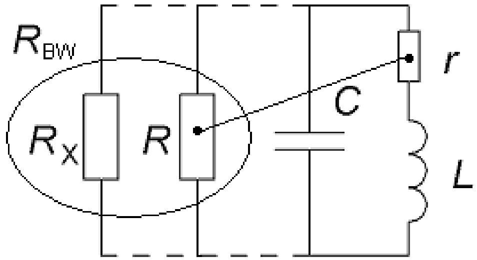
$$P = I^2 \cdot R \qquad \underline{I} = \frac{U}{\underline{Z}} = \frac{U}{R + j\omega L} \quad \Rightarrow \quad I = \frac{U}{\sqrt{R^2 + (\omega L)^2}}$$

$$P = R \cdot \frac{U^2}{R^2 + (\omega L)^2} = \frac{RU^2}{R^2 + (\omega L)^2}$$

SL accesscard (13.7)



$R_X = ?$



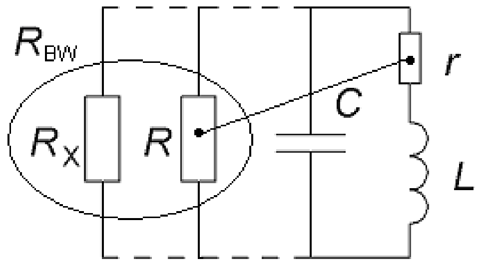
SL access-card contains a RFID-tag that communicates with the turnstyle reader on the frequency 13,56 MHz and uses the data transfer speed of 70 KHz.

To be able to read data in that speed then the resonance circuits inside the reader and the card must have a bandwidth at least twice this data speed: $2 \cdot 70$ = **140 kHz**.

SL accesscard (13.7)

RFID-tag in the card concists of a parallel resonance circuit $C||L||R||R_X$. The processor in the card consumes current from the resonance circuit. This is symbolized with the resistance R_X.

$R_X = ?$



f_0 = 13,56 MHz
BW = 140 kHz
L = 2,5 μH
r = 1,5 Ω
C = 55 pF

a) Calculate the value of R_X gives the card the desired bandwidth *BW*.

b) How big current at the voltage 3V can the processor (R_X) then take from the resonant circuit?

William Sandqvist william@kth.se

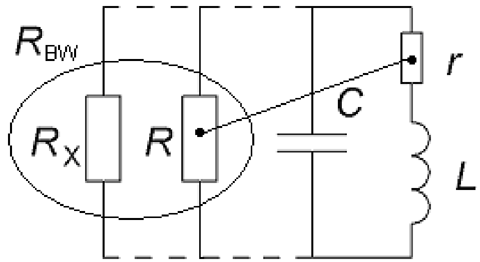
SL accesscard (13.7)

- The inductor Q-value:

$$Q = \frac{\omega L}{r} = \frac{2\pi f_0 \cdot L}{r} =$$

$$= \frac{2\pi \cdot 13{,}56 \cdot 10^6 \cdot 2{,}5 \cdot 10^{-6}}{1{,}5} = 142$$

$$R_X = ?$$



- Transformation of r

$$R = Q^2 \cdot r = 142^2 \cdot 1{,}5 = 30{,}25\ \text{k}\Omega$$

- Parallel resistance to give bandwith 140 kHz

$$Q_{\text{BW}} = \frac{f_0}{\Delta f} = \frac{13{,}56 \cdot 10^6}{140 \cdot 10^3} = 96{,}86 \quad Q_{\text{BW}} = \frac{R_{\text{BW}}}{2\pi \cdot f_0 \cdot L} \quad \Rightarrow$$

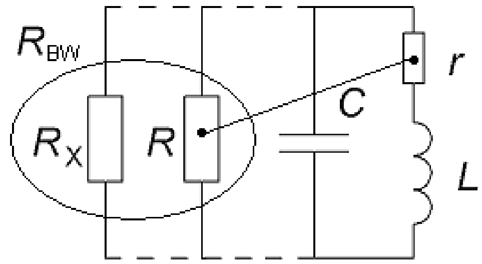
$$R_{\text{BW}} = Q_{\text{BW}} \cdot 2\pi \cdot f_0 \cdot L = 96{,}86 \cdot 2\pi \cdot 13{,}56 \cdot 10^6 \cdot 2{,}5 \cdot 10^{-6} = 20{,}63\ \text{k}\Omega$$

SL accesscard (13.7)

Parallel resistande for bandwidth 140 kHz

$R_X = ?$

$$R_{\mathrm{BW}} = 20{,}63\ \mathrm{k\Omega}$$



a) $R_{\mathrm{BW}} = R_X \parallel R \quad \Rightarrow \quad R_X = \dfrac{R \cdot R_{BW}}{R - R_{BW}}$

$$R_X = \frac{30{,}25 \cdot 20{,}63}{30{,}25 - 20{,}63} \cdot 10^3 = \boxed{64\ \mathrm{k\Omega}}$$

b) $U = I \cdot R_X \quad U = 3 \quad I = \dfrac{U}{R_X} = \dfrac{3}{64 \cdot 10^3} = \boxed{47\ \mu\mathrm{A}}$

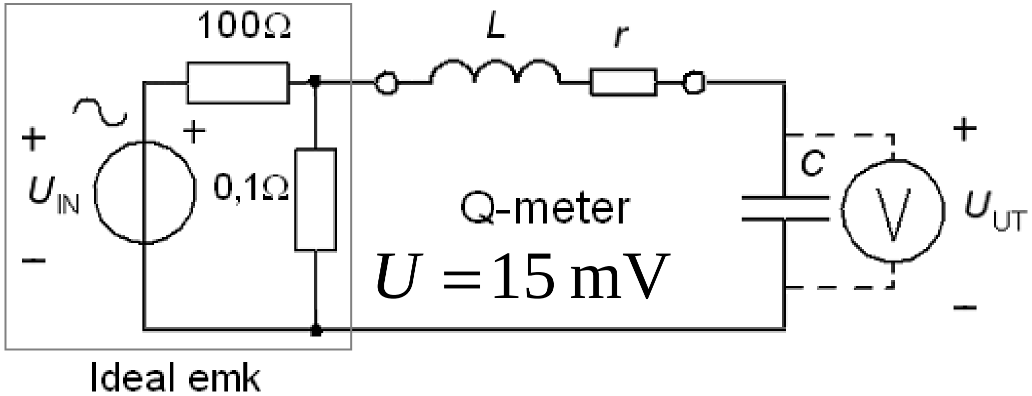
To measure Q-value

Radio controled clock is a clock that is automatically synchronized with a time code from a radio transmitter in Germany, on longvawe 77,5 kHz. The time signal consists of pulses encoded digitally. The signal strength is weak so such a receiver uses a tuned resonant circuit with L and C. The coil has a ferrite core, and this is also used as an antenna.
In a project we have to measure the Q-value this resonance circuit. How will this be done? Other values:

L = 1,5 mH
C = 2,8 nF

To measure Q-value

$U_{IN} = 15\ \mathrm{V}$



$U_{UT} = 1{,}73\ \mathrm{V}$

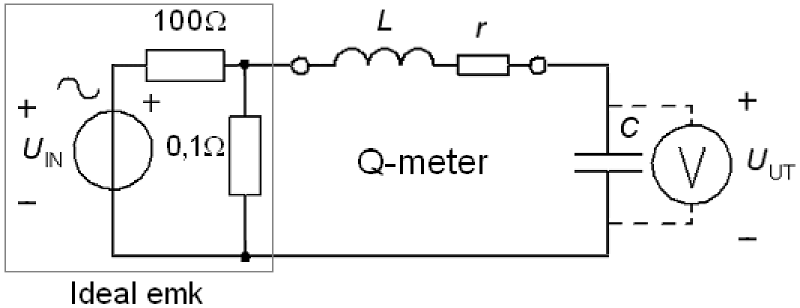
This is how to measure the inductor's Q-value.

U_{IN} = 15V is a sine voltage with the frequency 77,5 kHz (the resonance resonansfrequency) which is voltage divided to 15 mV. Over the capacitor we then measures the much bigger voltageU_{UT} = 1,73 V.

a) What is the inductor's *Q*-value?

b) What is the value of the inductor's internal resitance *r* (will also include other losses)?

To measure Q-value



$$f_0 = \frac{1}{2\pi\sqrt{L \cdot C}} = \frac{1}{2\pi\sqrt{1{,}5 \cdot 10^{-3} \cdot 2{,}8 \cdot 10^{-9}}} = 77{,}5 \cdot 10^3$$

Check of resonance frequency 77,5 KHz

The voltage divider: $U_r = 15\frac{0{,}1}{100} = 0{,}015$ V

a) $$Q = \frac{2\pi f \cdot L}{r} \cdot \frac{I}{I} = \frac{U_L}{U_r} = \{U_L = U_C = U_{UT}\} = \frac{U_{UT}}{U_r} = \frac{1{,}73}{0{,}015} = \boxed{115}$$

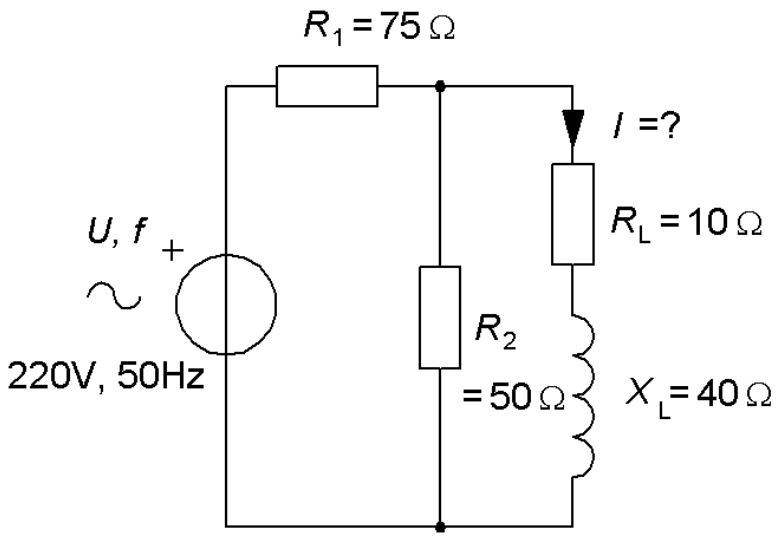
b) $$r = \frac{2\pi f \cdot L}{Q} = \frac{2\pi \cdot 77{,}5 \cdot 10^3 \cdot 1{,}5 \cdot 10^{-3}}{115} = \boxed{6{,}33\ \Omega}$$

Big compared with 0,1Ω from voltage divider.

Thevenin equivalent with inductor (12.4)

Determine the value of the current *I*.

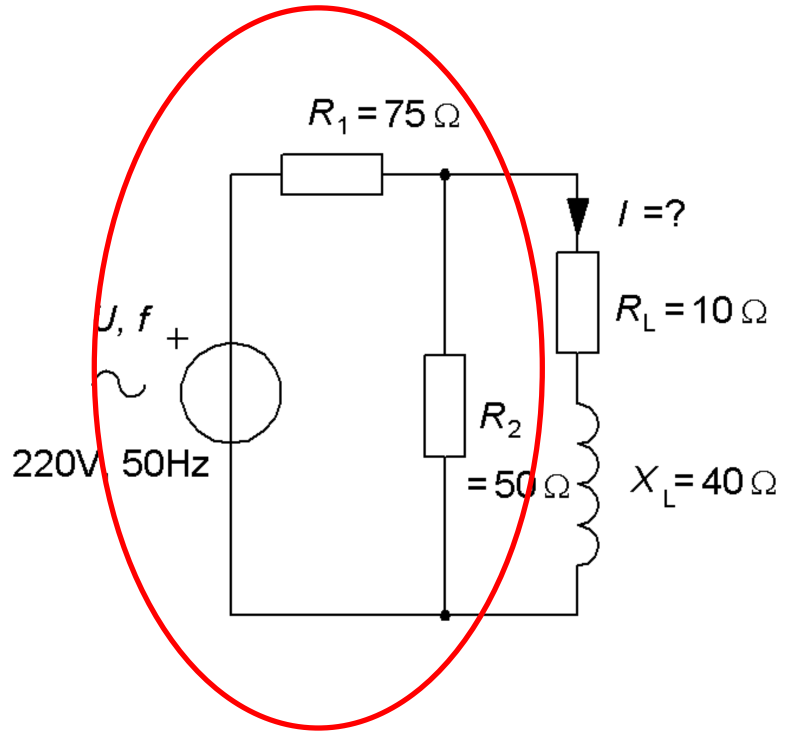
Use Thevenin equivalent.



Thevenin equivalent with inductor (12.4)

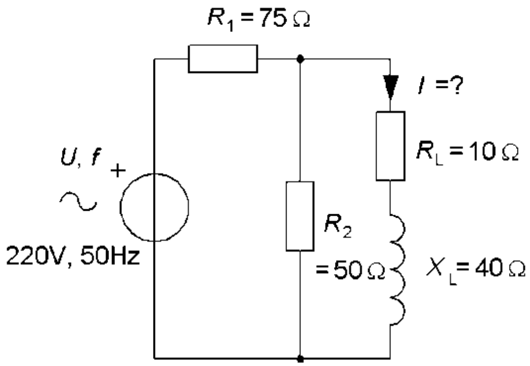
Determine the value of the current *I*.

Use Thevenin equivalent.



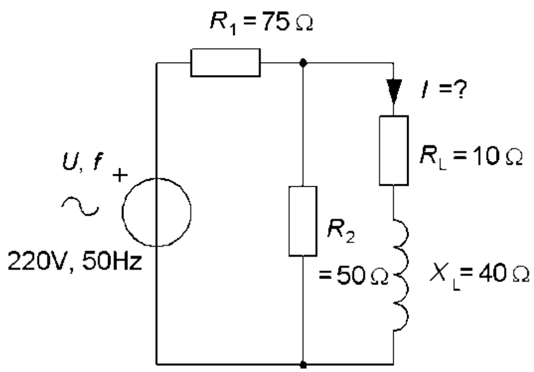
Thevenin equivalent with inductor (12.4)

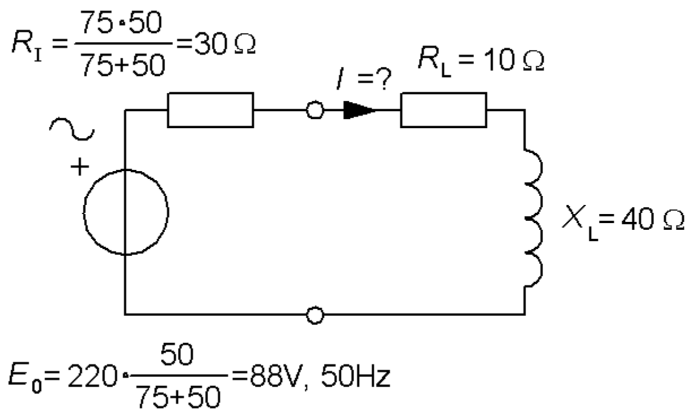
Calculate the Thevenin equivalent E_0 and R_I of this circuit.



Thevenin equivalent with inductor (12.4)

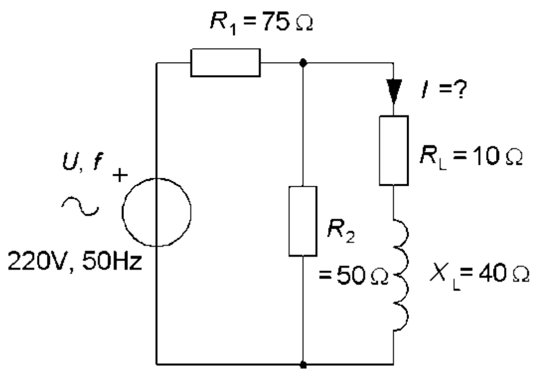
Calculate the Thevenin equivalent E_0 and R_I of this circuit.

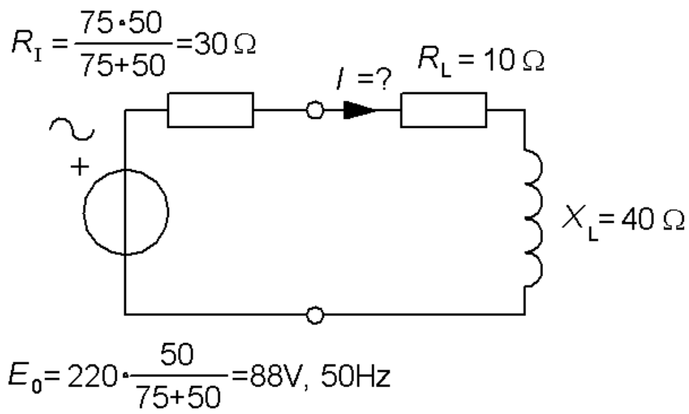




Thevenin equivalent with inductor (12.4)

Calculate the Thevenin equivalent E_0 and R_I of this circuit.





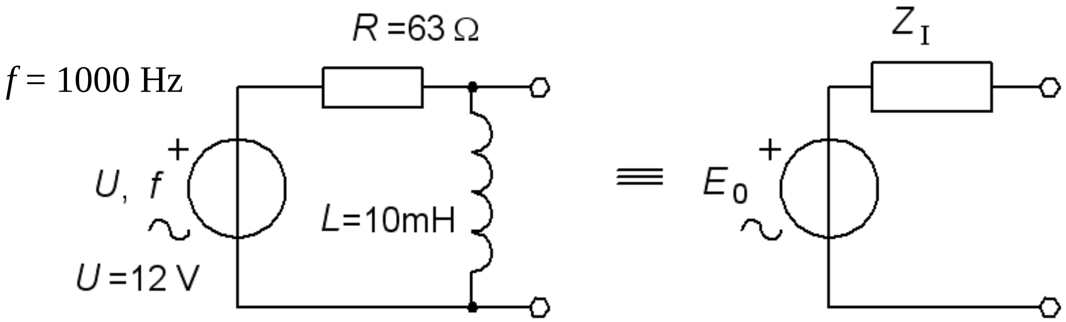
The emf and resistors – this time as with DC circuits …

$$R_I = \frac{75 \cdot 50}{75+50} = 30\,\Omega \quad E_0 = 220\frac{50}{75+50} = 88\text{ V}$$

The inductor – now it must be considered an AC circuits …

$$\underline{I} = \frac{U}{\underline{Z}} \quad \Rightarrow \quad I = \frac{88}{|(30+10)+\text{j}40|} = \frac{88}{\sqrt{(30+10)^2+40^2}} = 1{,}56\text{ A}$$

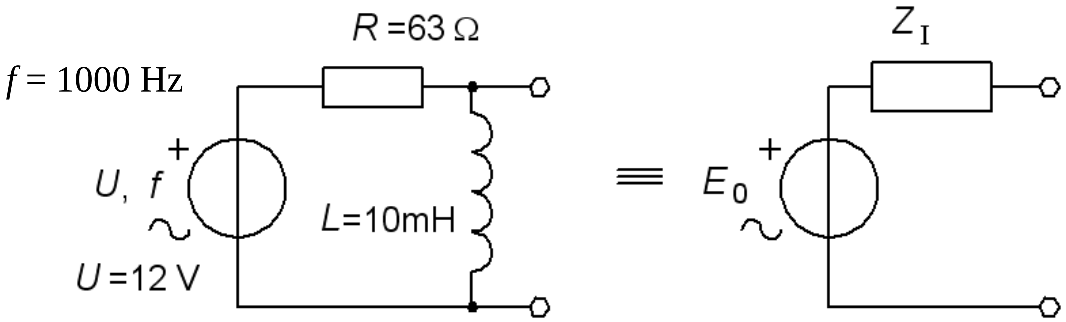
Example. Complex equivalent



a) Derive the equivalent complex circuit with E_0+Z_I.

b) Suppose that we can load the circuit with an arbitrary chosen impedance – how should this be composed if one wishes the power in the load to be the maximum? (Maximum power transfer theorem).

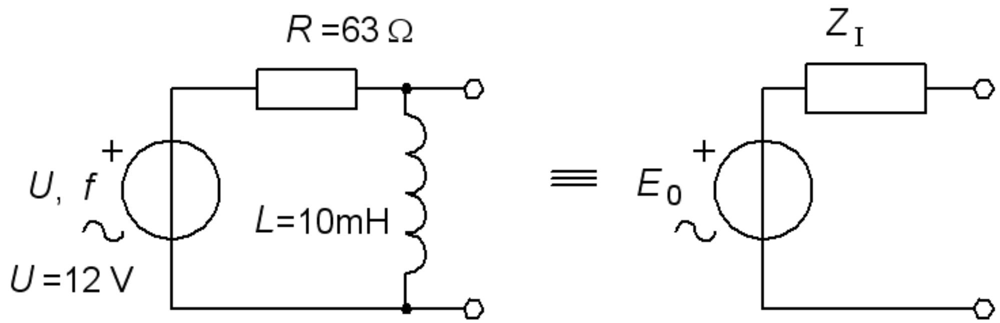
Example. Complex equivalent, E_0



E_0 is calculated as the divided voltage. If U is the reference phase we get E_0 8,47 V and gets the phase 45° to U.
If there are no other voltage sources or current sources in the circuit then we don't have to keep track on the phase, as E_0 might as well become the network's new reference phase!

$$\underline{E}_0 = U\frac{j\omega L}{R + j\omega L} = 12\frac{j2\pi 1000\cdot 0,01}{63 + j2\pi 1000\cdot 0,01} = 6 + 6j \quad E_0 = \sqrt{6^2 + 6^2} = 8,48\ \mathrm{V}$$

Example. Complex equivalent, Z_I

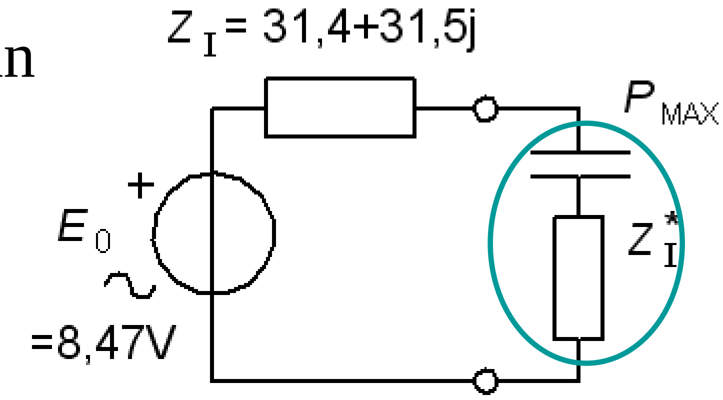


Z_I is the impedance we see if we turn down U.

$$\underline{Z}_I = \frac{R \cdot \mathrm{j}\omega L}{R + \mathrm{j}\omega L} = \frac{63 \cdot \mathrm{j}2\pi 1000 \cdot 0{,}01}{63 + \mathrm{j}2\pi 1000 \cdot 0{,}01} = 31{,}4 + 31{,}5\mathrm{j}$$

Maximum power, *X*

The equivalent circuit is 8,57 V an emf with internal impedance Z_I = 31,4+31,5j.



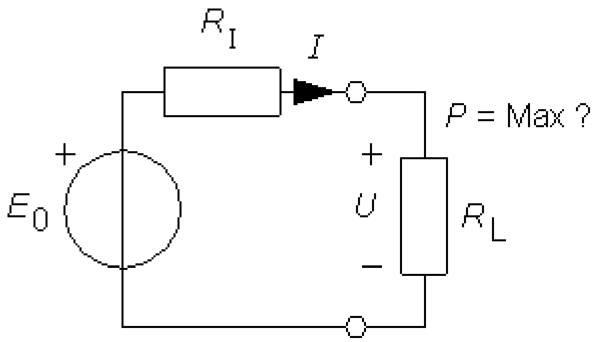
- **Maximum power**.

At resonance inductance and capacitance cancel each other. This will maximize the power in the load. Therefore, the load this time should be capacitive (-31,5j).

When the two reactances cancel each other the circuit becomes completely resistive. What load resistance will give the maximum power?

Maximum power, R_I

$$P = R_L \cdot I^2 \quad I = \frac{E_0}{R_I + R_L} \quad \Rightarrow \quad P = E_0^2 \cdot \frac{R_L}{(R_I + R_L)^2}$$



When do $P(R_L)$ have a maximum? (You get simpler calculations if you turn to the question to "where is 1/P minimum").

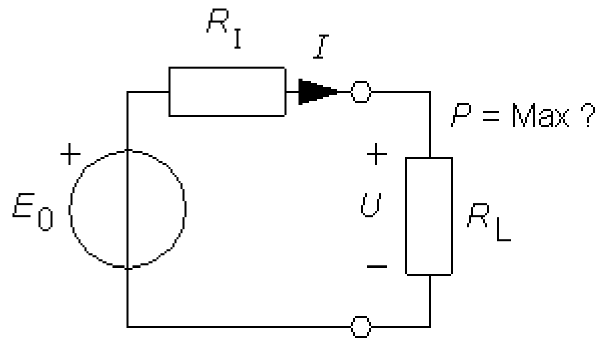
$$\frac{1}{P} = \frac{1}{E_0^2} \cdot \left(\frac{R_L^2}{R_L} + \frac{R_I^2}{R_L} + 2 \cdot \frac{R_I \cdot R_L}{R_L}\right) = \frac{1}{E_0^2} \cdot \left(R_L + 2 \cdot R_I + \frac{R_I^2}{R_L}\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}R_L}\left(\frac{1}{P}\right) = \frac{\mathrm{d}}{\mathrm{d}R_L}\left(\frac{1}{E_0^2} \cdot \left(R_L + 2 \cdot R_I + \frac{R_I^2}{R_L}\right)\right) = 1 - \frac{R_I^2}{R_L^2} = 0 \quad \Rightarrow \quad R_L = R_I$$

Maximum transfered power if you chose $R_L = R_I$.
(R_L = 31,4 Ω).

The maximum power

How big is the power for $R_L = R_I$ (Maximum power)?



$$P = E_0^2 \cdot \frac{R_L}{(R_I + R_L)^2} \quad R_I = R_L \quad \Rightarrow \quad P_{MAX} = \frac{E_0^2}{4 \cdot R_I}$$

How big are the losses inside the equivalent circuit?

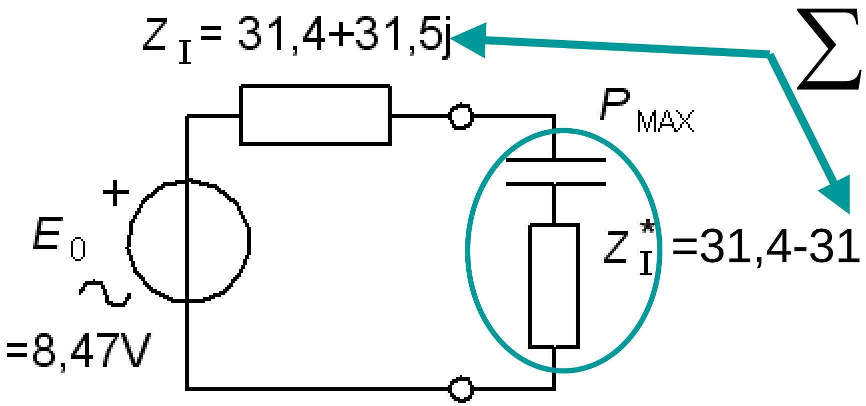
If $R_L = R_I$ the power is divided equal between the internal resistance and the load. This means that the Thermal efficiency will be 50% (= bad).

Maximum power transfer, impedance matching, is only used when neccessary, such as for radio transmitters.

Maximum power transfer

$$P_{\max} \Rightarrow$$

$$\underline{Z} = \underline{Z}_{\mathrm{I}}^{*}$$



$$\sum X = 0$$

At power match with a load equal to the complex conjugate of the internal impedance, the effect:

$$P_{\max} = \frac{\left|\underline{E}_0\right|^2}{4 \cdot \mathrm{Re}[\underline{Z}_{\mathrm{I}}]}$$

William Sandqvist william@kth.se