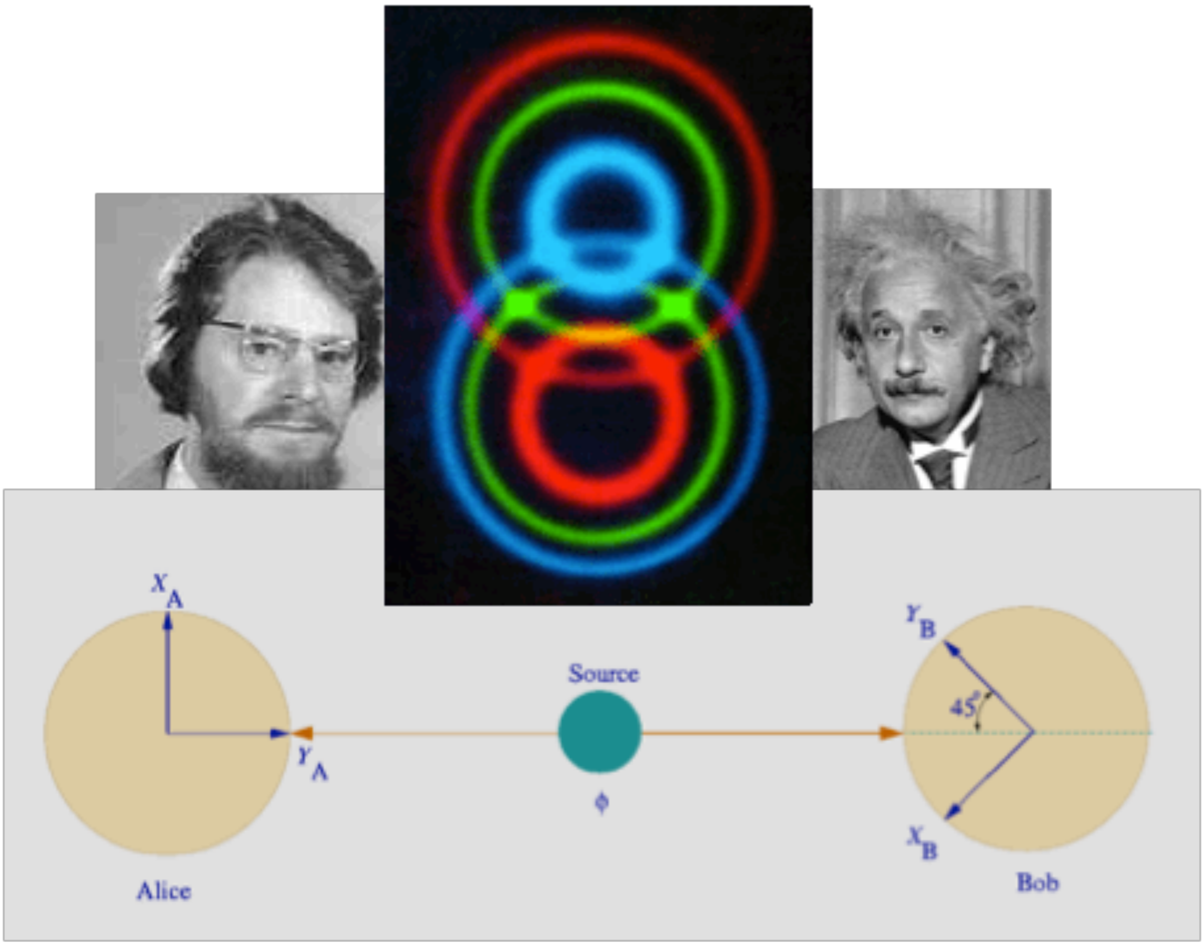
Violation of Bell's Inequality

Manual for the Quantum Entanglement Setup

Julia Cramer

Barbara Witek
Val Zwiller



1. Introduction

This manual is written for Applied Physics bachelor students at TU Delft to introduce quantum entanglement and to give instructions for quantum entanglement measurements that the students will perform.

Entanglement is a feature of quantum physics, the science of the very small, proposed by Einstein, Podolski and Rosen. It is possible to link together two quantum particles, two photons, for example, in a special way that makes them effectively two parts of the same entity. You can then separate them as far as you like, and measuring a property of one is instantly reflected in the other. This feature, which intends information traveling faster than light, is a fundamental aspect of quantum science. Erwin Schrödinger, who came up with the name “entanglement” called it “the characteristic trait of quantum mechanics.” Entanglement is fascinating in its own right, but what makes it really special are dramatic practical applications that have become apparent in the last few years.

In this experiment, you are going to perform a Bell inequality measurement with entangled photons, to verify if entanglement truly exists. To do this, you need some theoretical background and information about the setup. The first part of the manual contains theoretical information, and information about the parts of the setup.

In the second part, you will find a manual for the setup, and lab instructions to perform a Bell inequality test. There are also some theoretical exercises to prepare the final Bell test.

Table of Contents

2. Theory

2.1 Entanglement

Two particles are called 'entangled' when one particle's property is directly correlated to the other particle's property. In quantum mechanics, the state of a particle is described by a wavefunction, a function which gives the possible states of the system. The state is not determined, unless you perform a measurement. When you measure the state of the particle, the probability wave function 'collapses' to one of the possible states. When two particles are entangled, measuring one of them has direct consequences for the other particle. If you measure the state of one particle, the wavefunction for both entangled particles directly collapses. Therefore, the state of the other particle becomes known instantaneously, wherever the other particle might be. In this experiment, photons are involved, elementary particles of light. The polarizations of these photons are entangled. This means for example, in case of negative correlation, that when one of the photons is measured as vertically polarized, the other photon has horizontal polarization, and vice versa. When Alice and Bob each get one entangled photon, they can seperately measure the polarization of their photon.

Instead of saying that when Alice measures vertical polarization, Bob should measure horizontal polarization, and when Aice measures horizontal polarization, Bob should measure vertical polarization, we use the following notation:

$$\varphi = \frac{1}{\sqrt{2}}\left(\left|H_1V_2\right\rangle + e^{i\theta}\left|V_1H_2\right\rangle\right)$$

Where Alice has the first (1) photon and Bob the second (2) photon. In this case the total probability of the two possibilities (Alice measures H, Bob V and vice versa) is 1.

When the photons are positively correlated, the particles have the same polarization. So, when one photon is measured as horizontally polarized, the other photon has horizontal polarization too. We notate positive correlation as the following wave function:

$$\varphi = \frac{1}{\sqrt{2}}\left(\left|H_1H_2\right\rangle + e^{i\theta}\left|V_1V_2\right\rangle\right)$$

The exponential term, is the phase difference between the two possibilities *HH* and *VV*. In calculating probabilities, we use the the absolute square of the wave-function. Therefore, this phase difference does not affect the probability wave function.

2.1.1 EPR Paradox

In 1935, after quantum mechanics was developed, and entanglement became a concept, Einstein wanted to prove that quantum mechanics was incomplete. Together with Podolski and Rosen, he formulated the EPR-paradox[i] (referring to the last names of the three scientists). The EPR-paradox is a thought experiment to show that entanglement violates the principle of locality, a concept from Einstein's relativity theory. The principle of locality says that nothing can travel faster than light. Entanglement seems to violate locality, because for example when one of the entangled photons was sent to an other galaxy, say 70 light years away, it would take 140 years to communicate with that particle from earth, where the other particle is. The other photon stays in the lab on earth, just waiting for the other one to travel away. When we measure the photon on earth, at the same moment the probability wave function of the other one collapses to the correlated state. So, by measuring the photon on earth, we instantly know the state of the photon 70 light years from here. And here comes the paradox, because then there would have been 'communication' between the two photons, instantly, faster than light. With this paradox, Einstein, Podolski and Rosen did not claim that quantum mechanics should be completely rejected. They stated that the theory was not yet complete. Einstein, Podolski and Rosen introduced the hidden variable theory, they thought there might be hidden variables, which we are not aware of yet, that influence the measurements.

2.2 Cryptography

Cryptography is the practice of hiding information. Since the ancient world cryptography has been used mainly for military, diplomatic and general government purposes. When the digital age came up, and computers and digital communication systems became available for personal use, a broad demand for private secure communication was created. The success of the internet and electronic commerce made it necessary to make cryptography available to the masses. To secure a certain piece of information, the process of encryption is used. The safety is based on the secrecy of the key. The key is a way to 'hide' a message by, for example summing it with the real message. Therefore, cryptography can be done in both classical and quantum ways. The methods will be explained by a simple example. Two communication parties, Alice and Bob, want to transmit a message that has to be private. Eve (the eavesdropper) would like to intercept the message.

2.2.1 Classical Encryption

In the classical way of encrypting, Alice and Bob share a key of the same length as the message to be encrypted. In the following example (see figure 1), Alice just sums the message and the key to get the so called 'cipher text', that she sends to Bob. After that, Bob must, to decode the message, subtract the key from the message.

Message	P	A	Y	M	E	N	T	⋆	R	E	C	E	I	V	E	D
Plaintext	16	1	25	13	5	14	20	0	18	5	3	5	9	22	5	4
Key	5	23	12	20	9	25	1	6	3	26	3	10	2	23	0	7
Ciphertext	21	24	11	7	14	13	21	6	21	5	6	15	11	19	5	11

Figure 1. Encription with a key. The letters are coded into numbers and the key is added to the message. A receiver can only decode the message if he knows the key.

The cipher text is more generally obtained by using an algorithm to encrypt the message. When both Alice and Bob know the key, by sharing it through a secured channel, they can just share the encrypted message (ciphertext) via a public channel, without losing the real message to others. But when Alice and Bob use the key more often, the probability that someone, for example Eve, is able to intercept the key by comparing the cipher texts, gets bigger. And then Eve will be able to decode the subsequent cipher text. At this moment there are algorithms to encrypt messages that are safe enough for now, it takes too much time to decode a cipher without knowing the algorithm, even with the most powerful computers. However, once quantum computers are available, their computing power will make it a lot easier and faster to decode the message. There is already an algorithm developed by Peter Shor factorize large numbers, something that is used often in cryptography, when using the computing power of a big quantum computer. Therefore, we need other ways to safely encrypt our secret messages. And here comes quantum mechanics to the rescue.

2.2.1 Quantum Encryption

In quantum encryption, entangled particles (in our case photons) are used. How the security works, will be explained by an example of a new communication between Alice and Bob. Here, from a certain source that produces entangled photon pairs, Alice and Bob will each get one of the entangled photons, for every set. They each can randomly decide for every new photon to measure it in one of the two bases: Horizontal-Vertical or Diagonal- Antidiagonal. After a lot of photon detections, Alice and Bob contact each other through a public channel, and only share with each other the bases in which they measured.

When they did not use the same basis, the corresponding bits are discarded. They keep the bits that came from measurements in the same bases. In figure 2 the procedure is shown.

A basis	⊠	⊞	⊠	⊞	⊞	⊞	⊠	⊠	⊞	⊞
A bit value	1	0	1	0	1	1	0	0	1	1
A sends	|↖⟩	|→⟩	|↖⟩	|→⟩	|↑⟩	|↑⟩	|↗⟩	|↗⟩	|↑⟩	|↑⟩
B basis	⊞	⊠	⊠	⊞	⊠	⊞	⊠	⊞	⊞	⊠
B bit	1	1	1	0	0	1	0	0	1	1
same basis	n	n	y	y	n	y	y	n	y	n
sifted key			1	0		1	0		1	

Figure 2. Quantum cryptography. Alice and Bob measure entangled particled in two different bases. After measuring, they contact each other and share the bases in which they measured. They only keep the bits that they measured in the same basis. The result is a common key.

When someone, for example Eve, intercepts the photons that were sent to Bob, she can measure the photon in one of the bases, and send a photon with the same polarization in the base she used. But this will be found out easily by Alice and Bob. Eve is not able to polarize the photon in the right way for both bases, just because she does not know the polarization in the base she did not measure. Because Alice and Bob randomly choose to measure in two different bases, they compare the first part of their key. For example the key in figure 2, they both have thrown away the bits in which they did not measure the same basis. There are 5 bits left (sifted key). Alice and Bob compare the first two, do they both have '1 0'? If this is not the case, someone (Eve) intercepted Bob's photons, and was not able to copy the entangled photon. Therefore Eve broke the entanglement by intercepting, and Alice and Bob will immediately notice and just discard this key. Of cource there is some chance that Eve was able to send the right photon. Therefore Alice and Bob need to compare more bits, and use a longer key code, to protect themselves against Eve.

2.3 Bell's Inequality

The EPR paradox remained a philosophical question until John Bell (1965) devised a test, to verify whether there are hidden variables. Bell's test stated that a certain inequality (called Bell's inequality) must be obeyed under any local hidden variable theories. When Bell's test is violated, there can not be any hidden variables. In this way, Bell proved that no real physical theory or local hidden variables can reproduce the predictions of quantum mechanics. The theory was stated in 1965, but the first experiments on Bells Inequality were done in 1982.

2.3.1 CHSH form

There are many mathematical formulations of Bell's theorem. Most often CHSH form is used, proposed by Clauser, Horn, Shimony and Holt. This inequality test can be easily applied in an experiment.
This is a thought experiment[ii], to understand Bell's inequality. We start by forgetting entanglement.
Imagine a stream of photons in pairs of two that can be seperately measured. Alice gets one of the photons, Bob gets the other one. Both can measure in two basis, which we call A_1, A_2 for Alice, and B_1 and B_2 for Bob. They do not decide on beforehand in which basis they will measure, until they have received the photon. When a photon is horizontally polarized in the chosen basis, it gets a +1, vertically polarized gives a -1. So the possible outcomes can be:

Alices basis	**outcome**		**Bobs basis**	**outcome**	
A_1	H1	+1	B_1	H1	+1
	V1	-1		V1	-1
A_2	H2	+1	B_2	H2	+1
	V2	-1		V2	-1

Alice and Bob measure at the same time, and because physical influences can not propagate faster than light (we assume), their measurement do not influence each other.
We know that
$A_1 + A_2 = 0$ or ± 2
$A_1 - A_2 = \pm 2$ or 0
$B1 = \pm 1$ and $B2 = \pm 1$
Then we can say for sure that
$-2 < B_1 \cdot (A_1 + A_2) + B_2 \cdot (A_1 - A_2) < 2$
and we can rewrite this to:
$-2 \leq B_1 A_1 + B_1 A_2 + B_1 A_2 - B_1 A_2 \leq 2$

Now, we introduce *E*, which denotes the expectation value. Lets say that *p(a1, a2, b1, b2)* is the probability that the photons are in such state that the outcome of the measurements gives a1 for A_1, a2 for A_2 etcetera.
We have, using the rules of statistics:

$$E(|B_1 A_1 + B_1 A_2 + B_2 A_1 - B_2 A_2|) = \sum_{a1,a2,b1,b2} p(a1,a2,b1,b2) \cdot (b1a1 + b2a2 + b2a1 - b2a2)$$

$$E(|B_1 A_1 + B_1 A_2 + B_2 A_1 - B_2 A_2|) \leq \sum_{a1,a2,b1,b2} p(a1,a2,b1,b2) \cdot 2 = 2$$

Because all these states are independent, we can rewrite *E* seperately:

$$E(B_1 A_1 + B_1 A_2 + B_2 A_1 - B_2 A_2) = E(B_1 A_1) + E(B_1 A_2) + E(B_2 A_1) - E(B_2 A_2)$$

and we get:

$$S = \left| E(B_1 A_1) + E(B_1 A_2) + E(B_2 A_1) - E(B_2 A_2) \right| \leq 2$$

Which we from now on call the Bell inequality, where *S* is the combination of *E*-factors we use.
By repeating an experiment many times, and averaging over the number experiments, we can estimate E with a acceptable accuracy.
So for example the first *E*, $E(A_1B_1)$ (shifting A and B is no problem here because they are indistinguishable).

$$E(B_1 A_1) = \frac{N_{HH} + N_{VV} - N_{HV} - N_{VH}}{N_{HH} + N_{VV} + N_{HV} + N_{VH}}$$

Where Alice and Bob got both +1 when they both measured *H* or when they both measured *V* and -1 when they measured opposite polarization.
This is for positive correlation, when the photons have negative correlation (see chapter 2.1) the table would look like:

Alices basis	**outcome**		**Bobs basis**	**outcome**	
A_1	H1	+1	B_1	H1	-1
	V1	-1		V1	+1
A_2	H2	+1	B_2	H2	-1
	V2	-1		V2	+1

And similarly, *E* would become:

$$E(B_1 A_1) = \frac{N_{HV} + N_{VH} - N_{HH} - N_{VV}}{N_{HH} + N_{VV} + N_{HV} + N_{VH}}$$

Because they get +1 when they measure opposite polarization.

Let us proceed to the quantum mechanic approach. We now have the following Bell-state:

$$\varphi = \frac{1}{\sqrt{2}}\left(\left|HV\right\rangle + \left|VH\right\rangle \right)$$

One photon is sent to Alice, The other one to Bob and the photons are negatively correlated.
Alice measures her photons again in two chosen bases: the *HV*-basis, and the *DA*-basis. We give her a table again:

Alices basis	**outcome**	
$A_1=HV$	H	+1
	V	-1
$A_2=DA$	D	+1
	A	-1

But now, when we assume the photons to be entangled, we know that when Alice measures her photon to be *H*, Bob's photon has to be *V.*
But Bob measures in two other bases, which are just in between the *HV* basis and the *DA* basis. We write his bases as:

$$B_1 = \frac{1}{\sqrt{2}}\left(HV + DA \right)$$

$$B_2 = \frac{1}{\sqrt{2}}(HV - DA)$$

Now, we can again calculate the *E*-factors:

$$E(A_1,B_1) = 1 \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$E(A_1,B_2) = 1 \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$E(A_2,B_1) = 1 \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$E(A_2,B_2) = 1 \cdot -\frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

The 1 comes from Alice all the time, because we start with her photon. The $1/\sqrt{2}$ term comes from Bob, whose polarization we know because of Alices measurement, but only in Alices basis. So for example, when Alice measures +1in her *HV*-basis, there is a $1/\sqrt{2}$ term for Bob measuring +1 in his *HV* basis. Only the last *E*-factor gets -1 because now Alice measures for example +1 in her *DA* basis, and when we look at Bob's basis, we see that he has a $-1/\sqrt{2}$ term for *DA* measuring in his basis.

Now we calculate *S* again for this quantum mechanic alternative:

$$S = \left|E(B_1A_1) + E(B_1A_2) + E(B_2A_1) - E(B_2A_2)\right| = 2\sqrt{2} > 2$$

In this case, Bell's inequality is violated, the photons measurements must have influenced each other instantly. Information went faster than light, which means that at least one of the assumption in the relativity theory: realism or locality, are not correct, and there might not be hidden variables.

2.4 Light

Light is an electromagnetic wave. Photons are tiny packets of light (energy). Photons can either be described as electromagnetic waves or particles. This property is also called the wave-particle duality. Light as a wave can be described by its intensity, frequency/wavelength, polarization and phase. A light wave can be visualized as a transverse oscillating wave of electric and magnetic fields.

4.4.1 Polarization

Polarization of light is described by the direction in which the electric field component of the electromagnetic wave oscillates. The electric field is perpendicular to the direction of propagation, when light propagates in the z-direction, the electric field oscillates in the x-y plane. It is also a measurable quantum mechanical quantity. In describing polarization, the two directions in which the electic component of the electromagnetic wave oscillate are important. When it oscillates exactly in x-direction the polarization is horizontal, for y-direction it is vertical. When, for example the x and y amplitude are equal, the wave can be either diagonal or antidiagonal.

In the case of a simple harmonic wave, travelling in z-direction, the x-component and the y-component of the electric field have exactly the same frequency. But their amplitude and their phase can differ from each other. The polarization state can be described by the shape traced out in a fixed plane, where the electromagnetic plane waves pass. When the x- and y-component are propagating in phase, as shown in figure 3, the polarization is called linear. The direction of this line depends on the relative amplitude between the two components. When the two components have the same amplitude, but have a phase-shift of exactly 90 degrees, the polarization is called circular. In all other cases, the polarization is called eliptical, because the two components of the electric field wave trace out an eliptical shape in the propagation plane. All three cases are shown in figure 3.

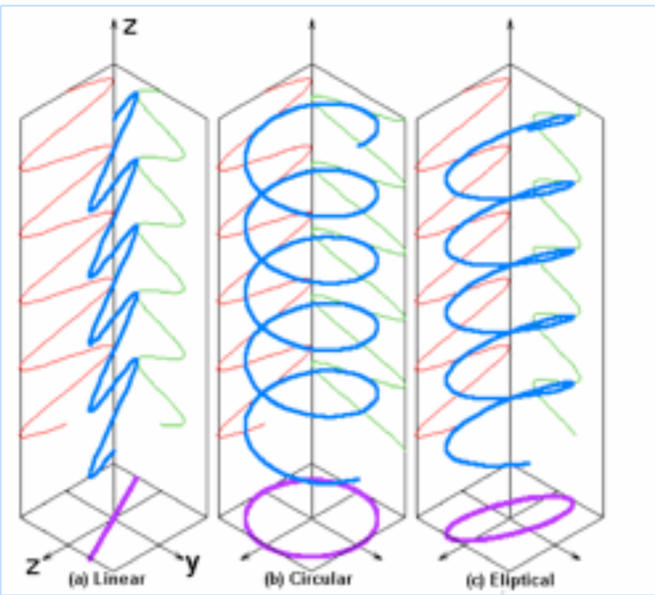


Figure 3. Three possible polarization states. (a) When E_x and E_y are in phase, the polarization is linear. The sum of $E_x$and E_y is always in the same plane. (b) When E_x and E_y are out of phase by $\pi/2$, the polarization is circular. The sum of $E_x$and E_y follows a circle. (c) For all other possible phase shifts, the light is eliptically polarization.

3. Setup

In figure 4 you see an overview picture of the complete setup. The setup is composed of 2 parts, in the first part (top), entangled photons are generated. In the second (bottom) part, the photons polarization can be measured. With optical fibers, the photons can be transmitted. In this chapter, each component of the setup will be explained. The setup is inspired by the work of Kwiat et al.[iii] In appendix A, the setup can be found with a desciption of each part, and the blue and red beams.

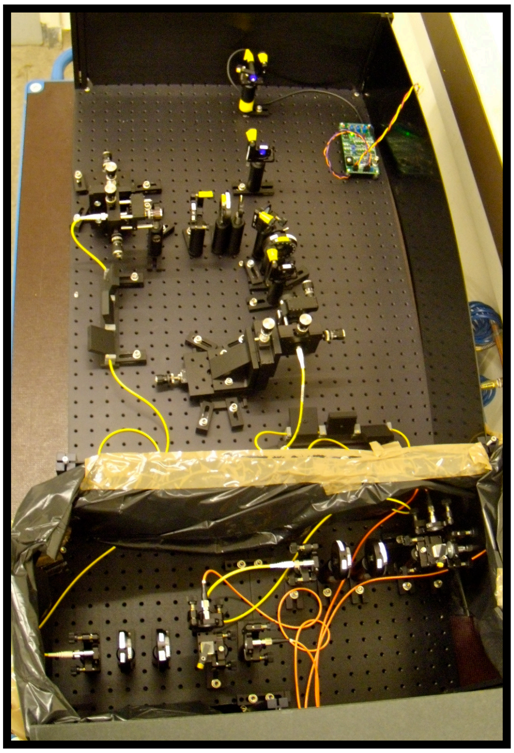


Figure 4. Setup overview. The laser source is at the top.

3.1 Laser Source

On top in the setup-picture is the laser source, shown in figure 5. The laser diode produces weak (±50 mW) blue light, with a wavelength of 390 nm. In the laser source there is also a lens, which focusses the laser light on the non-linear-crystal, 20 cm away from the laser source.

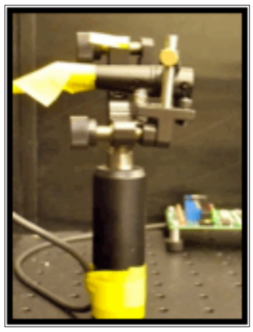


Figure 5. Laser source.

3.2 BBO Crystal - Downconversion

The second element in the setup, is the non-linear crystal, made of Barium Borate (BBO). In this crystal, a small fraction of the laser photons decay into pairs of photons by the process of spontaneous parametric down conversion. Because of energy conservation both photons have a wavelength of 780 nm as shown in the following derivation. The laser beam is referred to as laser 'l', the down converted photons are called horizontal 'h' and vertical 'v', because of their polarization.

Conservation of energy gives

$$E_l = E_h + E_v$$

As known

$$k = \frac{E}{\hbar c}$$

and therefore

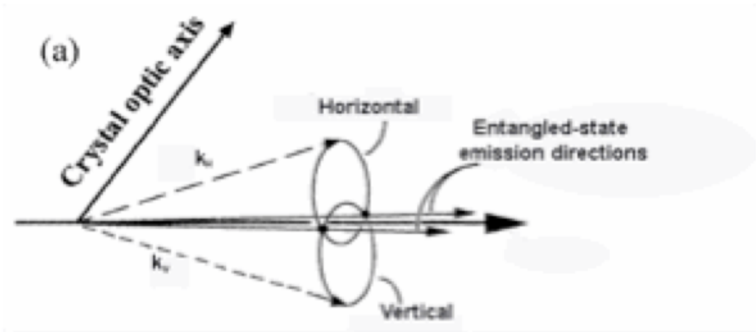
$$k_l = k_h + k_v$$

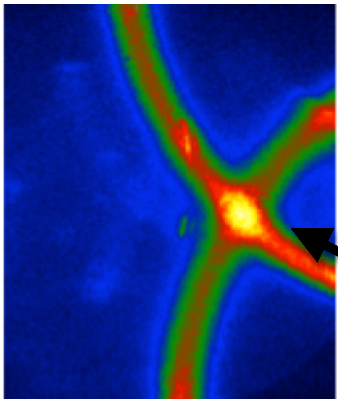
with the fact that

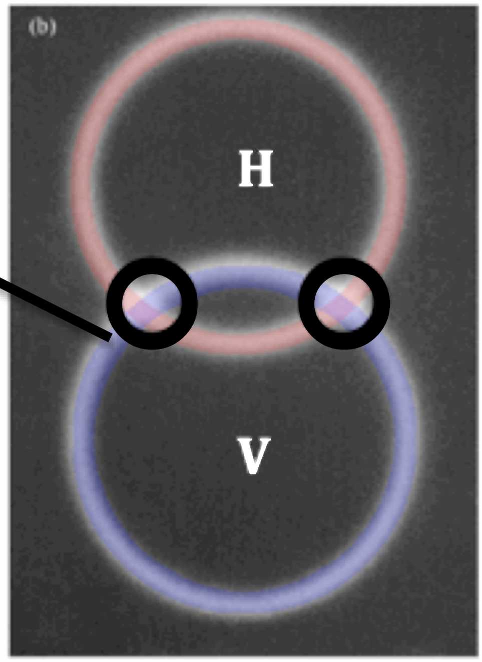
$$k = \frac{2\pi}{\lambda}$$

then it is clear that:

$$\frac{1}{\lambda_l} = \frac{1}{\lambda_h} + \frac{1}{\lambda_v}$$







The crystal was cut with the axis at 41,8° from the normal to the large face. The down converted photons leave the crystal in two cones, with a relative angle of 6° to each other. This is shown in figure 6. The pump-beam is the original beam, coming from the laser. The upper cone carries horizontal polarization, the lower cone vertical polarization. At the intersection points of the two cones, there is a superposition of horizontally and vertically polarized photons:

$$\varphi = \frac{1}{\sqrt{2}}\left(|H_1V_2\rangle + |V_1H_2\rangle\right)$$

Figure 6. (a &b) The spontaneous down-conversion cones. The photons in the upper cone are horizontally polarized, the photons in the lower cone are vertically polarized. Only photons from the intersection points are used in this Bell experiment. (c) A picture of the intersection points that is taken in the setup described in this manual.

3.2.1 Nonlinearity

The BBO crystal is a birefringent, nonlinear crystal, which means that there is a difference between the refraction indices of the horizontal (H) and vertical (V) ray. Both give a cone. Because of the difference in refraction index, the new photons travel at different velocities through the crystal. Therefore, when they come out of the crystal, a transversal walk-off of the extraordinary beam and the ordinary beam will occur. The transversal walk-off produces a shift between the ordinary and extraordinary cone, while the longitudinal walk-off introduces a time delay between horizontally and vertically

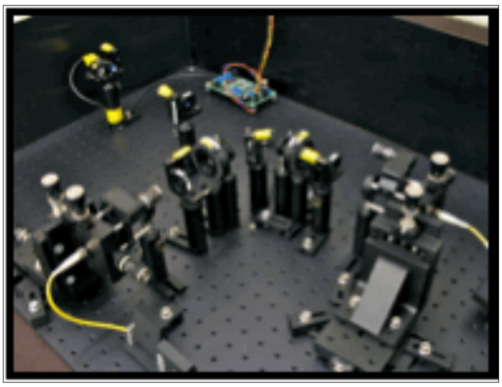


Figure 7. The compensation system consists of a wave plate and a BBO Crystal for each photon.

polarized photons. These effects lead to distinguishability of the photons as shown in figure 8, and therfore destroys the entanglement. To make sure that the photons will be indistinguishable, a compensation system is required, which will be explained in the following paragraph.

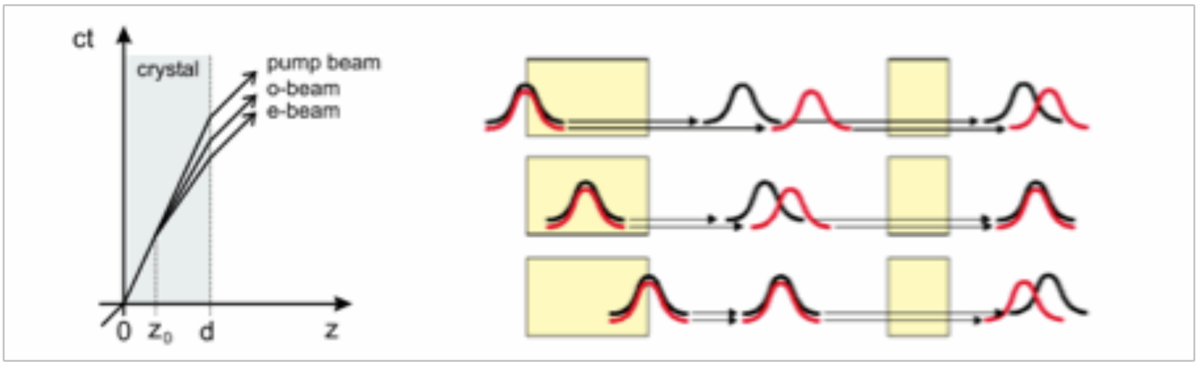


Figure 8. The walk-off compensation. The yellow squares are the BBO Crystals. Between the first and second crystal, a wave plate shifts the polarization by 90 degrees.

3.3 Time compensation system

In figure 7 the compensation system, is shown. The two photons enter this part of the setup from the top left of the picture. With a mirror, one beam is reflected to the left side of the compensation system, the other travels right across to the right side of the compensation system. The time compensation system consists of a wave plate and a second BBO Crystal for each path. After compensation, the two possible states $|H_1V_2\rangle$ and $|H_2V_1\rangle$ can no longer be distinguished by comparing the arrival times of individual photons.

3.3.1 Wave plates

The half wave plates, shown in figure 9, retard every photon in such way that their polarization shifts. For example a shift of 90 degrees takes care for the horizontally polarized photon becoming vertically polarized, and vice versa. A half wave plate is a non-linear optical crystal. It has different refraction index for horizontal and vertical incoming waves. They cause a relative phase shift between the horizontal and vertical wave by:

$$\Delta\varphi = \frac{2\pi}{\lambda_0} d(|n_v - n_h|)$$

Figure 9. Wave plate. By rotating the angle of the wave plate, the polarization shifts for twice this angle. Source: www.thorlabs.com

Due to the length and refraction indices of the wave plate, a polarization shifting scale is displayed on the wave plate. With $\Delta\varphi = 2\pi$, the same polarization is required, because the horizontal and vertical wave have an relative retardation of exactly one wavelength. The horizontal wave travels faster than the vertical wave. It is important to notice that when you turn the wave plate, for example, with α, the polarization is shifted with 2α, due to the influence to both the horizontal, and the vertical wave. In the time compensation system the wave plate shifts the polarization for 90 degrees; the horizontal polarized photon becomes vertical and vice versa.

3.3.2 BBO-Crystal

After the phase shift, both photons travel trough a BBO crystal, with half of the length of the first BBO crystal. Because both photons switched their polarization, the one that went faster trough the first one, now goes slower trough this crystal, and vice versa. The walk-off distance depends on the position in the first crystal where the blue photon was split into two red ones, as shown in the middle part of figure 8.[iv]

Because it is impossible to determine where in the crystal the red photons were formed, the walk-off length is unknown, and is different for every pair of photons. The second crystal is half the length of the first one, and brings together the photons that were formed exactly in the middle of the first BBO crystal. For the other photon pairs, there is still a walk-off, as shown in the right part of figure 8. But, because it is unknown where in the downconversion crystal the red photons were formed, it is unknown whether the vertically polarized or the horizontally polarized photon is the first, the upper and lower scheme of figure 8 are indistinguishable. Therefore, the photons are from now on indistinguishable and entangled, because it is a set of two photons with opposite polarization.

3.4 Detection part

The most important part for this Bell experiment is the part where the measurement takes place. The two photons can each be measured in the same way. We have two detection systems, which we call Alice and Bob. The detection part is shown in figure 10.
The photons come in from the right in figure 10. Each photon meets two wave plates. The first is a quarter wave plate, which will not be used by the students but is used to change circular polarization to linear polarization. The next wave plate, a half wave plate, can be set to different angles, to change the basis in which the measurement takes place. After choosing the basis, a polarizing beamsplitter transmits horizontally polarized light, but reflects vertically polarized light. The photons are detected by avalanche photodiodes, which are described in paragraph 3.4.4. These photodiodes are sensitive detectors that detect single photons by using the avalanche effect.
Alice and Bob can measure in any basis, by turning their half wave plate. Alice and Bob each have the exact same measurement system, so in this part, we will only describe Alices part. Through the single mode fibers, the down converted photons will be transmitted to the detection part.

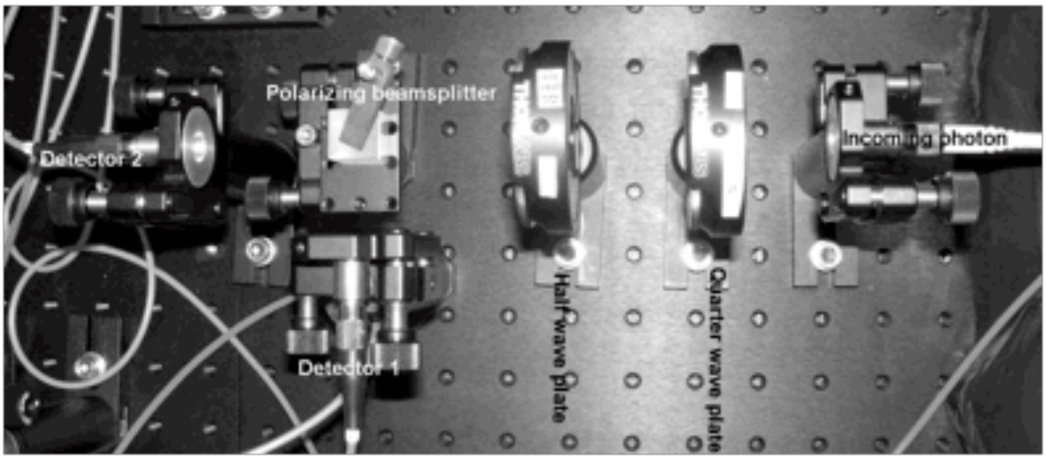


Figure 10. Detection part. One of the entangled photons comes in from the right. It meets two wave plates, that change the polarization of the photon. In the polarizing beamsplitter, the photons are transmitted to detector 2, or reflected to detector 1.

3.4.1 Optical fibers

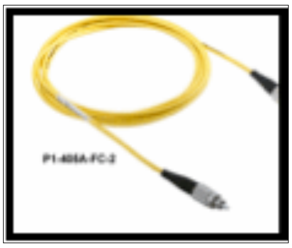


Figure 11. Optical singe-mode fiber. source: www.thorlabs.com

In figure 11, the single mode optical fibers that are used in the setup are shown. Optical fibers transport light, just as copper wires transport electricity. Light is kept in the core, the light reflects in the cladding as shown in figure 12. The core has a slightly higher refraction index that the cladding, which makes total internal reflection possible. Optical fibers can, depending on the wavelength, transmit light for kilometres, with negligable loss.
Single mode fibers have a core diameter of only 8 microns, they transmit only one mode of light. In the detection part, multimide fibers are used. These fibers have a core diameter of 50 microns. Therefore, they can detect much more light, and different modes. In figure 12, the cross-section of a single and a multimode fiber are drawn.

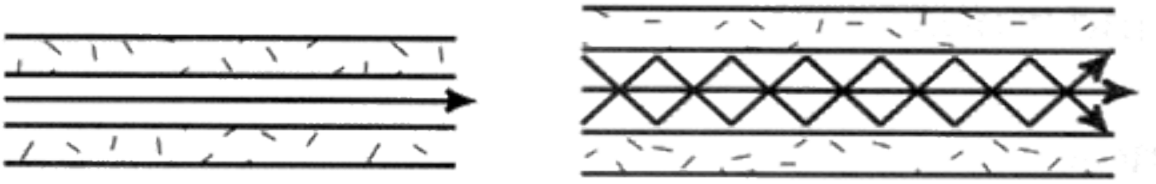


Figure 12. Left: a single mode fiber, only one mode can pass trough the fiber. Right: a multimode fiber, more light can pass trough the fiber. The light is captured in the core.

3.4.2 Polarization compensation system

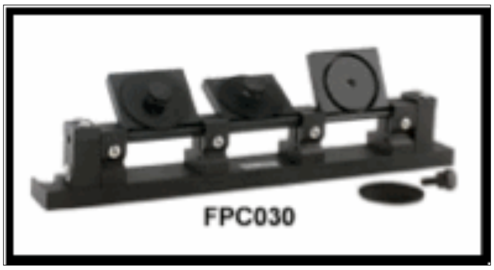


Figure 13. Fiber-based polarizing control modules. source: www.thorlabs.com

Because the optical fibers act as birefringent elements, they influence the polarization of the transported photons. The polarization shift depends on the way the fibers are moved. The compensation system we use here, is a fiber-based polarization control module, shown in figure 13. It compensates for the polarization shift caused by the optical fibers. A fiber is formed in three loops which can be rotated with respect to each other. The overall polarization rotation caused by this fiber element, depends on the angles between the fiber loops.

3.4.3 Polarizing beamsplitter

Figure 14. Polarizing beamsplitter. Source: www.thorlabs.com

For Alice, to know which polarization Bob's photon has, she has to measure the polarization of her photon. Therefore, she uses a polarizing beamsplitter, see figure 14, and a half wave plate. The polarizing beamsplitten is an optical device that transmits horizontally polarized light, and reflects vertically polarized light with an angle of 90 degrees. With the wave plate before the polarizing beamsplitter, Alice can choose the base in which she wants to measure. For example, if she wants to measure diagonal and antidiagonal polarization, she turns her wave plate for 45 degrees. Incident diagonal light is turned by the half wave plate, and enters the polarizing beamsplitter as horizontally polarized light and gets transmitted. Incident antidiagonal light is turned to vertical light, and gets reflected by the polarizing beamsplitter. After the beamsplitter, the photons of each H and V are detected by single photon detectors. This measurement method makes it possible to statistically distribute half of the incoming photons between two different measurement bases. In this setup, also Bob can measure the polarization of his photon, so he also uses a polarizing beamsplitter.

3.4.4 Single photon detectors and Correlator

The APD's (Avalanche Photo Diodes) are in a box placed under a sheet, because they are very sensitive detectors. An APD is a highly sensitive semiconductor electronic device. It converts light to electricity by the photoelectric effect. They are photodetectors that provide a gain through avalanche multiplication. This is a way to multiplate a current. The process is shown in figure 15. One photon exites an electron from E_v up to E_c. This electron is highly accelerated by the electric field and collides with the atoms in the material. There, some atoms get ionized and new electrons accelerate and collide with the matierial again. Therefore, a much higher current is obtained, by the avalanche principle. We get a large electrical pulse that can easily be detected, just by starting with one single photon exicting one electron.

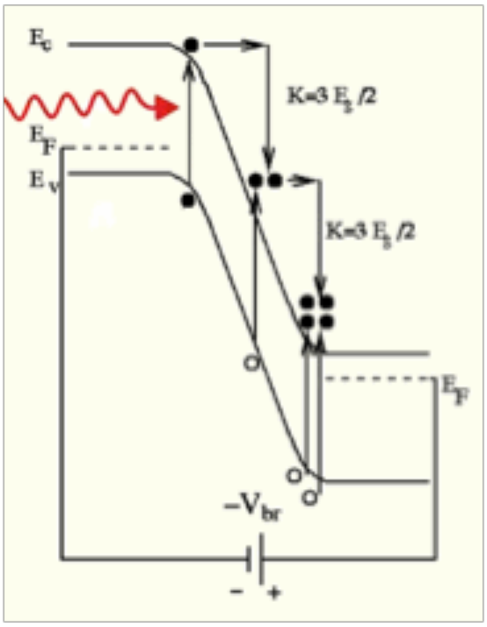


Figure 15. The avalanche effect in the APD's. The red photon is coming in from the left and excites the electron from the energy-level E_v to the level E_c.

In figure 16 a single APD is shown. This instrument is really sensitive for light. As explained, a single photon causes already a current. Therefore it is really important to <u>turn of the lights when you are working with the APD's.</u>

The electric pulses are transported to a correlater. This correlator measures a count and remains this information for 4 nanoseconds, when another electric pulse reaches the correlator within this 4 nanoseconds, the correlator gives not only information on those counts to the computer, but also a correlation between those counts.

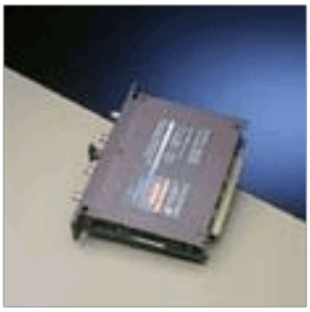
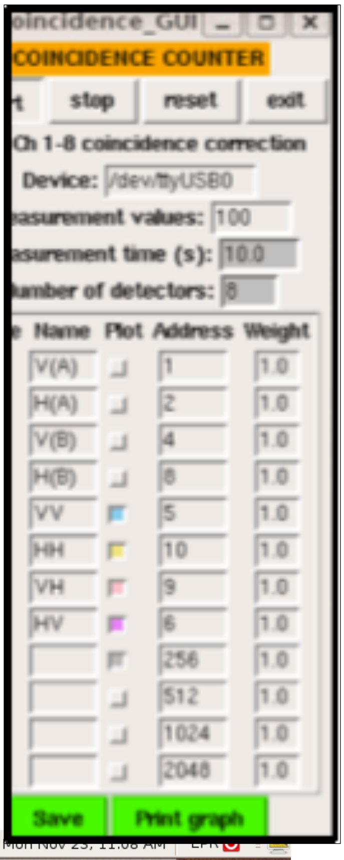


Figure 16. A single APD. This APD is the same as used in this setup. The APD'scomes from Perkin Elmer.

4. Bell Inequality Violation Test

4.1 The computer programme



The avalanche photodiodes, that detect the incoming photons, are connected to the computer. The password to start up the computer is 'epr'. You can find all the necessary files for this experiment in the folder called 'coincidence'. The measurement programme can be openend by clicking on 'coincidence_gui.tcl'. The programme will be described with help of figure 16, a screen shot of the programme while running.

The first thing to do is type [/dev/ttyUSB0] at the device. Do not forget to press enter after changing anything in this programme, otherwise you will see the changed text, but the programme does not carry it through. The programme saves your data in the 'coincidence'-folder. When you click at the green buttom called 'save', you can set your own file-name.

The first four measured outputs are the counts for Alice and Bob, in horizontal and vertical polarizations; the addresses correspond with the four APD's. You should change the second four items to measure the coincidences. Coincidences for example between H(A) and H(B) (both horizontal) get numer 10 (2 for the H of Alice, and 8 for H of Bob). See figure 17 for all items.

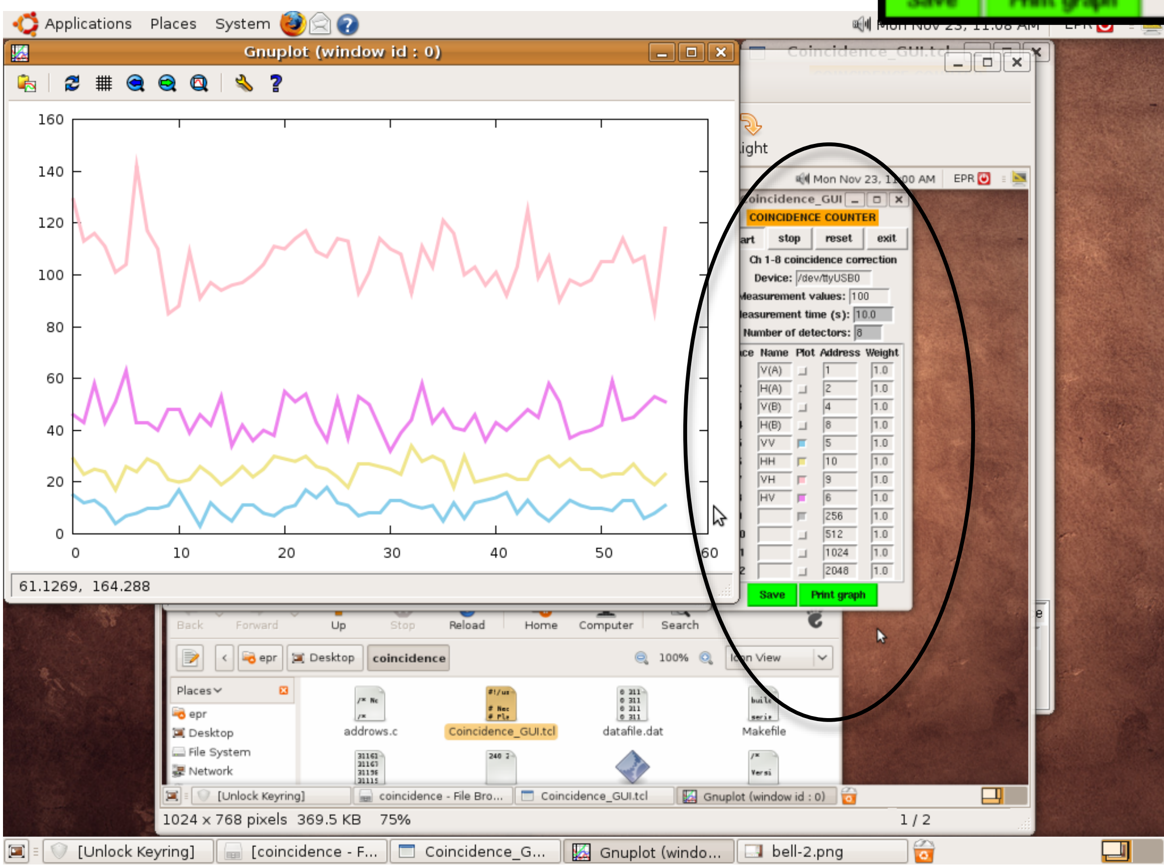


Figure 17. The computerscreen when the coincidence-programme is running.

1	V(A)	1
2	H(A)	2
3	V(B)	4
4	H(B)	8
5	V(A)V(B)	5
6	H(A)H(B)	10
7	V(A)H(B)	9
8	H(A)V(B)	6

Figure 18. Table with all measurable items. Item 1-4 are measurements of Alice and Bob, Item 5-8 are coincidences between Alices and Bobs measurements.

4.2.4 Computer

Turn on the computer and follow the steps mentioned below to start the programme.

1. password: epr
2. open folder: coincidence (on desktop)
3. start programme: coincidence_gui.trl
4. do not forget to press enter everytime you change some text in the programme
5. type in device: /dev/ttyUSB0
6. change the coincidence items
7. choose a filename
8. run the programme

4.2 Exercises

4.2.1 Important rules

1. Never put on the light in the room when the APD's are on, the detectors might die or get major damage.
2. Do not open the left side of the setup, the part that is covered by a plexiglass plate. The alignment for entanglement generation is very sensitive and was carefully aligned.

4.2.2 Theoretical exercises

The theoretical exercises can be found within this document. The exercises are based on a theoretical violation of Bell's inequality. Fill in the exercises. Based on the exercises, calculate E for every set of bases and calculate S. Is Bell's inequality violated?

4.2.3 Practical exercises

To understand the violation of the Bell inequality, there are first some understanding exercises, followed by the 'real' measurement. The understanding exercises can be found within this document.

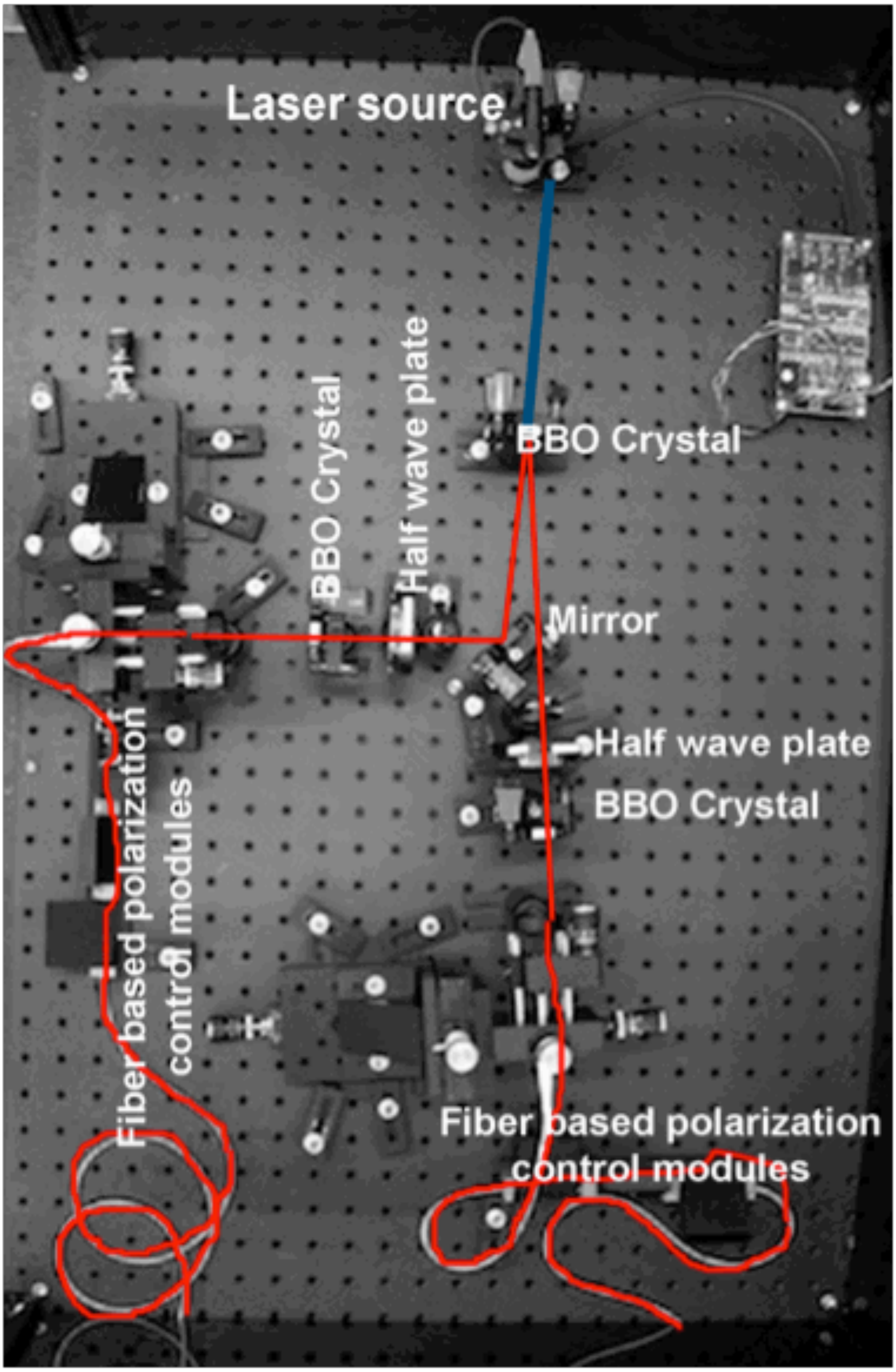
Show the theoretical exercises and the understanding exercises to your supervisor, before doing the Bell Inequality violation.

4.2.4 Bell Inequality Violation

Observe correlations, when you have Alices bases at 0 degrees and 45 degrees, and Bob's bases at 33,5 degrees and 67,5 degrees.

a. Calculate the uncertainty in your measurement, using the information obtained in exercise 1.
b. Calculate S. Is Bell's inequality violated?
c. Make two correlation curves: Keep Alices basis constant (one curve 0 degrees, one curve 45 degrees) and turn Bobs wave plate in small steps from 0 degrees to 180 degrees. Calculate E for all points and plot E versus Bob's angle for both cases in one graph. What can you conclude?

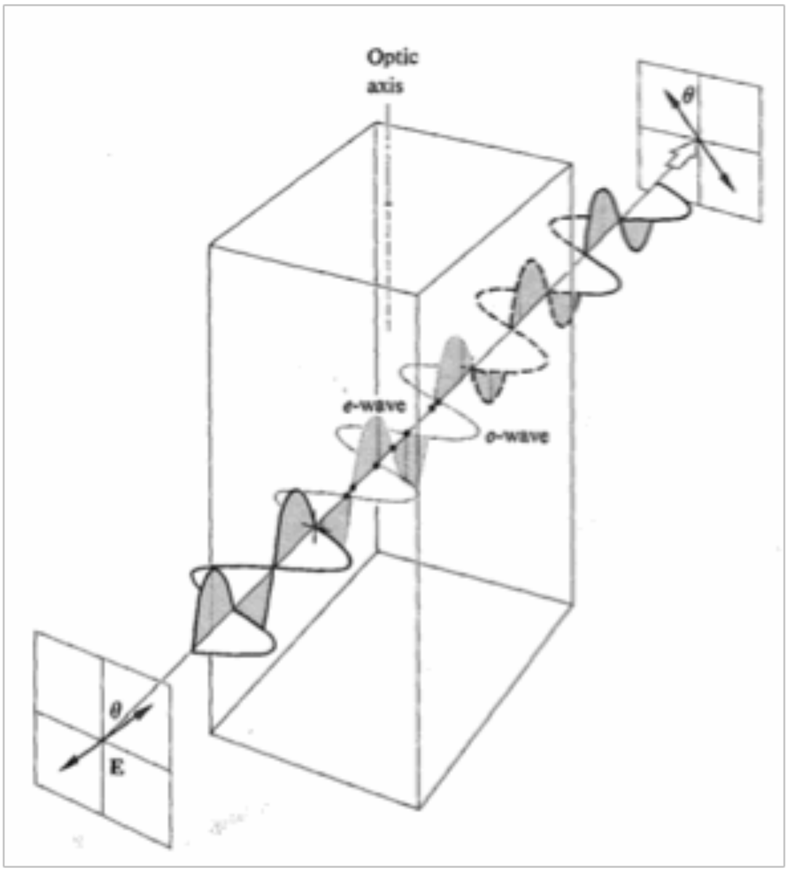
Appendix A: Setup



Appendix B

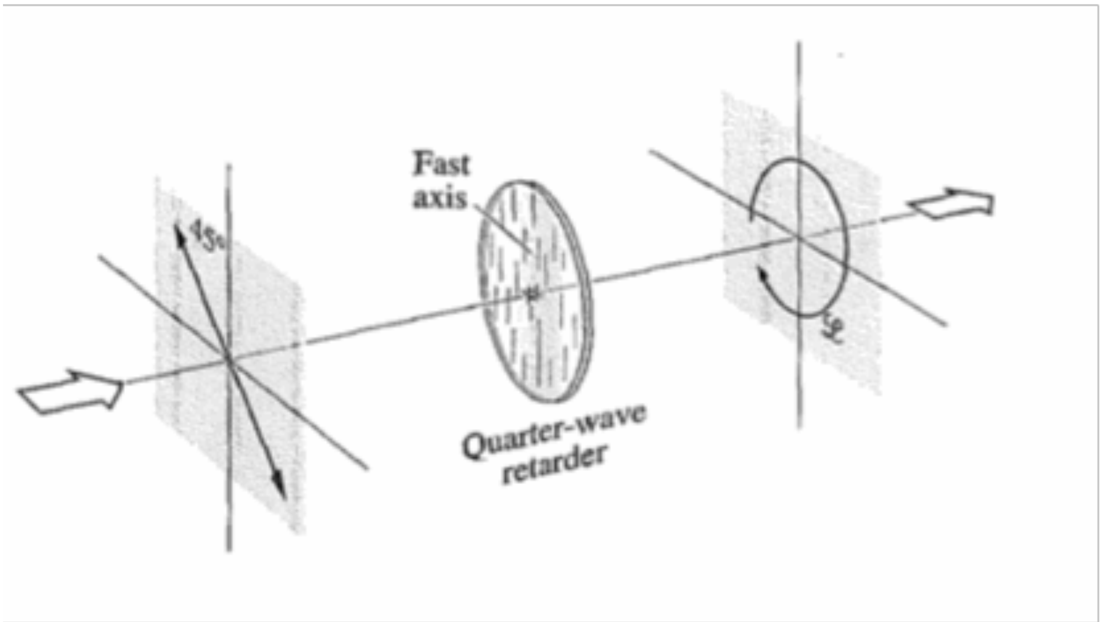
Half wave plate

$$d(|n_o - n_e|) = (2m+1)\lambda_0/2$$



Quarter wave plate

$$d(|n_o - n_e|) = (4m+1)\lambda_0/4$$



Error Analysis

The error analysis is based on the assuption that the error goes with the square root of the measured value. This fits the amplitude of the noise in the correlation count graphs quite well.
We say that:

$$error \cong \sqrt{n}$$

We furthermore know that, for high counts and negative correlations:

$$E(B_1A_1) = \frac{N_{HV} + N_{VH} - N_{HH} - N_{VV}}{N_{HH} + N_{VV} + N_{HV} + N_{VH}}$$

Where we separate

$$A = N_{HV} + N_{VH} - N_{HH} - N_{VV}$$
$$B = N_{HV} + N_{VH} + N_{HH} + N_{VV}$$

So we say

$$E = \frac{A}{B}$$

and

$$A_{error} = \sqrt{\left(N_{HV,error}\right)^2 + \left(N_{HV,error}\right)^2 + \left(N_{HV,error}\right)^2 + \left(N_{HV,error}\right)^2} = B_{error}$$

and by rules of differentiating, we state that

$$E_{error} = \sqrt{\left(\frac{A_{error}}{B}\right)^2 + \left(\frac{A}{B^2}B_{error}\right)^2}$$

In the same way the error in S can be obtained:

$$S_{error} = \sqrt{\left(E_{A1B1,error}\right)^2 + \left(N_{A1B2,error}\right)^2 + \left(N_{A2B1,error}\right)^2 + \left(N_{A2B2,error}\right)^2}$$

The standarddeviation in the errors of the correlation curves is for example obtained by using the function 'stdev' in excel.

Bibliography

[i] Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?, *A. Einstein, B. Podolsky, and N. Rosen*, Institute for Advanced Study, Princeton, New Jersey, Phys. Rev. 47, 777 - 780 (1935)
[ii] Qantum Computation and Quantum Information, *Nielsen and Chuang,* Cambridge University Press, 114-116 (2000)
[iii] New High-intensity Source of Polarization-Entangled Photon Pairs, *P.G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger,* Institüt for experimentalphysic, Universität Innsbruck, Phys. Rev. 75, 4337-4341 (1995)
[iv] A compact source for polarization entangled photons, *H.R. Böhm,* Atominstitut der Östereichischen Universitäten