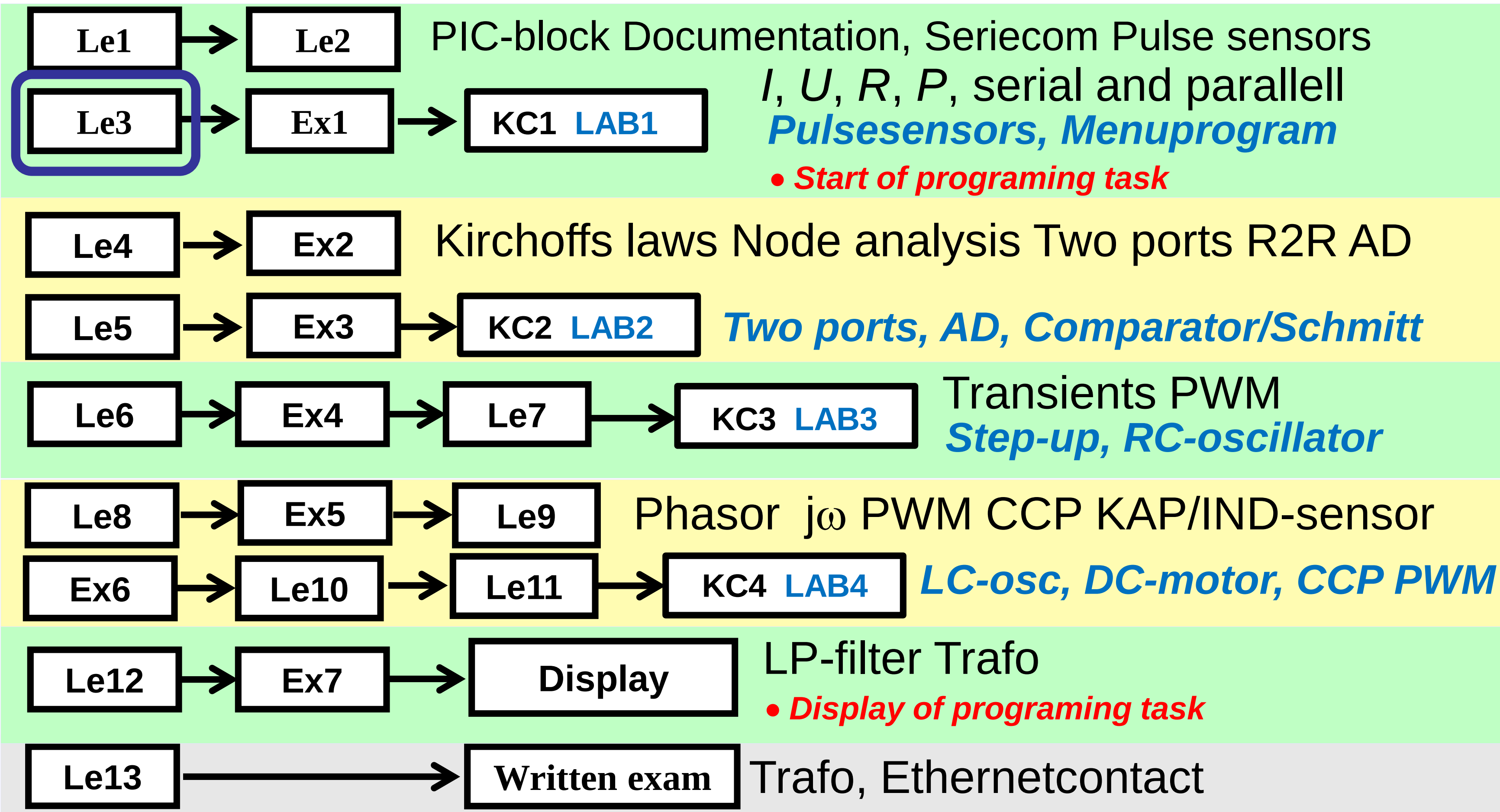
# IE1206 Embedded Electronics

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# Scircuit analysis
## resistors
## connected in series and parallel

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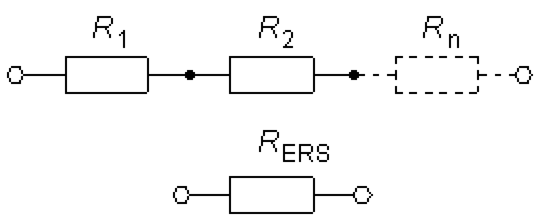
# Series-connected resistors – equivalent resistance

**Series connected resistors – equivalent resistance**
Series connected resistors $R_1$ $R_2$ ... $R_n$ can in calculating be replaced by a **equivalent resistance** $R_{ERS}$ which is the sum of the resistors.
The sum is obviously larger than the largest of the resistors.

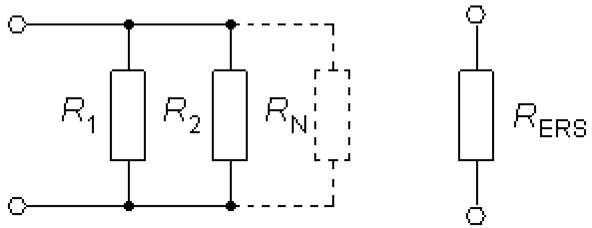



$$R_{ERS} = R_1 + R_2 + ... + R_N$$

Series-connected components, are characterized in that they are interconnected in **one** point.

# Parallel connected resistors – equivalent resistans

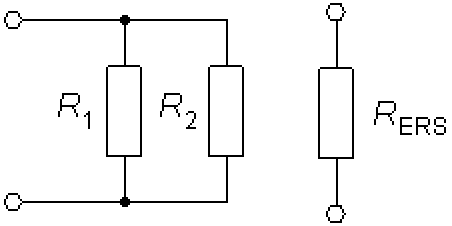
**Parallel connected resistors – equivalent resistance**
Parallel connected components have *both* connections in common with each other.
Parallel resistors $R_1$ $R_2$ ... $R_n$ can in calculating be replaced by a **equivalent resistance** $R_{ERS}$.




$$\frac{1}{R_{ERS}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N}$$

Parallel connected components, are characterized in that they have **both connections** in common with each other.

# Two Parallel connected resistors



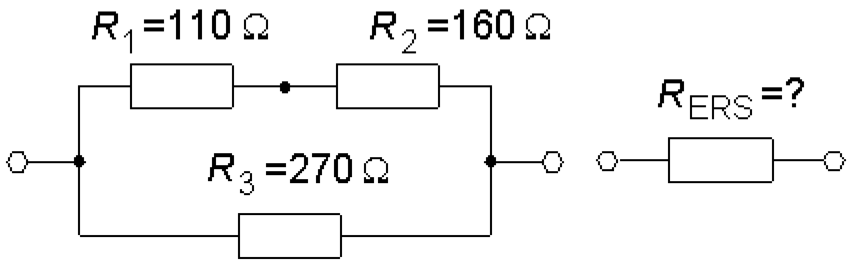

If one particularly has two parallel resistors $R_1$ and $R_2$ the formula can be reformulated as :

$$\frac{1}{R_{ERS}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2}{R_2} \cdot \frac{1}{R_1} + \frac{R_1}{R_1} \cdot \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 \cdot R_2}$$

$$\boxed{R_{ERS} = \frac{R_1 \cdot R_2}{R_1 + R_2}}$$

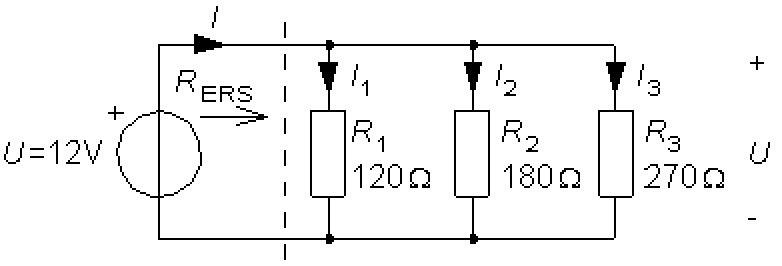
Do you have more parallel resistors than two, repeat this formula for two resistors at a time until you get the equivalent resistance for all.
In parallel connection the equivalent resistance always becomes smaller than the smallest of the constituent parallel connected resistors.

# Examle – series and parallel connection




$$R_{ERS} = \frac{R_3 \cdot (R_1 + R_2)}{R_1 + R_2 + R_3} = \frac{270 \cdot (110 + 160)}{110 + 160 + 270} = 135\,\Omega$$

# Parallel circuit

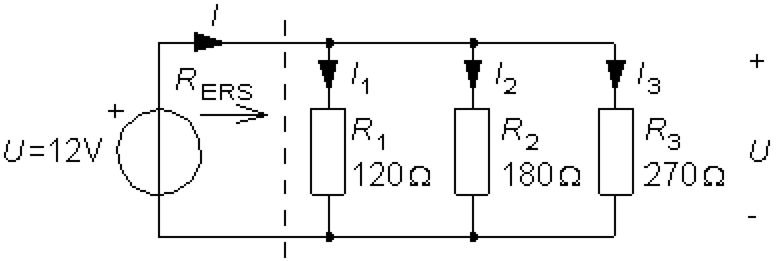



Same *U* over all resistors!

$$I_1 = \frac{U}{R_1} = \frac{12}{120} = 0{,}1 \qquad I_2 = \frac{U}{R_2} = \frac{12}{180} = 0{,}067 \qquad I_3 = \frac{U}{R_3} = \frac{12}{270} = 0{,}044$$

$$I = I_1 + I_2 + I_3 = 0{,}1 + 0{,}067 + 0{,}044 = 0{,}21 \text{ A}$$

# Equivalent resistance




From the emf *U* one only sees the current *I*, it could likely go to a lonely resistor, an equivalent resistance $R_{\text{ERS}}$. Ohms law gives:

$$R_{ERS} = \frac{U}{I} = \frac{U}{I_1 + I_2 + I_3} = \frac{U}{\frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{120} + \frac{1}{180} + \frac{1}{270}} = 56{,}8\,\Omega$$

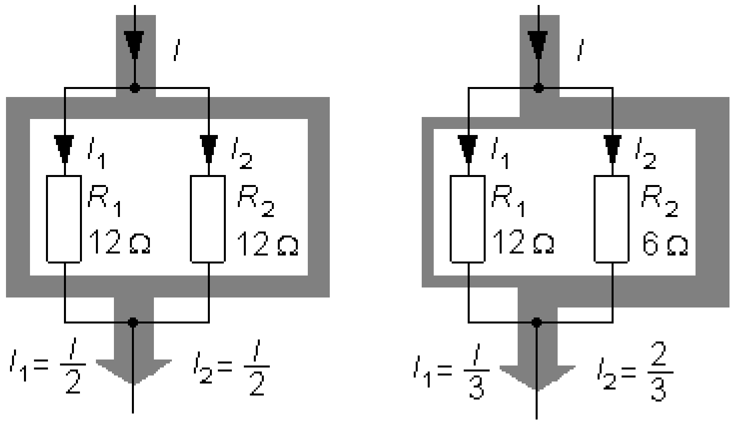
$$I = \frac{U}{R_{ERS}} = \frac{12}{56{,}8} = 0{,}21\,\text{A}$$

$$\frac{1}{R_{ERS}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The calculated equivalent resistor $R_{\text{ERS}}$ = 56,8 Ω gives the same total current *I* = 0,21 A as calculated earlier.

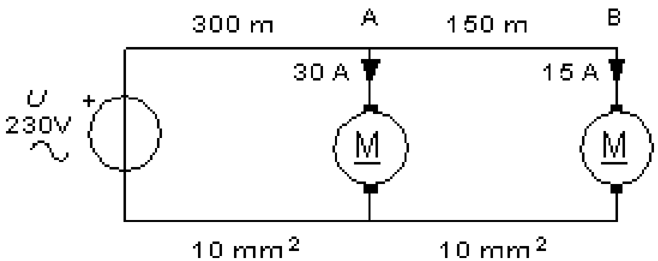
*This is how to derive the expression for the equivalent resistance.*

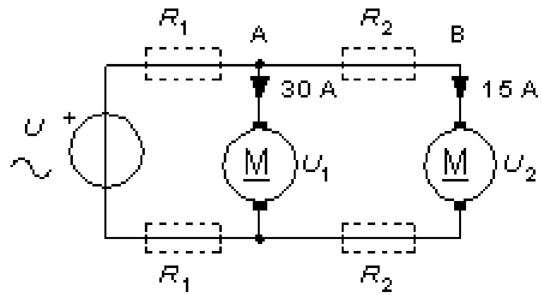
# Current branching




The current is divided between parallel branches inversely with the branch resistance (follows the least resistance).

# Example – *not* a parallel circuit

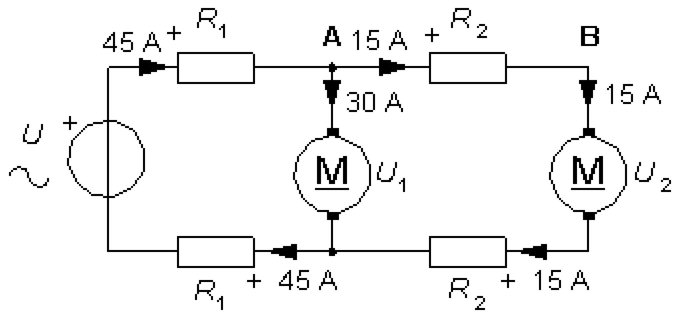



Two electric pumps A and B are placed 150 m from each other. A and then B, are powered by 230V from a socket 300 m away. A pump draws the current 30 A and B 15 A. See figure.

On paper it looks as if the motors are connected in parallel, but then you have not counted the resistance found in long lines. To the right of the figure, it is complemented with resistance symbols for the wiring resistances.

# Example – *not* a parallel circuit

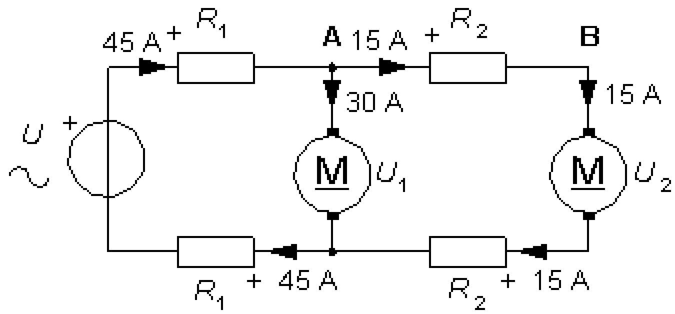



When the engines work, and thus consumes current, there will be a voltage drop in the cables : $U > U_1 > U_2$

How big will the voltages $U_1$ and $U_2$ be when *both* pumps are working?

The wires are of copper with the resistivity 0,018 [$\Omega$mm$^2$/m]. $R = \rho \cdot l / A$
$R_1 = 0{,}018 \times 300/10 = 0{,}54\ \Omega$
$R_2 = 0{,}018 \times 150/10 = 0{,}27\ \Omega$

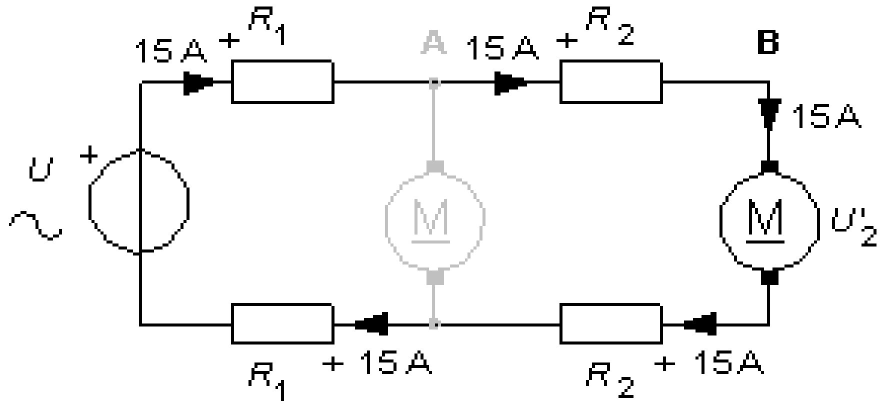
# Example – *not* a parallel circuit




$U_1 = U - 2 \times R_1 \times 45 = 230 - 2 \times 0{,}54 \times 45 = 181{,}4 \text{ V}$

$U_2 = U_1 - 2 \times R_2 \times 15 = 181{,}4 - 2 \times 0{,}27 \times 15 = 173{,}3 \text{ V}$

# If pump A is off?



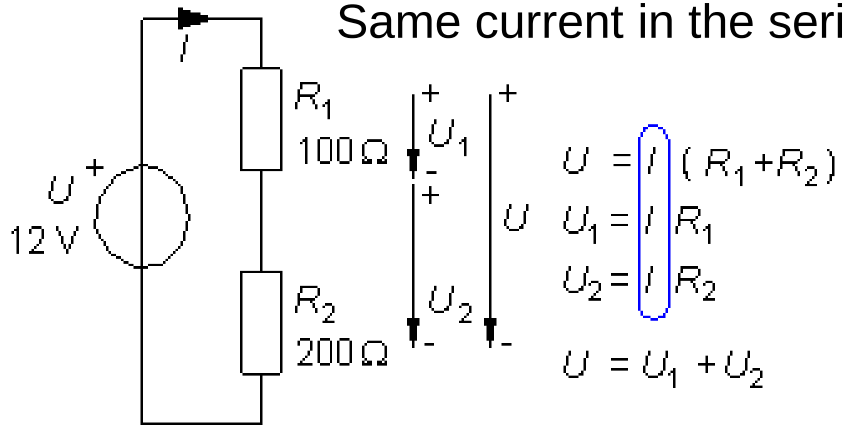

How big will the voltage be at pump B, $U'_2$, when pump A is off?

$$U'_2 = U - 2 \times 15 \times ( R_1 + R_2 ) = 230 - 2 \times 15 \times ( 0{,}54 + 0{,}27 ) = 205{,}7 \text{ V}$$

$U'_2 = 205{,}7$ V  ($U_2 = 173{,}3$ V)  – this change will be noticed!

# Series circuit

Same current in the series circuit

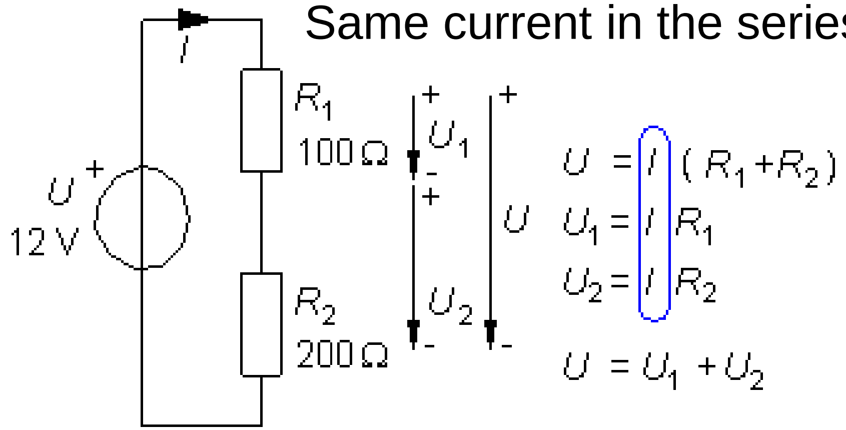



Same *I* through all resistors.

Series circuit is being characterized that it's the same current that goes through all the resistors. One example is the Christmas tree lights. If a bulb is broken so it is of course no current through it, and because it is a series circuit in this case same current in all resistors will mean that no other lamp will light!

# Series circuit

Same current in the series circuit




How big are voltages $U_1$ and $U_2$?

$R_{ERS} = R_1 + R_2 = 100 + 200 = 300$

$I = U/R_{ERS} = 12/300 = 0{,}04$ A

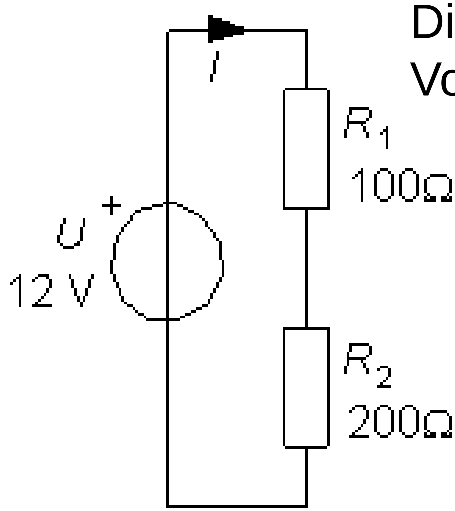
$U_1 = I \times R_1 = 0{,}04 \times 100 = 4$ V

$U_2 = I \times R_2 = 0{,}04 \times 200 = 8$ V

$U = U_1 + U_2 = 4 + 8 = 12$ V

# Voltage division formula

Since all resistors have the same current for series connection, the voltage falls proportional to their resistances. Using Ohm's law (twice), one can develop a formula, the voltage division formula, which can be used to quickly find out the voltage drop across a resistor in series with other resistors.




Divided Voltage | Total Voltage | Voltage division factor

$$U_1 = U \times \frac{R_1}{R_1 + R_2} = 12 \frac{100}{100+200} = 4\text{V}$$

$$U_2 = U \times \frac{R_2}{R_1 + R_2} = 12 \frac{200}{100+200} = 8\text{V}$$

According to the voltage divider formula you get a divided voltage, for example $U_1$ across the resistor $R_{1,}$ by multiplying the total voltage $U$ with a voltage division factor. This voltage division factor is the resistance $R_1$ divided by the sum of all the resistors that are in the series connection.

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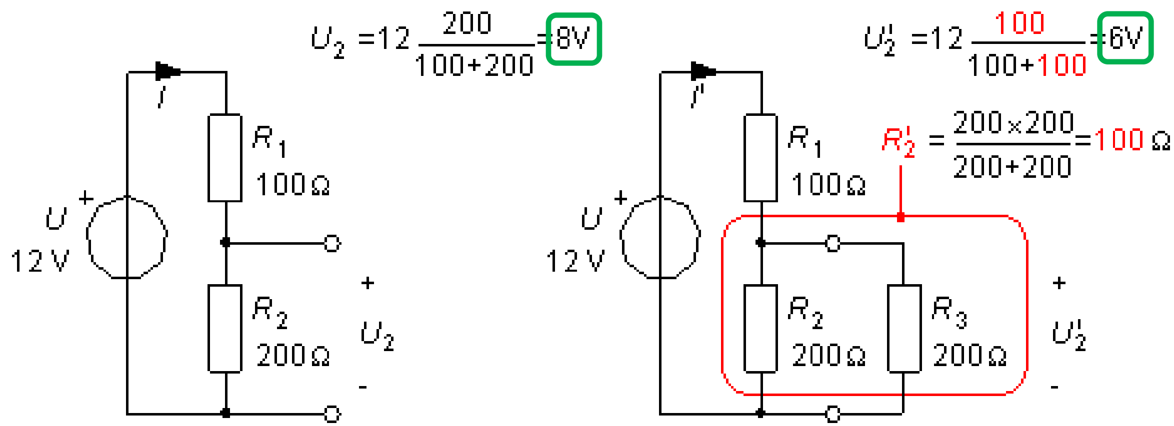
# *Voltage divider with a load*

In cars the battery voltage is 12 V. Suppose you need voltage 8 V to an electronic equipment in a car. One can then lower the voltage with a voltage divider.




In the figure above to right, the resistor $R_3$ = 200 Ω symbolizes the electronic equipment.

To use the voltage division formula one now has to see $R_2$ and $R_3$ as parallell connected. It is this equivalent resitance $R'_2$ that is in series with $R_1$.

The divided voltage $U'_2$ for a voltage divider with load is now calculated to 6 V, 2 V lower than for the voltage divider without load.

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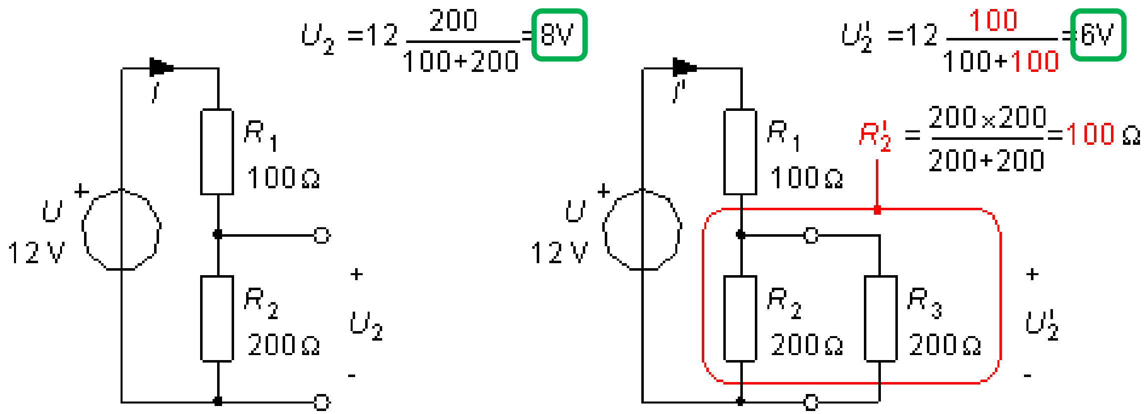
# *Voltage divider with a load*



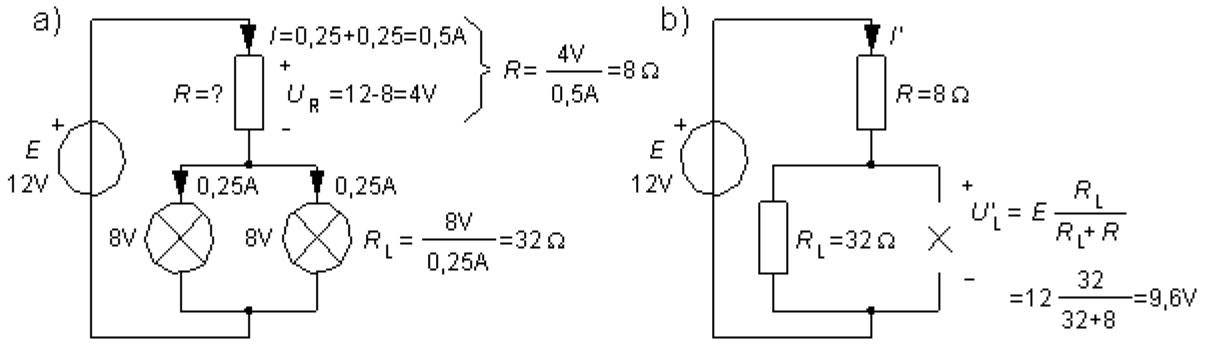

For a voltage divider to maintain it's divided voltage when it is loaded, it is required that the connected load has a much higher resistance than the resistors included in the voltage divider.

( In example $R_3$ = 2000 Ω would give $U_2$ = 7,74 which is closer to the unloaded value 8,0) .

($R_3$ = 20000 Ω would give $U_2$ = 7,97 which is even closer to 8,0).

# Example – voltage divider for lamps

Two 8 V 0,25 A lamps are used in a car that has a 12 V battery. Lamps are parallel connected and then in series via a resistor to the 12 V battery.



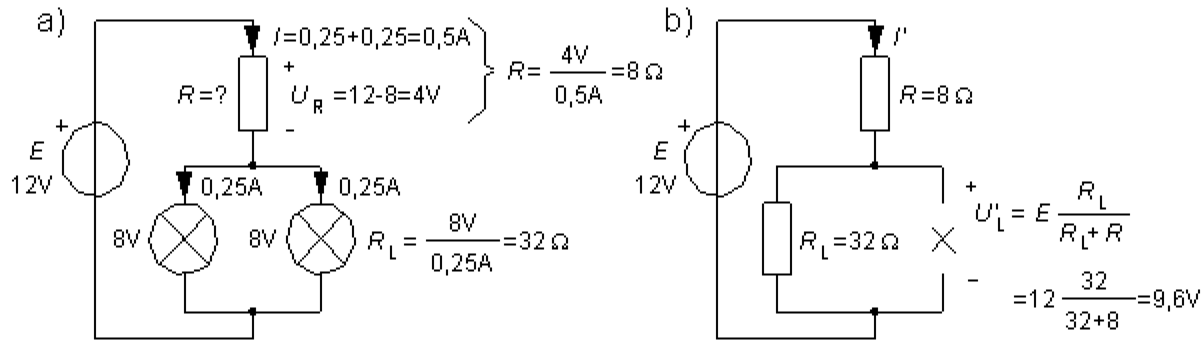

a) Calculate the series resistor *R* so that the lamp voltage will be correct, 8 V.

Current through series resistor will be the sum of the currents to the lamps.
$I = 0,25 + 0,25 = 0,5$ A

Voltage drop over resistor shall be 12 - 8 = 4 V

Ohms law gives: $R = 4/0,5 = 8\ \Omega$

# Example – voltage divider for lamps




b) Suppose that one of the lamps breaks – how big will then the voltage over the other lamp be?

The lamp resistance is calculated from rated data: $R_L = 8/0{,}25 = 32\ \Omega$

The series resistor and the working lamp forms a voltage divider. The lamp voltage is calculated with the voltage divider formula :

$$U'_L = E \times R_L / ( R + R_L ) = 12 \times 32/(32+8) = 9{,}6\ \text{V}$$

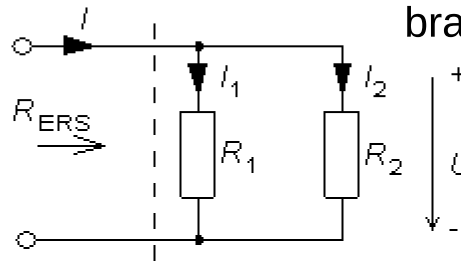
*What do you think will happen to the single lamp?*

# Current branching formula

In the same way as with the voltage division formula, one can derive a current branching formula.

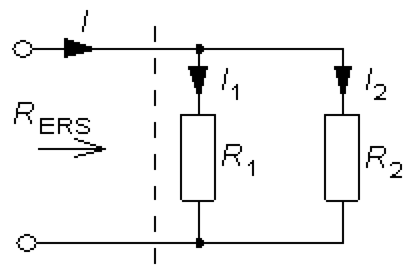
In practice, however, one has less advantage of a current branching formula.

Same voltage over parallel branches




$U = I \times R_{ERS}$

Branch current | Total current | Current Branching factor




$$I_1 = I \times \frac{R_{ERS}}{R_1} = I \times \frac{R_2}{R_1 + R_2}$$

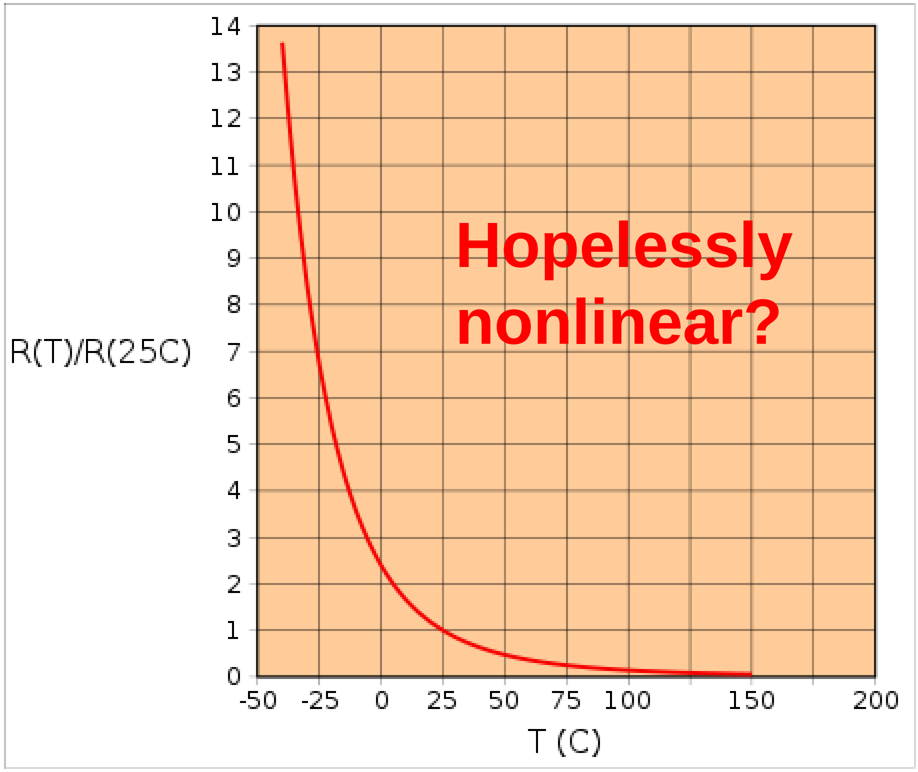
$$I_2 = I \times \frac{R_{ERS}}{R_2} = I \times \frac{R_1}{R_1 + R_2}$$

$$I_1 = \frac{\overbrace{I \cdot \frac{R_1 \cdot R_2}{R_1 + R_2}}^{U}}{R_1} = I \cdot \frac{R_2}{R_1 + R_2}$$

# An application of the voltage divider formula

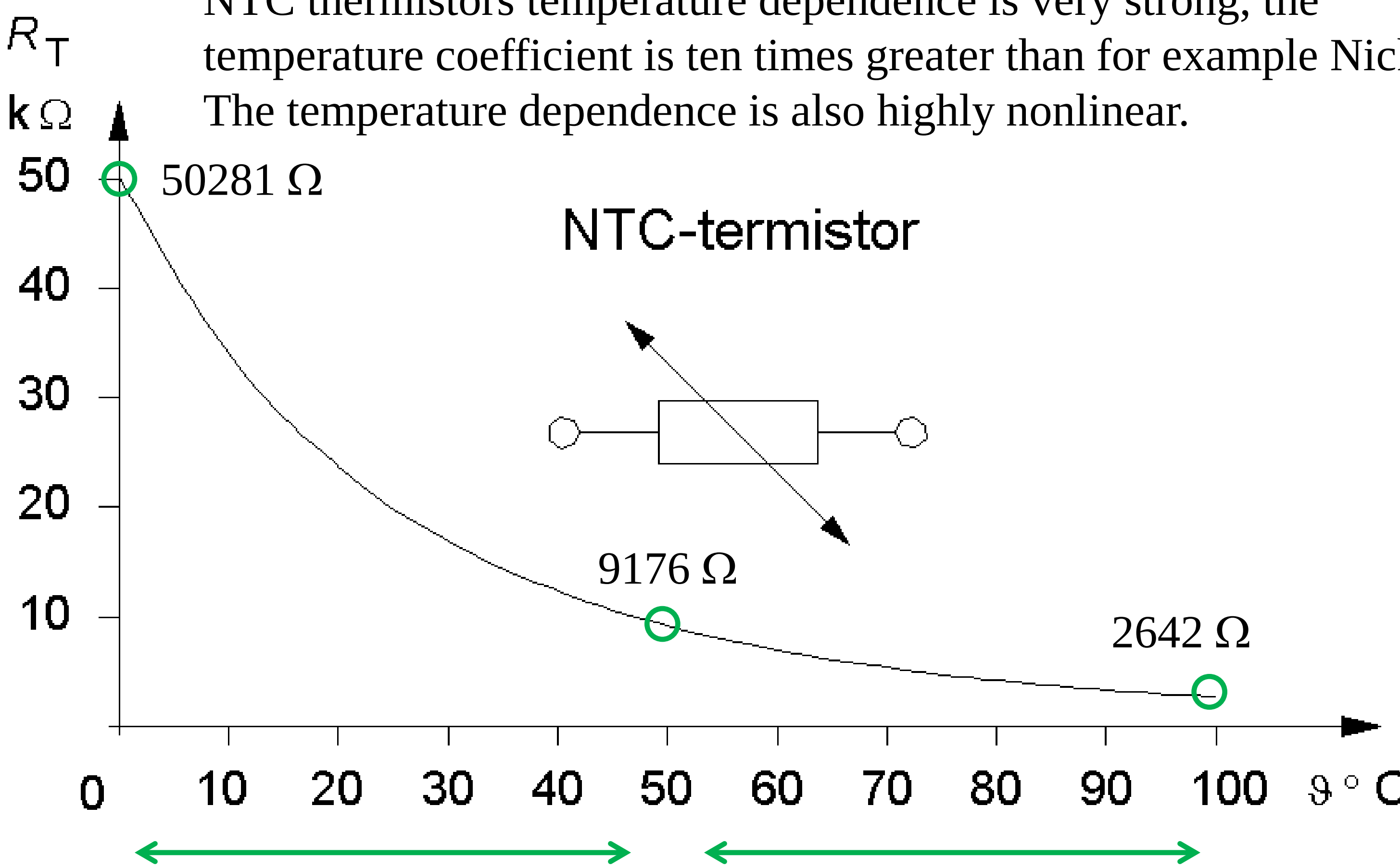
**NTC-termistor**

**Now you know the voltage division formula - then it's time to show the Linearisation method …**

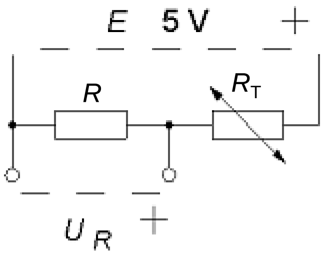


# Linearization of NTC thermistor

NTC thermistors temperature dependence is very strong, the temperature coefficient is ten times greater than for example Nickel. The temperature dependence is also highly nonlinear.



# Linearization




If a fixed resistor $R$ is connected in series with the NTC thermistor $R_T$ this combination will have a lesser nonlinearity than the thermistor alone. We will let the two resistors form a voltage divider.

If the temperature increases, the thermistor's resistance decrease, and then the portion of the voltage drops across the fixed resistor $U_R$ increases and therefore gives a proportional measure of temperature.

Voltage division formula: $$U_R = E \frac{R}{R_T + R}$$

$U_R(R_T)$ is a monotonic *decreasing* function of temperature, and $R_T(\vartheta)$ is also monotonic *decreasing*. The combined function $R_T(\vartheta)$ therefore has the potential of being somewhat linear, if you give $R$ a suitable value.

# Linearization example

We measure thermistor resistance at three **evenly distributed** temperatures, eg. 0 °C, 50 °C, och 100 °C. $R_{T0} = 50281\ \Omega$, $R_{T50} = 9176\ \Omega$, och $R_{T100} = 2642\ \Omega$.

If there is linearity then the voltages from the voltage divider $U_{R0}$, $U_{R50}$, and $U_{R100}$ also be "evenly distributed". From voltage division we get:

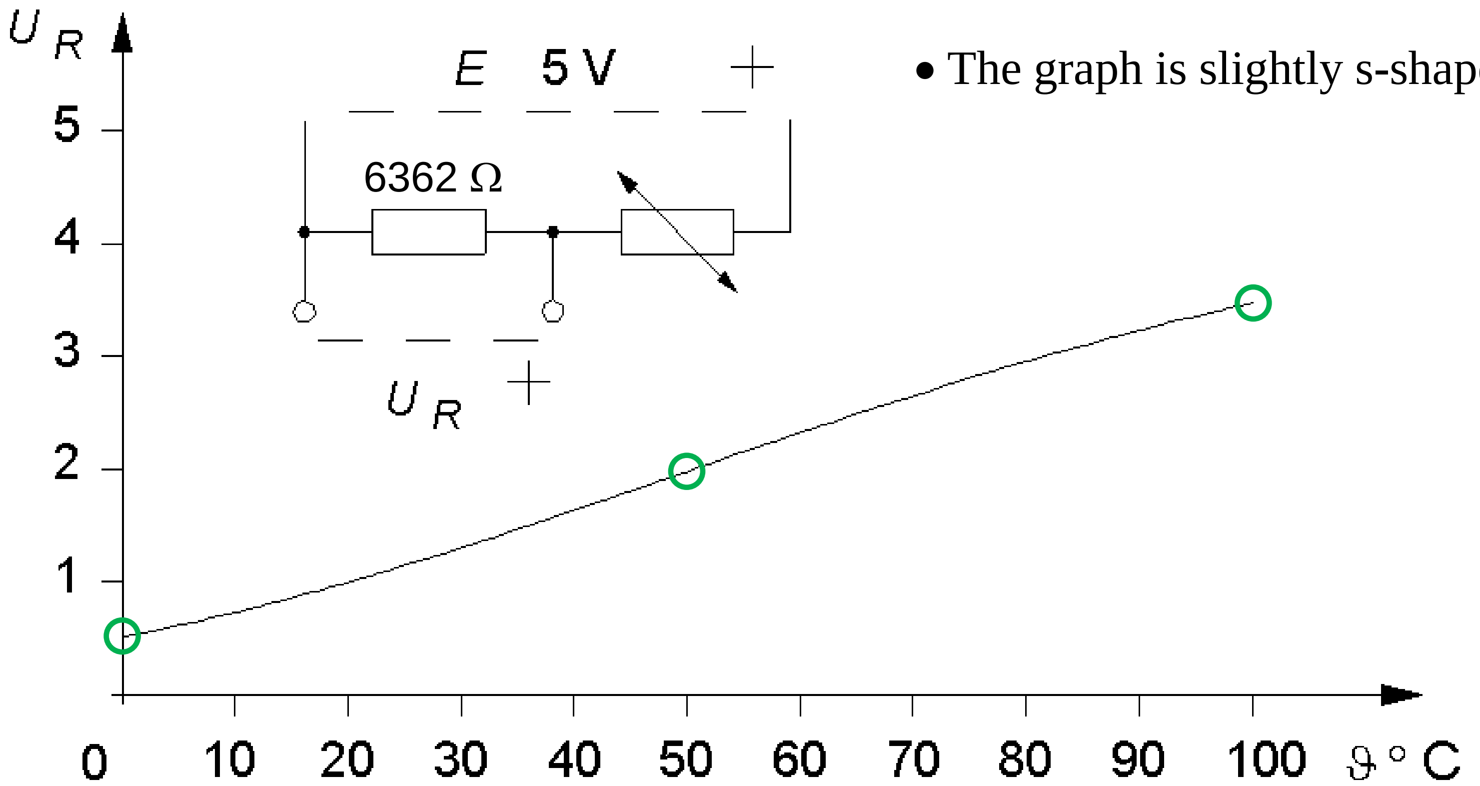
$$\cancel{E}\frac{\cancel{R}}{R_{T50}+R} - \cancel{E}\frac{\cancel{R}}{R_{T0}+R} = \cancel{E}\frac{\cancel{R}}{R_{T100}+R} - \cancel{E}\frac{\cancel{R}}{R_{T50}+R}$$

If $R$ is solved:

$$R = \frac{R_{T0}R_{T50} + R_{T50}R_{T100} - 2R_{T0}R_{T100}}{R_{T0} + R_{T100} - 2R_{T50}}$$

After insertion of our numerical values, we get $R = 6362\ \Omega$.

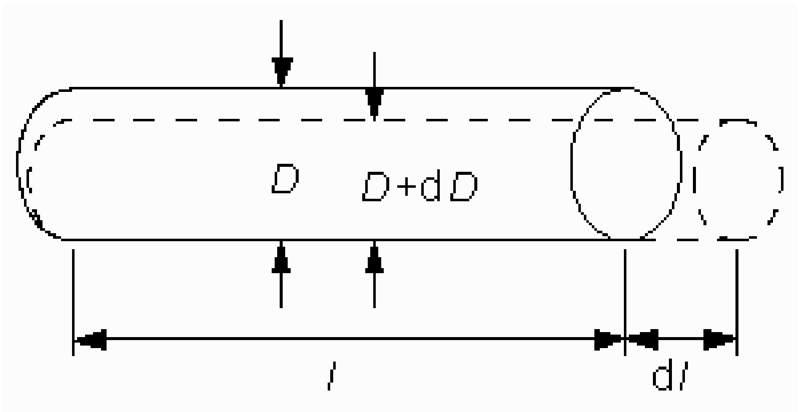
# The result - surprisingly linear!

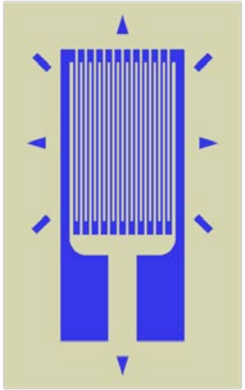


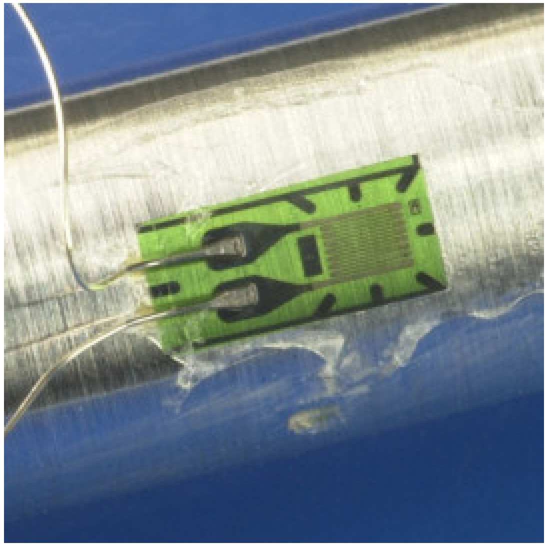

- The graph is slightly s-shaped.

*NTC-thermistors. All possible ( and impossible ) embodiments are available!*

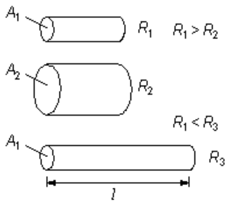
# Example - strain gauge





How do you measure such small resistance changes $\Delta R$?




$$R = \rho \frac{l \cdot 4}{D^2 \cdot \pi} \qquad \Delta R \approx k \cdot dl$$

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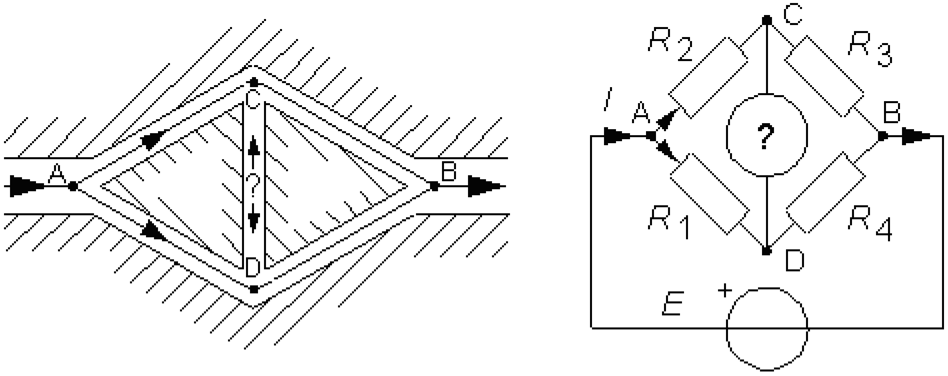
# Wheatstone bridge – branched river




Suppose
$R_4 = R_x$

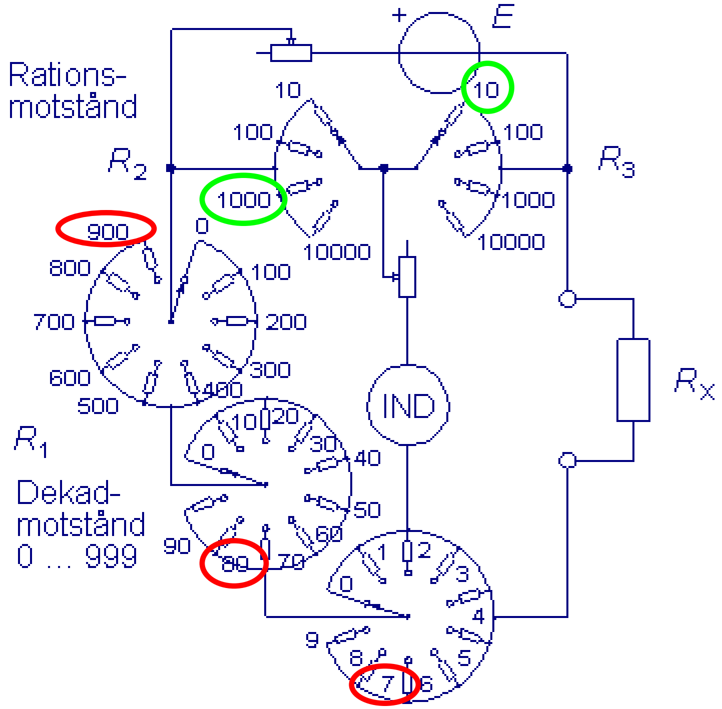
Balance, no current through the indicator $U_{R4} = U_{R3}$ .
Voltage division formula gives us:

$$E\frac{R_4}{R_1+R_4} = E\frac{R_3}{R_2+R_3} \quad \Leftrightarrow \quad R_4R_2 + R_4R_3 = R_3R_1 + R_3R_4$$

At balance: $$R_4 = R_1\frac{R_3}{R_2}$$

# $R_x = ?$

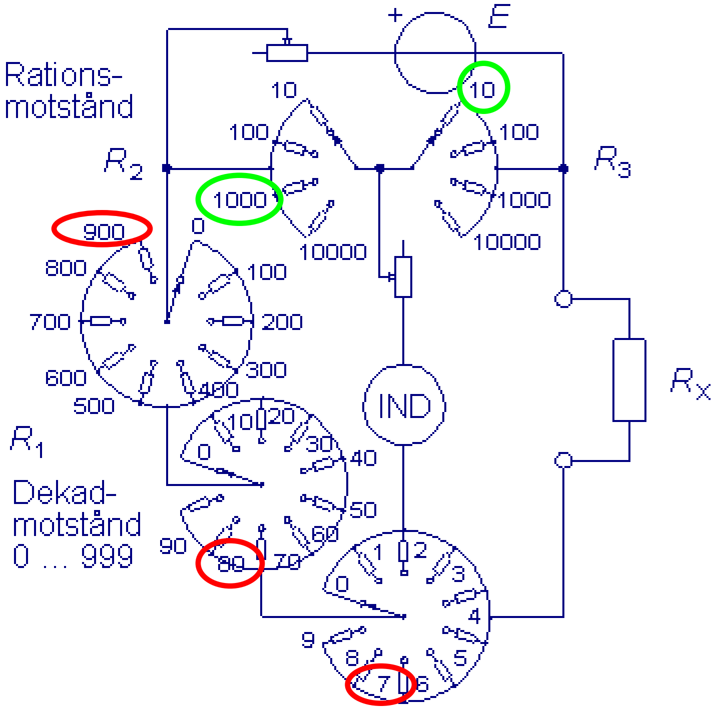
$$R_X = R_1 \frac{R_3}{R_2}$$



# $R_x = ?$

$$R_X = R_1 \frac{R_3}{R_2}$$

$$R_X = 987 \frac{10}{1000} = 9{,}87\ \Omega$$

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# $R_x = ?$

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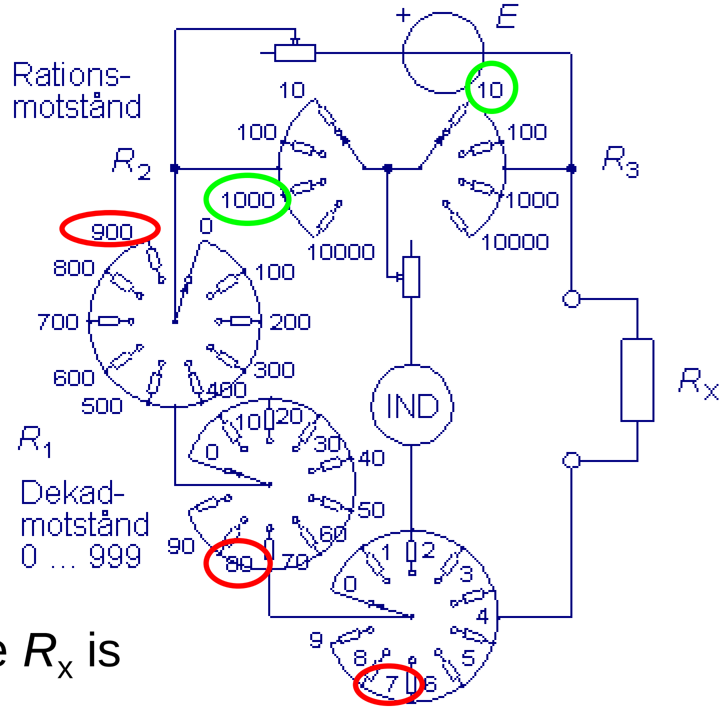


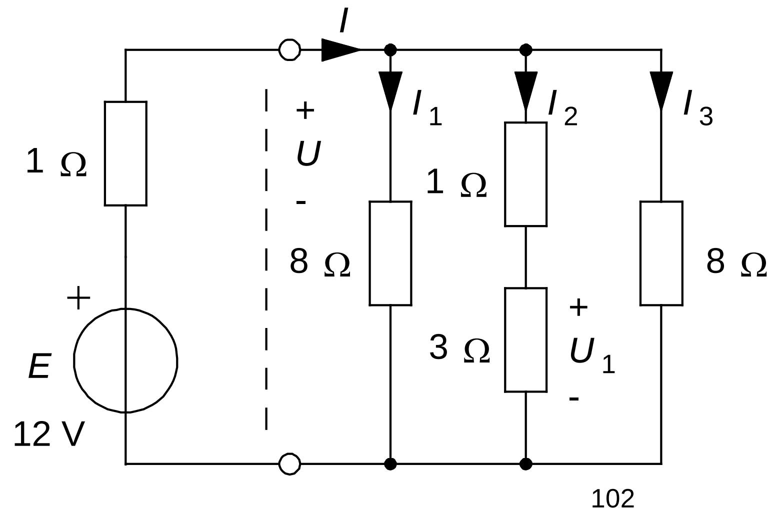

The balance method to determine $R_x$ is simple to use but slow.

It will work for temperature, but not if one would like to follow a resistive sensor that is used for mesuring a faster dynamical process.

# (3.2) OHM's law are often enough!

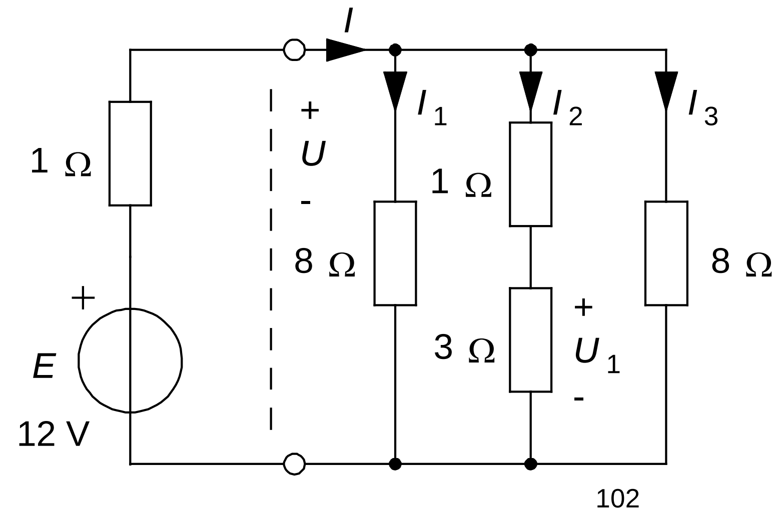
**a**) Calculate the resultant resistance $R_{ERS}$ for the three parallel connected branches.

**b**) Culculate current $I$ and voltage $U$.



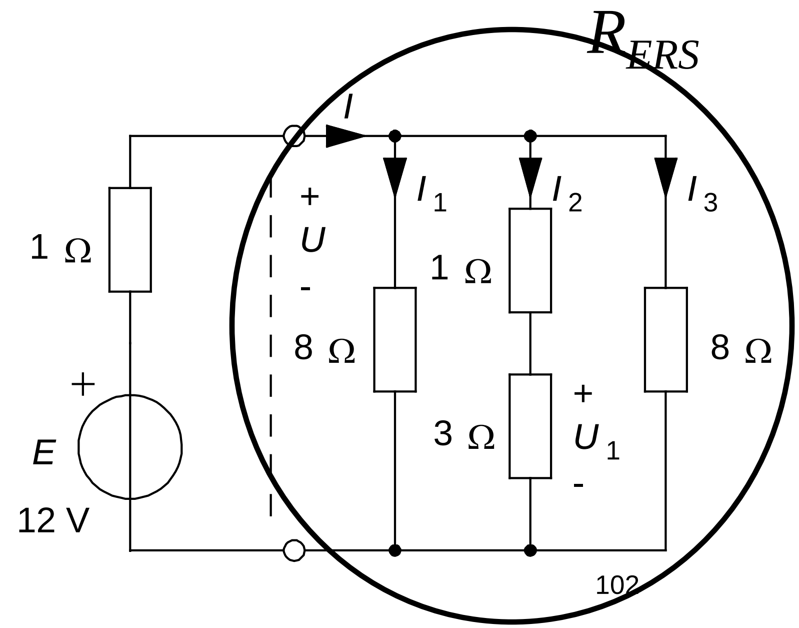

**c**) Calculate the three currents $I_1$ $I_2$ and $I_3$ together with the voltage $U_1$ over 3 Ω-resistor.

# OHM's law …



# OHM’s lag …

$$\frac{1}{R_{ERS}} = \frac{1}{8} + \frac{1}{1+3} + \frac{1}{8} = \frac{4}{8} \quad \Rightarrow \quad R_{ERS} = \frac{8}{4} = 2$$

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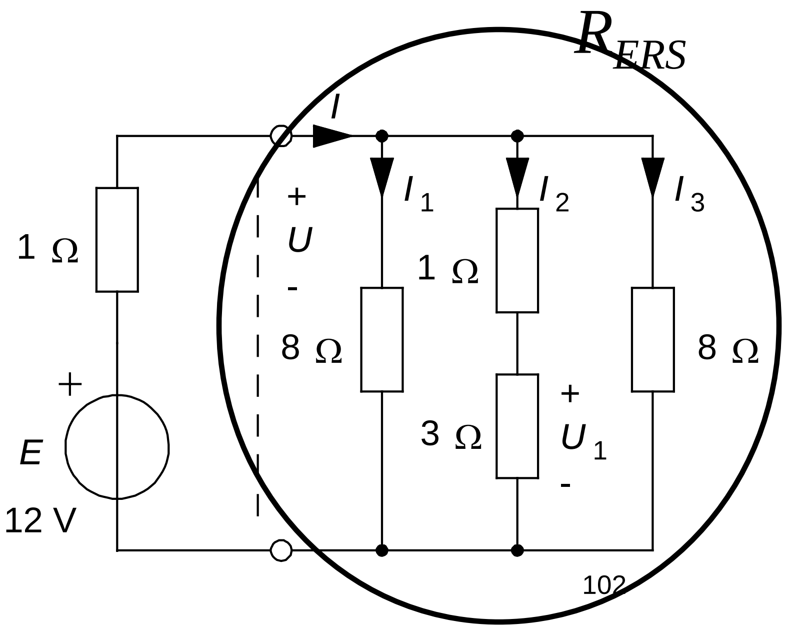
# OHM's lag …

$$\frac{1}{R_{ERS}} = \frac{1}{8} + \frac{1}{1+3} + \frac{1}{8} = \frac{4}{8} \quad \Rightarrow \quad R_{ERS} = \frac{8}{4} = 2$$

$$I = \frac{E}{1+R_{ERS}} = \frac{12}{1+2} = 4$$

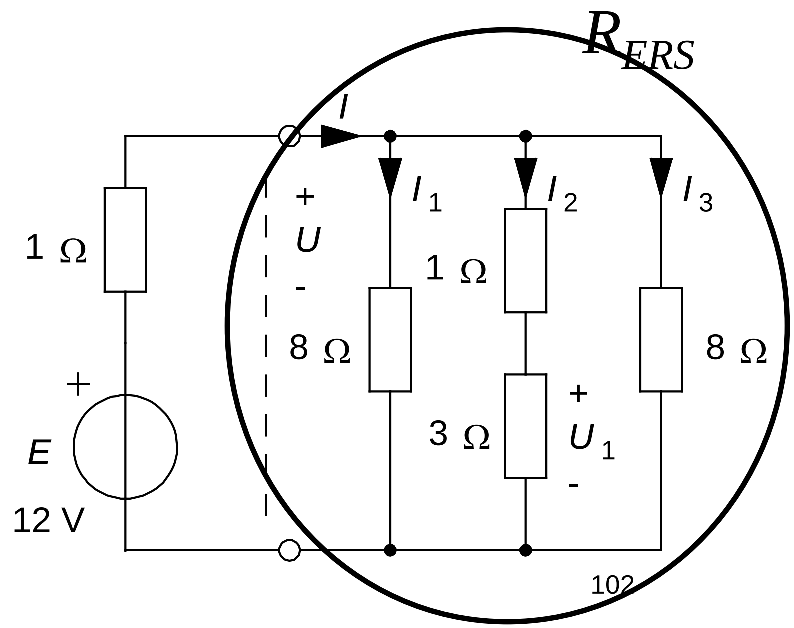


# OHM's lag …

$$\frac{1}{R_{ERS}} = \frac{1}{8} + \frac{1}{1+3} + \frac{1}{8} = \frac{4}{8} \quad \Rightarrow \quad R_{ERS} = \frac{8}{4} = 2$$

$$I = \frac{E}{1+R_{ERS}} = \frac{12}{1+2} = 4$$

$$U = I \cdot R_{ERS} = 4 \cdot 2 = 8$$



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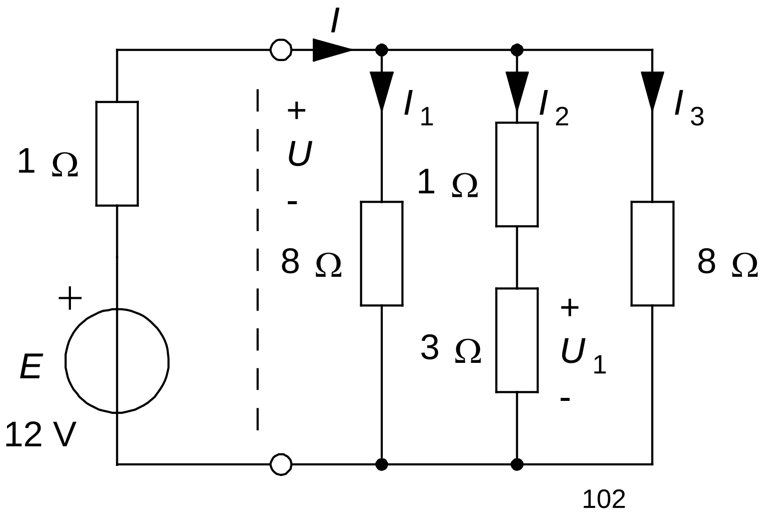


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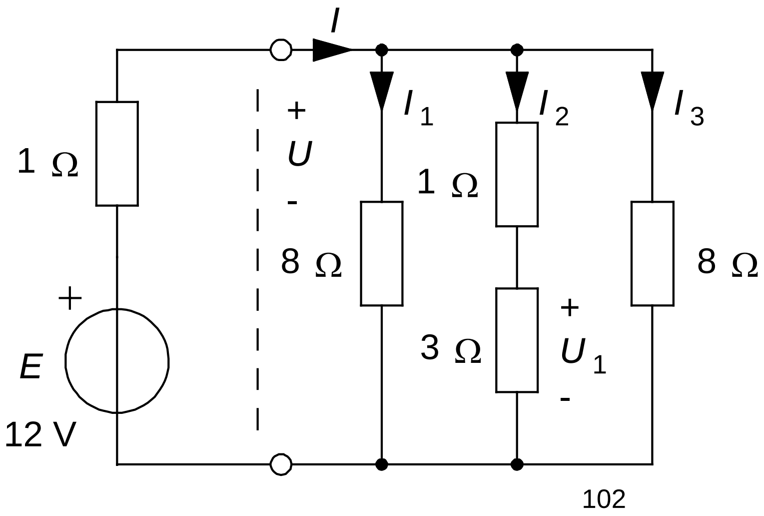


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$$I_1 = \frac{U}{8} = \frac{8}{8} = 1 \qquad I_2 = \frac{U}{1+3} = \frac{8}{1+3} = 2$$

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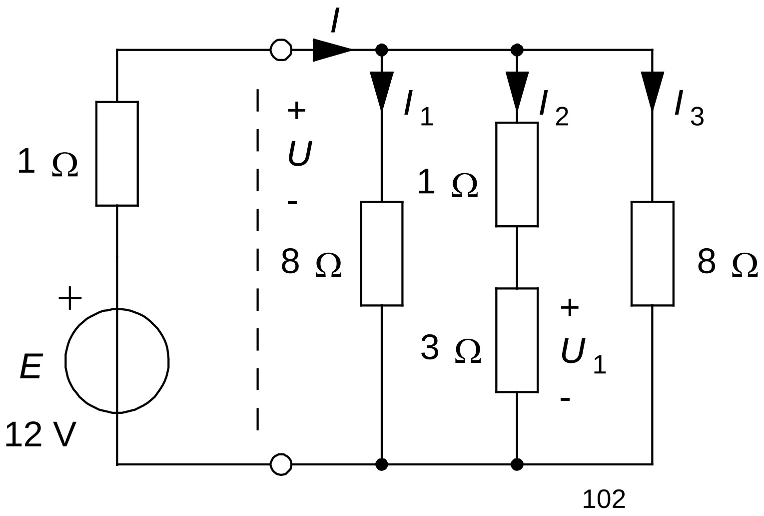


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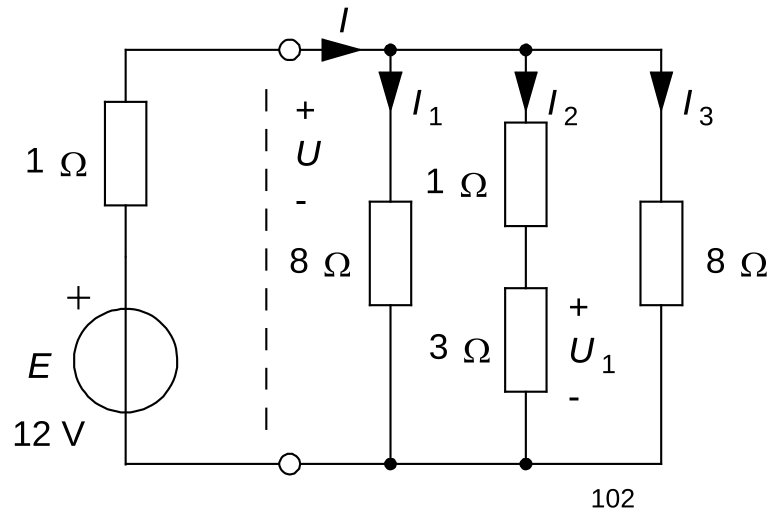


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$$U_1 = I_2 \cdot 3 = 2 \cdot 3 = 6$$

William Sandqvist william@kth.se

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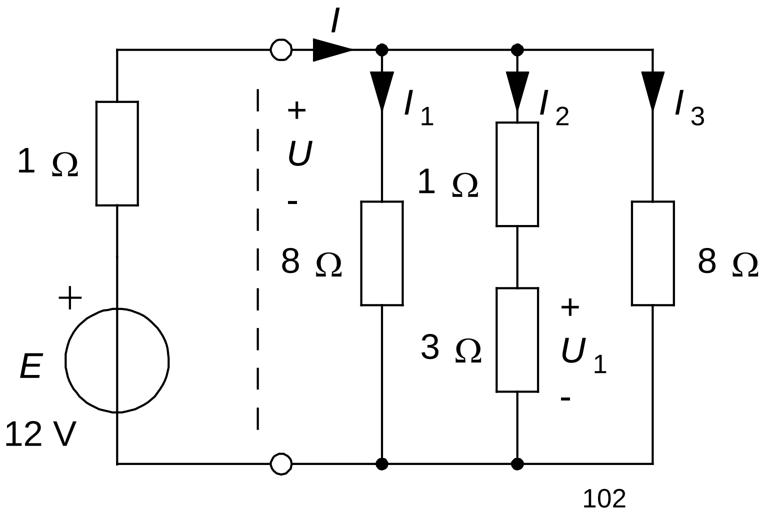


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*OHM's law was enough!*