ALBEVERIO FEST

September 30 - October 1, 2018

Stockholms universitet

Organizers:
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ALBEVERIO FEST

SEPTEMBER 30 - OCTOBER 1, 2018

ROOM 14, BUILDING 5, HOUSE 5,
DEPT. OF MATHEMATICS, STOCKHOLM UNIVERSITY

Program

Sunday

13:00–13:10 Opening

13:10–13:50 Cipriani:
The emergence of Noncommutative Potential Theory d'aprés Sergio Albeverio

14:00–14:25 Figari:
Regularized Quadratic Forms for a Three boson system with Zero-Range interactions

14:30–14:55 Blanchard:
The ETH approach to QM

Coffee break

15:30–15:55 Kondratiev:
Statistical dynamics in random time

16:00-16:25 Luger:
Herglotz-Nevanlinna functions in several variables

16:30–16:55 Boman:
Radon transforms supported in hypersurfaces and a conjecture by Arnold

17:00-17:25 Streit:
About Dirichlet Forms and Other Things I learnt from Sergio

17:30–17:55 Karwowski:
Random Processes on Ultrametric Spaces

Dinner at *Stallmästeregården*

Monday

09:00–09:25 Holden:

09:30–09:55 Khrennikov:
Modeling fluid's dynamics with master equations in ultrametric spaces representing the treelike structure of network of capillaries

10:00–10:25 Daletskii:
Non-equilibrium stochastic dynamics of infinite particle systems on unbounded degree graphs

10:30–10:55 Teta:
Efimov effect in a fermionic three-particle system

Coffee break

11:30–11:55 Hilbert:
Asymptotic Behaviour in Time of a Singular Stochastic Newton Equation

12:00-12:25 Kostenko:
Infinite Quantum Graphs

12:30–12:55 Rüdiger:
The Boltzmann (-Enskog) Process

Lunch

14:00–14:25 Shapiro:
Random matrix pencils and level crossings

14:30–14:55 Karabash:
On the multilevel internal structure of the asymptotic distribution of resonances

15:00–15:25 Gottschalk:
The Cosmological Semiclassical Einstein Equation as an Infinite Dimensional Dynamical System

Coffee break

16:00-16:25 Djehiche
Quenched mass transport of particles towards a target

16:30–16:55 Rohleder:
Eigenvalue inequalities for the Laplacian with mixed boundary conditions

17:00-17:25 Ugolini:
A stochastic approach to Bose-Einstein Condensation

17:30–17:55 :

Abstracts

The ETH approach to QM

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This is a joint work with B.-Fröhlich, Nuclear Physics B 912 (2016) 463-484.

Radon transforms supported in hypersurfaces and a conjecture by Arnold

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A famous lemma in Newton's Principia says that the area of a segment of a bounded convex domain in the plane cannot depend algebraically on the parameters of the line that defines the segment. Vassiliev extended Newton's lemma to bounded convex domains in arbitrary even dimensions. In odd dimensions the volume cut out from an ellipsoid by a hyperplane depends not only algebraically but polynomially on the position of the hyperplane. Arnold conjectured in 1987 that ellipsoids in odd dimensions are the only cases in which the volume function in question is algebraic. The special case when the volume function is assumed to be polynomial has been studied in several papers and was settled very recently. Motivated by a totally different problem I recently tried to construct a compactly supported distribution $f \neq 0$ whose Radon transform is supported in the set of tangent planes to the boundary surface ∂D of a bounded convex domain $D \subset \mathbb{R}^n$. However, I found that such distributions can exist only if ∂D is an ellipsoid. This result gives a new proof of the abovementioned special case of Arnold's conjecture.

The emergence of Noncommutative Potential Theory d'aprés Sergio Albeverio

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The first aim of the talk is to describe the emergence of Noncommutative Potential Theory, motivated by the works of L. Gross in quantum field theory, represented by the works of S. Albeverio and R. Hoegh-Krohn who introduced Dirichlet forms on C*-algebras with traces and some of their basic constructions.

Then we will describe two recent development of the theory such as the first order differential calculus underlying a Dirichlet form, the notion of Multiplier of a Dirichlet space and the noncommutative Deny inequality.

Finally, we will review applications of the theory to different fields such as functional analysis (spectrum of Dirichlet forms and amenability of von Neumann algebras), probability (construction of Levy processes on Quantum Groups and in Free Probability), Riemannian geometry (curvature and the Dirac Dirichlet form) and Noncommutative Geometry (construction of Fredholm Modules and Spectral Triples).

Non-equilibrium stochastic dynamics of infinite particle systems on unbounded degree graphs

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We prove the existence and uniqueness of solutions of an infinite system of stochastic differential equations in $\mathbb{R}^\nu$, parameterized by elements x of a fixed countable set $\gamma \subset \mathbb{R}^d$, where the right-hand side of each x-equation depends on a finite but in general unbounded number n_x of variables (a row-finite system). Such systems describe in particular (non-equilibrium) dynamics of spins $q_x \in \mathbb{R}^\nu$ of a collection of particles labelled by points $x \in \gamma$. Two spins q_x and q_y are allowed to interact via a pair potential if the distance between x and y is no more than a fixed interaction radius $r > 0$. In contrast to the case where γ is a regular lattice (e.g. $\mathbb{Z}^d$), the number n_x of particles interacting with particle x can be unbounded in x. Our main example of a "growing" configuration γ is a typical realization of a Poisson (or Gibbs) point process. As a technical tool, we use a generalization of the Ovsyannikov method.

Quenched mass transport of particles towards a target

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We consider the stochastic target problem of finding the collection of initial laws of a mean-field stochastic differential equation such that we can control its evolution to ensure that it reaches a prescribed set of terminal probability distributions, at a fixed time horizon. Here, laws are considered conditionally to the path of the Brownian motion that drives the system. We establish a version of the geometric dynamic programming principle for the associated reachability sets and prove that the corresponding value function is a viscosity solution of a geometric partial differential equation. This provides a characterization of the initial masses that can be almost-surely transported towards a given target, along the paths of a stochastic differential equation.

Regularized Quadratic Forms for a Three boson system with Zero-Range interactions

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We investigate a family of zero range Hamiltonians for a system of three bosons in a Hilbert space of tensorial wave functions. The Hamiltonians are associated to quadratic forms that are proved to be bounded from below.

The Cosmological Semiclassical Einstein Equation as an Infinite Dimensional Dynamical System

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The semiclassical Einstein equation couples the expected value of the energy-momentum tensor of a quantised field to classical gravity. Even for the case of free fields on cosmological (curved) space times, the renormalization of the energy-momentum tensor is non trivial, as ordinary Wick ordering wrt a quantum state breaks general covariance of the Einstein equation. However, a generally covariant renormalization scheme is possible using some generalization of heat kernel expansions on semi-Riemannian manifolds, known as Hadamard parametrices.

In this work, we rephrase the dynamics of the cosmological, semiclassical Einstein equation in terms of renormalized quantities and develop a theory for the existence and stability of solutions. Some implications to cosmology are discussed as well.

Asymptotic Behaviour in Time of a Singular Stochastic Newton Equation

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The work discusses singular modifications of Stochastic Newton Equations, which were studied by S. Albeverio, H. H. and collaborators, see e.g. Stochastics and Stochastic Reports **39**. It was motivated by the article of Christiansen et al. [1], Phys. Rev. E **54**, who introduced a focusing formal 2D non-linear Schrödiñger equation, perturbed by a damping term, and driven by multiplicative noise. The question in focus was whether collapse of the wave function occurs, i.e. whether the width of the wave function vanishes in finite time, while its L^2 value is conserved, which means explosion of solutions. Being a formal equation only, its rigorous meaning would need further discussion in the first place. This difficulty was by-passed in [1] by introducing the following family of trial wave functions:

$$\Psi(u,t) := C\|\Psi(\cdot,0)\|_{L_2}\tfrac{1}{x(t)}f\left(\tfrac{|u|}{x(t)}\right)\exp\left[i\tfrac{\dot{x}(t)|u|^2}{4x(t)}\right],$$

where $f : \mathbf{R} \to (0,\infty)$ is a rapidly decreasing function and x is an unknown stochastic process describing the width of the corresponding non-linear wave function. In order to answer the question in focus a function f and a process x need to be specified to **i**) match the trial functions to the non-linear wave function and **ii**) study whether an appropriate x reached the origin a.s. in finite time. This work answers part **ii**), with a process x satisfying the singular stochastic Newton equation:

$$dx(t) = y\,dt, \qquad dy(t) = \frac{1}{x^\alpha}dt - \frac{\gamma}{x^{2\beta}}dt + \frac{\sqrt{2T\gamma}}{x^\beta}dW_t,$$

where the physical relevance of the constants may be found in *arXiv:1405.0151v1*. Beyond existence an uniqueness of solutions we focus on the large time asymptotics of the solutions. For the existence proof methods of Cerny and Engelbert are extended to the 2D system.

Based on joint work with S. Assing, Warwick University.

On the multilevel internal structure of the asymptotic distribution of resonances

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The goal of the talk is to show that the asymptotic distribution of resonances has a multilevel internal structure for the several classes of Hamiltonians H including Schrödinger operators with point interactions in $\mathbb{R}^3$ and non-compact quantum graphs. In the case of point interactions the set of resonances $\Sigma(H)$ essentially consists of a finite number of sequences with logarithmic asymptotics. We show how the leading parameters μ of these sequences are connected with the geometry of the set $Y = \{y_j\}_{j=1}^N$ of interaction centers $y_j \in \mathbb{R}^3$. The minimal parameter $\mu^{\min}$ corresponds to the sequences with 'more narrow' and so more observable resonances. The asymptotic density of such narrow resonances can be expressed via the multiplicity of $\mu^{\min}$, which occurs to be connected with the symmetries of Y. In the case of quantum graphs, the decomposition of $\Sigma(H)$ into a finite number of asymptotic sequences is proved under additional commensurability conditions. The obtained results and effects will be compared with those of obstacle scattering.

The talk is based on the joint paper with Sergio Albeverio (arXiv:1807.02889 [math-ph]).

Random Processes on Ultrametric Spaces

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We briefly describe 3 papers by S. Albeverio et al. and indicate some points of their influence on study of random processes on non-Archimedean spaces.

Modeling fluid's dynamics with master equations in ultrametric spaces representing the treelike structure of network of capillaries

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We present a new conceptual approach for modeling of fluid flows in random porous media based on explicit exploring of treelike geometry of complex capillary networks. Such patterns can be represented mathematically as ultrametric spaces and the dynamics of fluids by ultrametric diffusion. The images of p-adic fields, extracted from the real multiscale rock samples and from some reference images, are depicted. In this model the porous background is treated as the environment contributing to the coefficients of evolutionary equations. For simplest trees, these equations are essentially less complicated than those with fractional differential operators which are commonly applied in geological studies looking for some fractional analogs to conventional Euclidean space but with anomalous scaling and diffusion properties. It is possible to solve the former equation analytically and, in particular, to find stationary solutions. The main aim of this paper is to attract the attention of researchers working on modeling of geological processes to the novel utrametric approach and to show some examples from the petroleum reservoir static and dynamic characterization, able to integrate the p-adic approach with multifractals, thermodynamics and scaling. We also present a non-mathematician friendly review on trees and ultrametric spaces and pseudo-differential operators on such spaces.

Statistical dynamics in random time

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Statistical Markov dynamics of interacting particle systems (IPS) may be described by means of forward Kolmogorov (aka Fokker–Planck) equations for states of considered systems. Let $X(t)$ be the corresponding Markov process. For given random time process $\xi(t)$ (independent of $X(t)$) consider $Y(t) = X(\xi(t))$ and denote ν_t the distribution of $Y(t)$. There appear the following questions:

(Q_1): properties of $Y(t)$

(Q_2): is there an evolution equation for ν_t (generalized Fokker-Planck equation)?

(Q_3): explicit form of ν_t in terms of X and ξ (subordination)

(Q_4): possible effects in IPS models

As random times we use inverse subordinators and then $Y(t)$ is not more a Markov process. But, nevertheless, the time evolution of states ν_t may be described by evolution equations with generalized fractional time derivatives (or convolution-type derivatives in another terminology). We will discuss resulting ergodic properties of time changed dynamics and fractional kinetic for IPS. We show intermittency properties for kinetic statistical dynamics caused by random times. Note that the random time change is especially important for biological models where a concept of a biological time plays an essential role.

Infinite Quantum Graphs

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The notion of quantum graph refers to a graph considered as a one-dimensional simplicial complex and equipped with a differential operator ("Hamiltonian"). We will review the basic spectral properties of infinite quantum graphs (graphs having infinitely many vertices and edges). In particular, we will discuss recently discovered fruitful connections between quantum graphs and discrete Laplacians on graphs.

The talk is based on joint works with P. Exner, M. Malamud, H. Neidhardt, and N. Nicolussi.

Herglotz-Nevanlinna functions in several variables

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Functions mapping the complex upper-half plane analytically into itself are very well studied by now and play an important role at many places, both inside mathematics as well as in applications. Also their analogues in several variables show up in applications, e.g. in connection with homogenisation, however, on the mathematical side many questions are still open.

Here we give an overview on what has been done so far, with particular focus on those aspects, which have surprised us. This talk is based in joint work with Mitja Nedic.

Eigenvalue inequalities for the Laplacian with mixed boundary conditions

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Inequalities for the eigenvalues of the Laplacian subject to mixed boundary conditions on polyhedral and more general bounded domains are established. The eigenvalues subject to a Dirichlet boundary condition on a part of the boundary and a Neumann boundary condition on the remainder of the boundary are estimated in terms of either Dirichlet or Neumann eigenvalues. The results are joint work with Vladimir Lotoreichik.

The Boltzmann (-Enskog) Process

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We derive a McKean-Vlasov equation for which the solution is distributed according to the Boltzmann (-Enskog) equation. We call its solution the Boltzmann (-Enskog) process. Under suitable conditions the Existence of the Boltzmann (-Enskog) process is proven for all cases including hard spheres, soft and hard potentials.

The results are obtained in two joint works with S. Albeverio, P. Sundar, and a joint work with M. Friesen and P. Sundar.

Random matrix pencils and level crossings

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We discuss some very recent results related to the following fundamental question.

Given two random matrices A and B independently sampled in some matrix ensemble, consider the random matrix pencil $A + tB$, where t is a complex parameter. Find the distribution of the level crossings of $A+tB$ in the couple plane of parameter t, where a level crossing point is a value of t for which $A+tB$ has a multiple eigenvalue. We will explain that for a number of complex random matrix ensembles, the resulting distribution of level crossing is uniform in the standard metric of CP^1 compactifying the t-plane.

About Dirichlet Forms and Other Things I learnt from Sergio

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We review applications of Dirichlet forms in physics and present some recent results.

Efimov effect in a fermionic three-particle system

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We consider a three-particle quantum system in dimension three made of two identical fermions of mass one and a different particle of mass m. The particles interact via two-body short range potentials. We assume that the Hamiltonians of all the two-particle subsystems do not have bound states with negative energy and, moreover, that the Hamiltonians of the two subsystems made of a fermion and the different particle have a zero-energy resonance. From the physical literature it is known that, under these conditions and for $m < m^* = (13.607)^{-1}$, the Efimov effect occurs, i.e., there are infinitely many negative eigenvalues for the three-particle Hamiltonian H. We give a rigorous proof of this fact. More precisely, we prove that: i) for $m > m^*$ the number of negative eigenvalues of H is finite, ii) for $m < m^*$ the number $N(z)$ of negative eigenvalues of H below $z < 0$ has the asymptotic behaviour $N(z) \sim C(m)|\log|z||$ for $z \to 0^-$. Moreover, we give an upper and a lower bound for the positive constant $C(m)$.

A stochastic approach to Bose-Einstein Condensation

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