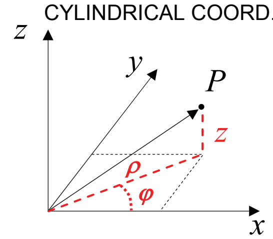
CURVILINEAR COORDINATES (u_1, u_2, u_3)

Base vectors	$\hat{e}_i = \frac{1}{h_i}\frac{\partial \bar{r}}{\partial u_i}$ *with scale factor* $h_i = \left\lvert \frac{\partial \bar{r}}{\partial u_i} \right\rvert$
Position vector differential	$d\bar{r} = \sum_{i=1}^{3} h_i du_i \hat{e}_i$
Surface element	$dS_3 = h_1 h_2 du_1 du_2 \quad (dS_3 \text{ surface perpendicular to } u_3 \text{ axis})$
Volume element	$dV = h_1 h_2 h_3 du_1 du_2 du_3$
Gradient	$\nabla\phi = \sum_i \frac{1}{h_i}\frac{\partial\phi}{\partial u_i}\hat{e}_i$
Divergence	$\nabla\cdot\bar{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(A_1h_2h_3) + \frac{\partial}{\partial u_2}(A_2h_3h_1) + \frac{\partial}{\partial u_3}(A_3h_1h_2)\right]$
Curl	$\nabla\times\bar{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1A_1 & h_2A_2 & h_3A_3 \end{vmatrix}$

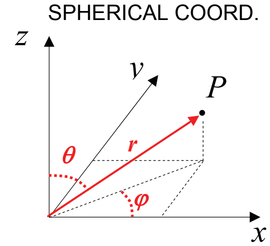
CYLINDRICAL COORDINATES

Position vector	$\bar{r} = (\rho\cos\varphi, \rho\sin\varphi, z) = \rho\cos\varphi\,\hat{e}_x + \rho\sin\varphi\,\hat{e}_y + z\,\hat{e}_z$ $\bar{r} = \rho\,\hat{e}_\rho + z\,\hat{e}_z$
Scale factors	$h_\rho = 1, \qquad h_\varphi = \rho, \qquad h_z = 1$
Base vectors	$\hat{e}_\rho = (\cos\varphi, \sin\varphi, 0) = \cos\varphi\,\hat{e}_x + \sin\varphi\,\hat{e}_y$ $\hat{e}_\varphi = (-\sin\varphi, \cos\varphi, 0) = -\sin\varphi\,\hat{e}_x + \cos\varphi\,\hat{e}_y$ $\hat{e}_z = (0,0,1)$
Surface element	$dS_\rho = \rho\, d\varphi\, dz$ $dS_z = \rho\, d\varphi\, d\rho$
Volume element	$dV = \rho\, d\varphi\, d\rho\, dz$
Gradient	$\nabla\phi = \left(\frac{\partial\phi}{\partial\rho}, \frac{1}{\rho}\frac{\partial\phi}{\partial\varphi}, \frac{\partial\phi}{\partial z}\right) = \frac{\partial\phi}{\partial\rho}\hat{e}_\rho + \frac{1}{\rho}\frac{\partial\phi}{\partial\varphi}\hat{e}_\varphi + \frac{\partial\phi}{\partial z}\hat{e}_z$
Divergence	$\nabla\cdot\bar{A} = \left(\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}\right)$
Curl	$\nabla\times\bar{A} = \left(\frac{\partial A_z}{\rho\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{e}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\hat{e}_\varphi + \left(\frac{\partial(\rho A_\varphi)}{\rho\partial\rho} - \frac{\partial A_\rho}{\rho\partial\varphi}\right)\hat{e}_z$
Laplacian	$\nabla^2\phi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\phi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{\partial^2\phi}{\partial z^2}$

CYLINDRICAL COORD.



SPHERICAL COORD.



SPHERICAL COORDINATES

Position vector	$\bar{r} = (r\sin\theta\cos\varphi, r\sin\theta\sin\varphi, r\cos\theta) = r\sin\theta\cos\varphi\,\hat{e}_x + r\sin\theta\sin\varphi\,\hat{e}_y + r\cos\theta\,\hat{e}_z$ $\bar{r} = r\,\hat{e}_r$
Scale factors	$h_r = 1, \qquad h_\theta = r, \qquad h_\varphi = r\sin\theta$
Base vectors	$\hat{e}_r = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) = (\sin\theta\cos\varphi)\hat{e}_x + (\sin\theta\sin\varphi)\hat{e}_y + \cos\theta\,\hat{e}_z$ $\hat{e}_\theta = (\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta) = (\cos\theta\cos\varphi)\hat{e}_x + (\cos\theta\sin\varphi)\hat{e}_y - \sin\theta\,\hat{e}_z$ $\hat{e}_\varphi = (-\sin\varphi, \cos\varphi, 0) = -\sin\varphi\,\hat{e}_x + \cos\varphi\,\hat{e}_y$
Surface element	$dS_r = r^2\sin\theta\, d\theta\, d\varphi$
Volume element	$dV = r^2\sin\theta\, d\theta\, d\varphi\, dr$
Gradient	$\nabla\phi = \left(\frac{\partial\phi}{\partial r}, \frac{1}{r}\frac{\partial\phi}{\partial\theta}, \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}\right) = \frac{\partial\phi}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}\hat{e}_\varphi$
Divergence	$\nabla\cdot\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(A_\varphi)$
Curl	$\nabla\times\bar{A} = \left(\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\varphi) - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial\varphi}\right)\hat{e}_r + \left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi)\right)\hat{e}_\theta + \left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial\theta}\right)\hat{e}_\varphi$
Laplacian	$\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\phi}{\partial\varphi^2}$