Advanced Topics in Proof Theory and the Foundations of Mathematics – a PhD-level (reading) course 7.5 cp

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A leading motive in Mathematical Logic is Hilbert's programme and its later modifications. The original programme was to prove the consistency of foundational systems, like set-theory, to be consistent by finitistic means, more specifically to study proofs in such systems as finite combinatory objects, and show that no proofs of inconsistencies can occur, e.g. a proof of $1 = 0$ in the system. However the orginal aim of the programme, to prove that stronger systems are consistent by means of weaker systems, was shown to be, in general, impossible by Gödel's theorem in 1931. The programme has continued in various modified forms.

One form is *Ordinal Analysis Proof Theory,* initiated by Gerhard Gentzen, proving the consistency of Peano Arithmetic with finitistic means, extended with a single and specific infinitistic method: induction on a concretely given ordinal ε_0. Ordinal analysis aims to find similar characteristic ordinals of other foundational systems, mostly the fragments of Second Order Peano Arithmetic. Despite considerable efforts during the 20th century no satisfactory analysis has been found for the full second order theory. This is often referred to as *Hilbert's second problem.*

Another form of the programme is *Reverse Mathematics,* initiated by Harvey Friedman in the 1970s, which is to find out which axioms are necessary to prove certain fundamental theorem in mathematics, i.e. it aims to, so to say, "reverse-engineering" mathematical theorems. The background theory is often a very weak second order theory of arithmetic. A fairly robust classification into five main axiom systems has emerged.

Both these lines of developments of the orginal programme can be studied from a constructive, predicative point of view. Important parts of current

research is concentrated on this.

Prequisites: 90 cp in Mathematics, including Logic II (MM7022) and Foundations of Analysis (MM5021), or equivalent courses.

Examination: homework problems and seminar assignments. Examiner is Erik Palmgren.

Course literature:

Wohlfram Pohlers: Proof theory - The first step into the impredicative. Springer 2009.

Steven G. Simpson: Subsystems of Second-Order Arithmetics. 2nd Edition. Cambridge University Press 2010.

Research articles.

Course literature: Lectures will take place weeks 46-50, 2018 and weeks 5-9, 2019. Examinations is planned for weeks 10-12, 2019. "Reading course" means reduced number of lecture hours. There will be approximately ten 60 minutes lectures. Exact schedule to be determined — see course page.

Course page: http://staff.math.su.se/palmgren/PTFM.html