Advanced Digital Communications (EQ2410)

Lecture 4, Period 3, 2015

Task 1 In groups, verify the construction of the Tanner graph in the example below.

Example 7.3.1 (Hamming code) A generator matrix for the (7, 4) Hamming code is given by

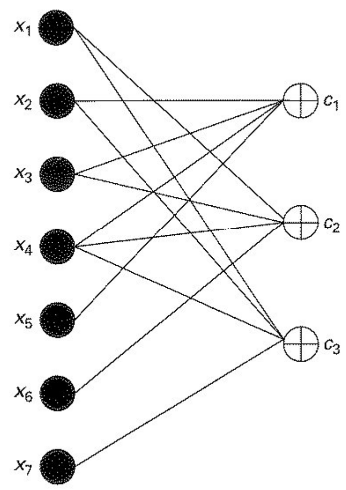
$$\mathbf{G} = \begin{pmatrix} 1\,0\,0\,0\,0\,1\,1 \\ 0\,1\,0\,0\,1\,0\,1 \\ 0\,0\,1\,0\,1\,1\,0 \\ 0\,0\,0\,1\,1\,1\,1 \end{pmatrix}. \quad (7.90)$$

This generator matrix is in *systematic* form, i.e., four out of the seven code bits are the information bits, without any modification. The remaining three bits are the parity check bits, formed by taking linear combinations of the information bits. A parity check matrix for the (7, 4) Hamming code is given by

$$\mathbf{H} = \begin{pmatrix} 0\,1\,1\,1\,1\,0\,0 \\ 1\,0\,1\,1\,0\,1\,0 \\ 1\,1\,0\,1\,0\,0\,1 \end{pmatrix}. \quad (7.91)$$

It can be checked that the inner product (using binary arithmetic) of any row of the generator matrix with any row of the parity check matrix is zero.

For any vector space of dimension n, a k-dimensional subspace can be specified either directly, or by specifying its orthogonal complement of dimension $n-k$ within the vector space. Thus, the code $\mathcal{C}$ can be specified compactly by either specifying a generator matrix $\mathbf{G}$ or a parity check matrix $\mathbf{H}$.



[Madhow, Fundamentals of Dig. Comm., 2008]

Task 3 Give the degree distributions $\lambda(x), \rho(x)$ for the code in the example above.

Task4 Assume that the symbols x_i of the code in the example above are transmitted over a binary symmetric channel (BSC), and the output is given by $y = [1, 0, 0, 0, 0, 0, 1]$. Go through the first few iterations of Gallager's Algorithm A to decode the codeword (use the table on the next page).

Iteration Edge	1 $v_i^{(1)}$	1 $u_j^{(1)}$	2 $v_i^{(2)}$	2 $u_j^{(2)}$	3 $v_i^{(3)}$	3 $u_j^{(3)}$
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						