Exam VT 2011 23/5

1. a) Wikipedia: Electromagnetic spectrum

For IR: 1 mm – 10 μm absorbed by rotational modes in gas-phase molecules, molecular motion in liquids, phonons in solids

Mid IR: 10 μm – 2,5 μm absorbed by molecular vibrations

Near IR: 2500 nm – 750 nm valence electrons

visible: 380 nm – 760 nm –||–

UV: 10 nm – 400 nm valence electrons (inner shell below ~30 nm)

Greenhouse gas = absorb longer wavelength IR radiation – vibrations – some of which is then reradiated back to earth

need a dipole moment to absorb/emit light – need separation of different charges which mono-atomic molecules don't have ⇒ cannot absorb photons through vibrations ⇒ IR-inactive (N_2)

b) 385 μm = Far IR → rotational transitions

atom moving in 3-D has 3 degrees of freedom (translational) ⇒ N atoms have 3N in total, the center of mass can move along 3 axes & the molecule can rotate around the 3 axes (nonlinear) 2 axes (linear molecules) – no moment of inertia around axis through bond. mr^2

⇒ ~~degrees~~ Vibrational degrees of freedom: $3N-6$ (nonlinear)
$3N-5$ (linear)

we have a nonlinear molecule with $N=3$ ⇒ $3\cdot 3-6 = 3$ vibrational degrees of freedom
3 rotational degrees of freedom

c) 385 μm for IR ⇒ rotational transition, eq. 3.1.7 $E_r = BJ(J+1)$ incorrect expression in formula sheet $B = \frac{\hbar^2}{2I}$? $I = \sum_i m_i r_i^2$ distance to rotation axis through center of mass — not in formula sheet!

$\Rightarrow E_r \propto \frac{1}{I}$

$I_1 = m_O r_O + 2m_H r_H$

$I_2 = \frac{m_O}{2} r_O' + \frac{2m_H}{2} r_H' = m_O r_O' + m_H r_H'$ assume $r_O' \approx r_O \approx 0$ because O is heavier than H ⇒ center of mass should be closer to O

$m_{proton} \approx m_{neutron}$ – remove one neutron from D →

$\Rightarrow I_1 \approx 2m_H r_H$
$I_2 \approx m_H r_H'$ assume $r_H' \approx r_H \Rightarrow \boxed{I_2 \approx \frac{I_1}{2}}$

$\Rightarrow \frac{E_{r_2}}{E_{r_1}} = \frac{1/(I_1/2)}{1/I_1} = 2 \Rightarrow E_{r_2} = 2E_{r_1}$ $E = h\nu = h\frac{c}{\lambda} \Rightarrow \frac{hc}{\lambda_2} = 2\frac{hc}{\lambda_1} \rightarrow \lambda_2 = \frac{\lambda_1}{2} = 192.5$ μm

(142.5 in solutions does not follow from this! requires $\lambda_1 = 285$ μm)

2. hom. broad. = all ions/atoms' energy levels are affected the same – all have the same resonances

inhom. broad. = different ions/atoms' energy levels are affected differently – have different resonances which makes it possible for different modes to deplete different parts of the gain – this is called spectral hole burning.

Nd: Glass – amorphous – ions affected differently → inhom. ⇒ spectral hole burning
HeNe – gas ⇒ Doppler ⇒ inhom. → spectral hole burning
Nd: YAG – crystal → hom. – NO spectral hole burning

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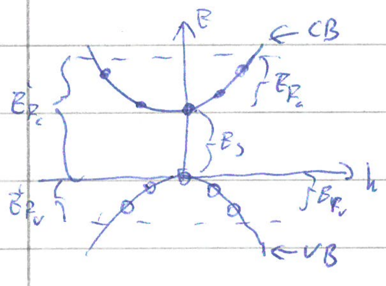
b) spatial hole burning = standing waves have peaks and nodes at different positions in the active medium ⇒ gain depletes more at some locations (around peaks) ~~[illegible]~~
– this would be present in all materials in standing wave resonators!

c) • gain ≈ 0 @ $E \approx 1.424$ eV because this is the bandgap energy, above which the population in CB will increase

• the gain is given by $g = \alpha[f_c(E_2') - f_v(E_1')]$ ← eq. 3.2.37

$f_c(E_c) = \frac{1}{1 + e^{(E_c - E_{F_c})/kT}}$ 3.2.10 a, b

$f_v(E_v) = \frac{1}{1 + e^{(E_{F_v} - E_v)/kT}}$ probabilities to find electron/hole at given energy



E_{F_v} & E_{F_c} changes with the number of injection carriers
⇒ max point in $f_c(E_c)$ & $f_v(E_v)$ changes – i.e. the gain maxima shifts at different levels of injection carriers!

• increasing the injection carriers ⇒ increases population in CB – pushes E'_{F_c} up & E'_{F_v} down

⇒ $E_g \le h\nu \le (E'_{F_c} - E'_{F_v})$ ← increases ⇒ broadens width!

↑ maximum energy difference ⇒ top of populated CB to bottom of depopulated VB
where net gain is possible

3. a) given: $\sigma_e = 2\cdot 10^{-16}\,\text{cm}^2$, $n = 1.5$, $\lambda = 600$ nm, $\Delta\nu_0 = 3$ THz, hom. broad.

eq. 2.4.18 ⇒ $\sigma_h = \frac{2\pi^2}{3n\epsilon_0 ch}|\mu|^2 \nu g(\nu - \nu_0) = \left[g(\nu-\nu_0) = \frac{2}{\pi\Delta\nu_0}\cdot\frac{1}{1+\left(2\frac{\nu-\nu_0}{\Delta\nu_0}\right)^2}\right.$ max when $\left.\nu = \nu_0 \Rightarrow \frac{2}{\pi\Delta\nu_0}\right] \Rightarrow$

↑ eq. 2.4.8

$\Rightarrow \sigma = \frac{2\pi^2}{3n\epsilon_0 ch}|\mu|^2 \nu_0 \frac{2}{\pi\Delta\nu_0}$

eq. 2.3.15 $\tau_{sp} = \frac{3h\epsilon_0 c^3}{16\pi^3\nu_0^3 n|\mu|^2} \to |\mu|^2 = \frac{3h\epsilon_0 c^3}{16\pi^3\nu_0^3 n\tau_{sp}} \Rightarrow \sigma_h = \frac{2\pi^2}{3n\epsilon_0 ch}\cdot\frac{3h\epsilon_0 c^3}{16\pi^3\nu_0^3 n\tau_{sp}}\cdot\nu_0\frac{2}{\pi\Delta\nu_0} \to$

$\Rightarrow \tau_{sp} = \frac{c^2}{4\pi^2 n^2\nu_0^2\Delta\nu_0\sigma_h} \approx 9.65$ ns

↑ $= 2\sigma_e$

fluorescence quantum yield: eq. 2.6.22 $\phi = \frac{\int (P(t)/h\nu)\,dt}{N_2(0)V} = \frac{\tau}{\tau_r}$ ← radiative lifetime $\Rightarrow \frac{1}{\tau_r} = \frac{\phi}{\tau}$

#photons emitted; lifetime; atoms initially raised to level 2

eq. 2.6.18: $\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}} = \frac{\phi}{\tau} + \frac{1}{\tau_{nr}} \to \tau_{nr} = \frac{1}{\frac{1}{\tau} - \frac{\phi}{\tau}} = \frac{\tau}{1-\phi} = \left[\frac{1}{\tau_r} = \frac{\phi}{\tau} \to \tau = \phi\tau_r,\ \tau_r = \tau_{sp}\right] = \frac{\phi\tau_{sp}}{1-\phi}$

$\Rightarrow$ 4.136 ns

b) natural linewidth: finite lifetime in level ⇒ frequency spread

4. a) modes in a plane parallel cavity consists of standing waves $L = n\frac{\lambda}{2} \Rightarrow \nu = \frac{c}{\lambda} = \frac{c}{\frac{2L}{n}}$

$\hookrightarrow \Delta\nu = \frac{c}{2L}$

single modes $\Delta\nu$ $\Delta\nu_0$ $\rightarrow \frac{\Delta\nu_0}{2} \leq \Delta\nu = \frac{c}{2L} \Rightarrow \boxed{L \leq \frac{c}{\Delta\nu_0}}$

b) $R_2 > R_1 \geq 0$ stability conditions eq. 5.4.11 $0 < g_1 g_2 < 1$ eq. 5.4.16 $g = 1 - \frac{L}{R}$

$\Rightarrow 0 < \left(1-\frac{L}{R_1}\right)\left(1-\frac{L}{R_2}\right) < 1$

① $0 < \left(1-\frac{L}{R_1}\right)\left(1-\frac{L}{R_2}\right) \rightarrow$ a) $\begin{cases} 1-\frac{L}{R_1} > 0 \\ 1-\frac{L}{R_2} > 0 \end{cases} \rightarrow \begin{matrix} L < R_1 \\ L < R_2 \end{matrix}$

$R_2 > R_1 \Rightarrow \boxed{L < R_1}$

b) $\begin{cases} 1-\frac{L}{R_1} < 0 \\ 1-\frac{L}{R_2} < 0 \end{cases} \rightarrow \begin{matrix} L > R_1 \\ L > R_2 \end{matrix}$

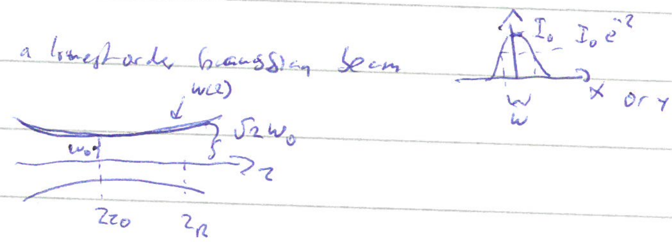
$R_2 > R_1 \Rightarrow \boxed{L > R_2}$

② $\left(1-\frac{L}{R_1}\right)\left(1-\frac{L}{R_2}\right) < 1 \rightarrow 1 - \frac{L}{R_2} - \frac{L}{R_1} + \frac{L^2}{R_1 R_2} < 1 \rightarrow$

$\hookrightarrow \frac{L}{R_1 R_2} < \frac{1}{R_2} + \frac{1}{R_1} = \frac{R_1 + R_2}{R_1 R_2} \Rightarrow \boxed{L < R_1 + R_2}$

$\Rightarrow$ $L < R_1$ for g_1 & $g_2 > 0$ or $R_2 < L < R_1 + R_2$ for g_1 & $g_2 < 0$

c) Yes it's a lowest order Gaussian beam



$I(x,y) = I_0 e^{-2\frac{x^2+y^2}{w^2(z)}}$

$w^2(z) = w_0^2\left(1 + \frac{z}{z_R}\right) \leftarrow$ eq. 4.2.13a

d) Lasers have high brightness - mostly due to great directionality, coherent light

applications - surgery/micromachining - need high intensities
- LIDar - need directionality
- measuring gravitational waves - need stable coherent source in an interferometer
- jeans processing - can control vintage looks, precise breaks...

5. a) $\frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau}$ ← eq. 7.2.16a

$\frac{dN}{dt}$: change in inversion; R_p: pump rate; $B\phi N$: stim. em.; $\frac{N}{\tau}$ ← spont. em.

$\frac{d\phi}{dt} = BV_a\phi N - \frac{\phi}{\tau_c}$ ← 7.2.16b not in formula sheet

$\frac{d\phi}{dt}$: changes in number of cavity photons; $BV_a\phi N$: stim. em.; τ_c: photon cavity life time

b) N_c when $\frac{d\phi}{dt} = 0 \rightarrow BN_c V_a \phi - \frac{\phi}{\tau_c} = 0 \rightarrow N_c = \frac{1}{BV_a\tau_c} = \left[B = \frac{\sigma L c}{V_a L_e} \quad \tau_c = \frac{L_e}{\gamma c}\right] = \frac{1}{\frac{\sigma L c}{V_a L_e} \cdot V_a \cdot \frac{L_e}{\gamma c}} = \frac{\gamma}{\sigma L}$

R_{pc} when $\frac{dN}{dt} = 0$ & $\phi = 0 \rightarrow R_{pc} - \frac{N_c}{\tau} = 0 \rightarrow R_{pc} = \frac{N_c}{\tau} = \frac{\gamma}{\sigma L \tau}$

c) steady-state $N = N_c$ additional pump energy increases the photon number, 7.3.4b: $\phi_0 = V_a \tau_c (R_p - R_{pc})$

R_p: actual pump rate; R_{pc}: critical/threshold pump rate

d) incoherent pumping eq. 6.2.5: $\eta_p = \eta_r \eta_t \eta_a \eta_{pq}$

η_r: emitted power in relevant wavelength range (can be absorbed); η_t: transfer - how much reaches the active medium; η_a: how much is absorbed; η_{pq} ← how much pump energy exceeds energy to raise electron directly to upper laser level

$\eta_{pq} = \frac{h\nu_{mp}}{h\nu_p}$

mode area

→ η_s depends on η_p, output coupling, how much the pumped region that is being used $\frac{A_b}{A}$

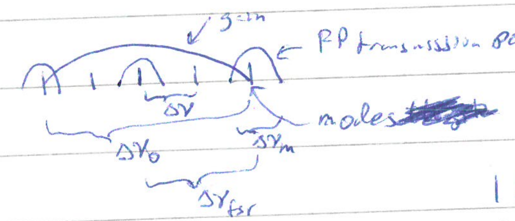
& difference in energy between pump & laser photons $\frac{h\nu_L}{h\nu_p}$

pumped area

6. a) spectral hole burning makes it possible for modes that have peaks where others have nodes to extract gain from the active medium ⇒ multi longitudinal modes! – not in ring resonators (no standing waves?)

spectral hole burning means that modes interact with different parts of the gain ⇒ multi longitudinal modes!

b)



1 mode through FP transmission peak if: $\frac{\Delta\nu_m}{2} \le \Delta\nu$ (1)

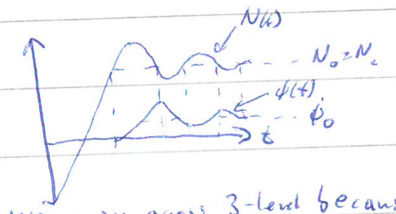
1 transmission peak under gain curve if:

$\frac{\Delta\nu_0}{2} \le \Delta\nu_{fsr} = \left[\text{eq. 4.5.12: } \Delta\nu_m = \frac{\Delta\nu_{fsr}}{F}\right] = \Delta\nu_m F$ (2) →

↳ ⇒ $\frac{\Delta\nu_0}{2F} \overset{(2)}{\le} \Delta\nu_m \overset{(1)}{\le} 2\Delta\nu$ → $\frac{\Delta\nu_0}{2F} \le 2\Delta\nu$, from task 4a $\Delta\nu = \frac{c}{2L}$ ⇒ $\frac{\Delta\nu_0}{2F} \le 2\frac{c}{2L} = \frac{c}{L}$ →

↳ ⇒ $L \le \frac{2Fc}{\Delta\nu_0}$ compared to $L \le \frac{c}{\Delta\nu_0}$ in 4a can have cavity a factor 2F longer!

c)



ϕ increases as $N \ge N_0 = N_c$ until stim. em. dominates the pump and N has decreased back to N_0 again ⇒ ϕ start decreasing and N can eventually start to recover..., N(t) leads by a half period because need inversion before stim. em. can start to build up smaller amplitudes after each period – tend towards steady-state N_0 & ϕ_0

$N(0) < 0$ in quasi 3-level because lower laser level is a part of the ground state which is populated...

d) Q-switching = modulate cavity losses by inserting a shutter into the cavity – when closed the inversion builds up far above N_c, when the shutter opens the gain is very high and the stored energy will be released as a pulse comparable to the cavity lifetime, close shutter and repeat...

Mode-locking = lock the phases of the cavity modes to form a pulse – utilize the whole gain bandwidth in inhom. materials ⇒ very short pulses

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2. a) $\lambda_1 = 750\ nm$, $\lambda_2 = 325\ nm = \frac{\lambda_1}{2}$, equal cavity losses & transition dipole moment elements

Which laser requires the highest threshold? $\boxed{\text{only spectroscopic difference is } \lambda_1 \ \& \ \lambda_2}$ $\Rightarrow$ implies the same cross-sections!

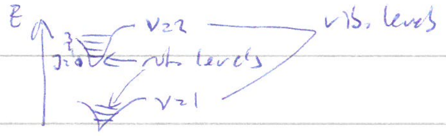
eqs. 6.3.20, 6.3.21, 6.3.25 & 7.4.4 have different expressions for P_{th} depending on: losses, pump geometry, pump wavelength, cross-sections & life times, we only need to care about pump wavelength/frequency & life times $\Rightarrow \boxed{P_{th} \propto \frac{h\nu_p}{\tau}}$

eq. 2.3.15 $\tau_{sp} = \frac{3h\varepsilon_0 c^3}{16\pi^3 \nu_0^3 n |\mu|^2} \Rightarrow \frac{P_{th2}}{P_{th1}} = \frac{h\nu_{p2} \Big/ \frac{3h\varepsilon_0 c^3}{16\pi^3 \nu_2^3 n_2 |\mu|^2}}{h\nu_{p1} \Big/ \left(\frac{3h\varepsilon_0 c^3}{16\pi^3 \nu_1^3 n_1 |\mu|^2}\right)} = \left\{ \begin{array}{l} \lambda_2 = \frac{\lambda_1}{2} \Rightarrow \nu_2 = 2\nu_1 \\ \text{assume } \nu_p \text{ scales the same} \Rightarrow \nu_{p_2} = 2\nu_{p1} \\ \& \text{ assume } n_1 \approx n_2 \end{array} \right\} =$

$\Leftrightarrow = \frac{2\nu_{p1} \cdot (2\nu_1)^3}{\nu_{p1} \cdot \nu_1^3} = 16 \Rightarrow \boxed{P_{th2} = 16 P_{th1}}$

c) we want an inhomogeneously broadened material so that different wavelengths interact with different laser active ions! - choose the fiber amplifier!

3.



eq. 3.1.7 ; $E_r = BJ(J+1)$, $J = 0, 1, 2, \ldots$

$\Rightarrow E_{tot} = E_v + E_r = E_v + BJ(J+1)$

energy absorbed because of transitions $\Delta E = E_{v_2} + E_{r_2} - (E_{v_1} + E_{r_1}) = E_{v_2} + BJ_2(J_2+1) - E_{v_1} - BJ_1(J_1+1) \Leftrightarrow$

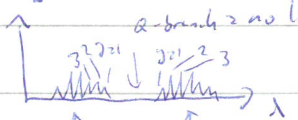
$\Leftrightarrow = \underbrace{E_{v_2} - E_{v_1}}_{\Delta E_v} + B\left[J_2^2 + J_2 - J_1^2 - J_1\right]$

selection rules : $\Delta J = J_2 - J_1 = \pm 1 \leftarrow$ electronic dipole allowed

$0 \leftarrow$ electronic dipole forbidden

if $J_1 = J$, $J_2 = J+1$, $\Delta J = 1 \leftarrow$ R-branch $\Rightarrow \Delta E = \Delta E_v + B\left[\underbrace{(J+1)^2}_{J^2+2J+1} + J + 1 - J^2 - J\right] = \Delta E_v + 2B(J+1)$

if $J_1 = J$, $J_2 = J-1$, $\Delta J = -1 \leftarrow$ P-branch $\Rightarrow \Delta E = \Delta E_v + B\left[\underbrace{(J-1)^2}_{J^2-2J+1} + J - 1 - J^2 - J\right] = \Delta E_v - 2BJ$



Q-branch = no lines because electric dipole forbidden

R-branch - shorter wavelength = higher energy

P-branch - longer λ = lower energy

$\Delta E = h\nu \leftarrow$ photon energy, difference in photon energy between 2 lines $\Delta(\Delta E) = 2B$

$B = \frac{\hbar^2}{2I} \leftarrow$ eq. 3.1.7 $\leftarrow$ note exponent 2 correct in formula sheet!

$\underbrace{\Delta E_v + 2B(1+1)}_{J=1 \text{ in R-branch}} + \underbrace{(\Delta E_v - 2B \cdot 1)}_{J=1 \text{ in P-branch}} 2 = 3\Delta E_v \rightarrow$ can estimate ΔE_v