Partiella derivator

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Partiella derivator

- Defintion

$$\frac{\partial f}{\partial x} = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h},$$
$$\frac{\partial f}{\partial y} = \lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h}.$$

- Partiell derivierng: Derivera i en variabel med övriga variabler fixerade.
- Högre derivator

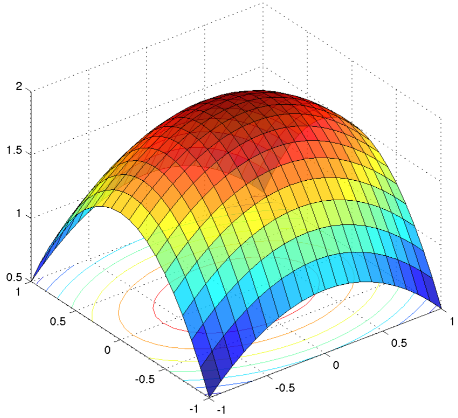
$$\frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial x \partial y}, \quad \frac{\partial^2 f}{\partial y \partial x}, \quad \frac{\partial^2 f}{\partial y^2}.$$

- Ordningen på blandade derivator kan kastas om (för "snälla" funktioner)

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

Normalvektorer och tangentplan

- Normalvektor: $n = (f_x(a,b),\ f_y(a,b),\ -1)$,
- Tangentplan: $z = f(a,b) + (x-a)f_x(a,b) + (y-b)f_y(a,b)$.



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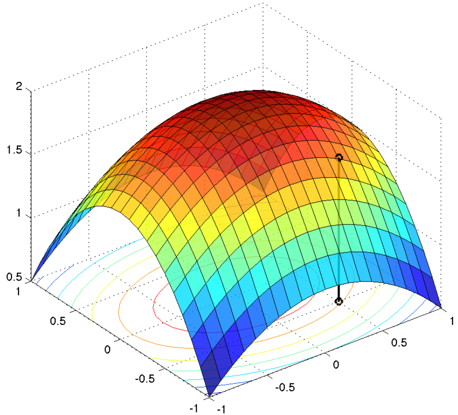

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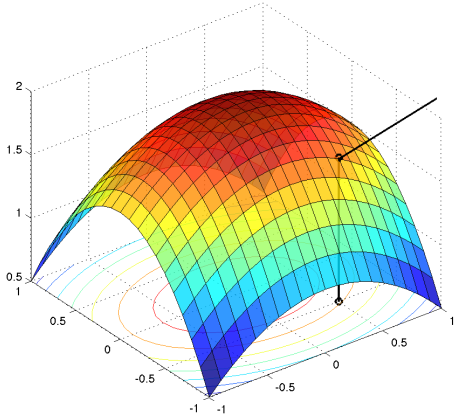

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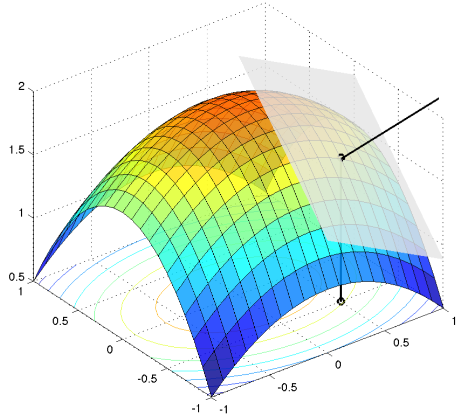

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- $z = \frac{19}{8} - \frac{1}{2}x + y$