Lecture 1
Course Description and Introduction to Ciphers

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Introduction and Administration

What is cryptography?

Cryptography is concerned with the conceptualization, definition, and construction of computing systems that address security concerns.

– Oded Goldreich, Foundations of Cryptography, 1997

Applications of Cryptography

Historically.

- Military and diplomatic secret communication.
- Communication between banks, e.g., credit card transactions.

Modern Time.

- Protecting satellite TV from leaching.
- Secrecy and authenticity on the Internet, mobile phones, etc.
- Credit cards.

Applications of Cryptography

Today.

- Distributed file systems, authenticity of blocks in bit torrents, anonymous remailers, Tor-network, etc.
- RFID tags, Internet banking, Försäkringskassan, Skatteverket, "e-legitimation".

Future.

- Secure distributed computing (multiparty computation): election schemes, auctions, secure cloud computing, etc.
- Variations of signatures, cryptosystem, and other primitives with special properties, e.g., group signatures, identity based encryption, etc.

Goal

The goal of the course is to

- give an overview of modern cryptography

in order that students should

- know how to evaluate and, to some extent, create cryptographic constructions, and
- to be able to read and to extract useful information from research papers in cryptography.

Prerequisites

- *DD1352 Algorithms, data structures and complexity*, or *DD2354 Algorithms and complexity*.
- Knowledge of mathematics and theory of algorithms corresponding to the required courses of the D or F-programmes at KTH.

Tentative Plan of Content (1/2)

- Administration, introduction, classical cryptography.
- Symmetric ciphers, substitution-permutation networks, linear cryptanalysis, differential cryptanalysis.
- AES, Feistel networks, Luby-Rackoff, DES, modes of operations, DES-variants.
- Entropy and perfect secrecy.
- Repetition of elementary number theory,
- Public-key cryptography, RSA, primality testing, textbook RSA, semantic security.

Tentative Plan of Content (2/2)

- RSA in ROM, Rabin, discrete logarithms, Diffie-Hellman, El Gamal.
- Security notions of hash functions, random oracles, iterated constructions, SHA, universal hash functions.
- Message authentication codes, identification schemes, signature schemes, PKI.
- Elliptic curve cryptography.
- Pseudorandom generators.
- Guest lecture.
- Make-up time and/or special topic.

Course Requirements

Presentations. a) Choose a research topic, and **b)** summarize the topic in a 12-min oral presentation.

Gives P-points ($P = 0$ or $30 \leq P \leq 80$), which is the sum of:

- (20P) Choice of content.
- (20P) Understanding of the content
- (20P) Quality of slides (or whiteboard)
- (20P) Presentation skills.

Up to 6 talks in 2 hour-sessions. Listen to the talks in your session.

Detailed rules and advice are found on the course homepage.

Course Requirements

Homework 1-4. Each homework is a set of problems giving I-points and T-points ($I \geq 10$ and $I + T \geq 50$).

- Solved in groups of up to three students, which may differ for each homework.
- Only informal discussions are allowed.
- Each student writes and submits his own solution.

Detailed rules and advice are found on the course homepage.

Course Requirements

Oral Exam. Purpose is to give a fair grade.

Discussion starting from submitted solutions and presentation.

Gives (possibly negative) I-points and T-points and a single O-point if passed.

Grading

To earn a given grade the requirements of all lower grades must be satisfied as well, with $A = I + T + P + O$.

Grade	Requirements
E	$I \geq 30$, $T \geq 40$, $P \geq 30$, and $O \geq 1$.
D	$A \geq 120$.
C	$A \geq 140$ and $P \geq 50$.
B	$A \geq 170$.
A	$A \geq 210$ and $P \geq 60$.

Kattis

Kattis is a judging server for programming competitions and for grading programming assignments. We use it for all exercises where code is submitted as a solution.

We assume that your Kattis id is the same as your KTH user name. If this is not the case, then email us your Kattis user name and use the subject `Krypto15 Kattis`.

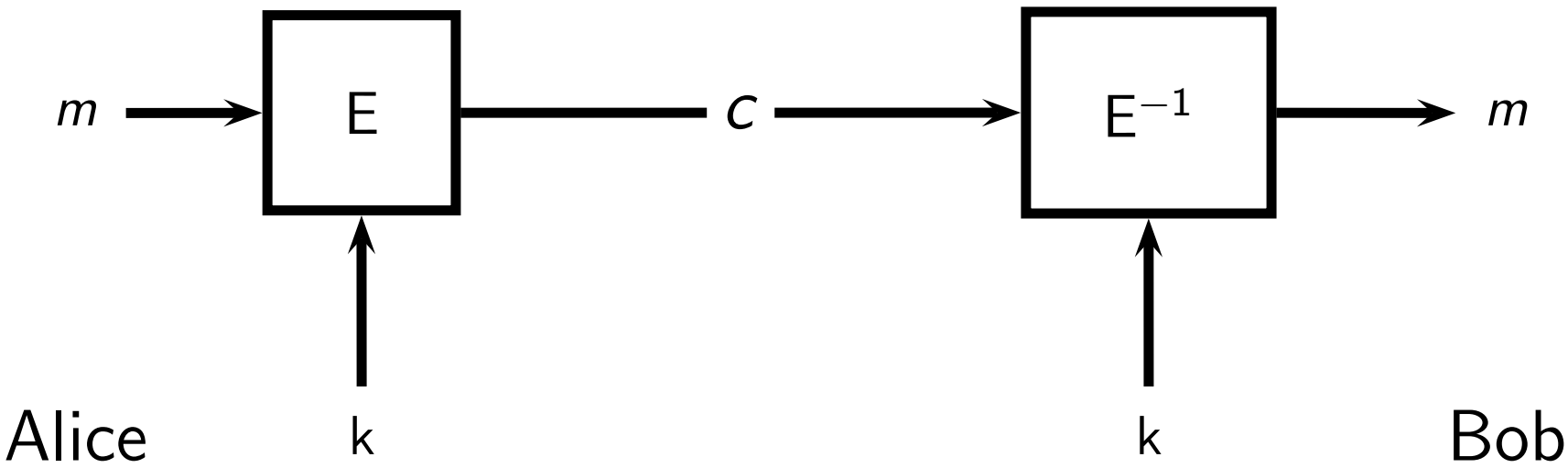
Latex

- Latex is the standard typesetting tool for mathematics.
- It is the fastest way to produce mathematical writing. You must use it to typeset your solutions.
- The best way to learn it is to read:
  `http://tobi.oetiker.ch/lshort/lshort.pdf`

Introduction to Ciphers

Cipher (Symmetric Cryptosystem)

$$c = \mathrm{E}_k(m) \qquad m = \mathrm{E}_k^{-1}(c)$$



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such that $\mathsf{E}_\mathsf{k}^{-1}(\mathsf{E}_\mathsf{k}(m)) = m$ for every message $m \in \mathcal{P}$ and $\mathsf{k} \in \mathcal{K}$. The set $\mathcal{C} = \{\mathsf{E}_\mathsf{k}(m) \mid m \in \mathcal{P} \wedge \mathsf{k} \in \mathcal{K}\}$ called the **set of ciphertexts**.

Attacks

Throughout the course we consider various attacks on cryptosystems. With small changes, these attacks make sense both for symmetric and asymmetric cryptosystems.

- Ciphertext-only attack.
- Known-plaintext attack
- Chosen-plaintext attack
- Chosen-ciphertext attack

Ceasar Cipher (Shift Cipher)

Consider English, with alphabet A-Z_, where _ denotes space, thought of as integers 0-26, i.e., $\mathbb{Z}_{27}$

- **Key.** Random letter $k \in \mathbb{Z}_{27}$.
- **Encrypt.** Plaintext $m = (m_1, \ldots, m_n) \in \mathbb{Z}_{27}^n$ gives ciphertext $c = (c_1, \ldots, c_n)$, where $c_i = m_i + k \bmod 27$.
- **Decrypt.** Ciphertext $c = (c_1, \ldots, c_n) \in \mathbb{Z}_{27}^n$ gives plaintext $m = (m_1, \ldots, m_n)$, where $m_i = c_i - k \bmod 27$.

Ceasar Cipher Example

Encoding.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	_
00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Key: $G = 6$

Plaintext.	B	R	I	B	E	_	L	U	L	A	_	T	O	_	B	U	Y	_	J	A	S
Plaintext.	01	17	08	01	04	26	11	20	11	00	26	19	14	26	01	20	24	26	09	00	18
Ciphertext.	07	23	14	07	10	05	17	26	17	06	05	25	20	05	07	26	03	05	15	06	24
Ciphertext.	H	X	O	H	K	F	R	_	R	G	F	Z	U	F	H	_	D	F	P	G	Y

Statistical Attack Against Ceasar (1/3)

Decrypt with all possible keys and see if some English shows up, or more precisely...

Statistical Attack Against Ceasar (2/3)

Written English Letter Frequency Table $F[\cdot]$.

A	0.072	J	0.001	S	0.056
B	0.013	K	0.007	T	0.080
C	0.024	L	0.035	U	0.024
D	0.037	M	0.021	V	0.009
E	**0.112**	N	0.059	W	0.021
F	0.020	O	0.066	X	0.001
G	0.018	P	0.017	Y	0.017
H	0.054	Q	0.001	Z	0.001
I	0.061	R	0.053	_	**0.120**

Note that the same frequencies appear in a ciphertext of written English, but in shifted order!

Statistical Attack Against Ceasar (3/3)

- Check that the plaintext of our ciphertext has similar frequencies as written English.
- Find the key k that maximizes the inner product $T(\mathrm{E}_k^{-1}(C)) \cdot F$, where $T(s)$ and F denotes the frequency tables of the string s and English.

This usually gives the correct key k.