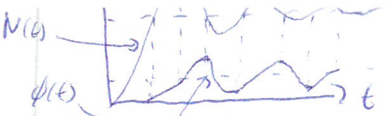
session 3



Applying a constant pump to a laser increases the inversion, once the losses are overcome at N_c photons can efficiently be generated through stimulated emission ⇒ increases the photon number ϕ and eventually dominate the pump rate ⇒ the inversion drops, once it drops below N_c ϕ will start to decrease ⇒ eventually the pump ~~increase~~ the inversion above N_c ... (losses > gain) $N(t)$ leads $\phi(t)$ by ~ half a period since there has to be an inversion exceeding the losses before the number of photons can build up

perturbing laser parameters can lead to pulsing – for instance pump modulations ⇒ osc. in output – becomes resonant if pump modulation matches relaxation oscillations!

Q-switching = change cavity loss (and hence the Q-factor) by inserting a "shutter" in the cavity. When closed there's no feedback for stim. em. ⇒ pump can achieve inversion far beyond N_c, open shutter, gain ≫ losses stored energy will be released as pulse that grows until losses match gain (which decreases due to stim. em. depopulating upper level). Fast switching (faster than the time it takes for the pulse to reach its peak value ~10–100 ns) the losses match the gain when $N = N_c$ | for slow switching the pulse grows until the losses at the current opening matches the losses – when it opens further it can give rise to additional pulses (Fig 8.4)

- Q-switched pulses are comparable to the photon life time in the cavity ~ ns
- To get high energy pulses a long upper level lifetime is needed to store the energy, if pumped by a pulse it should be shorter or comparable to the lifetime – otherwise spontaneous emission will depopulate upper level before the pump pulse ends!
- Active Q-switch usually uses acousto-optic or electro-optic modulators
- Passive Q-switch uses saturable absorber = absorbs laser light (closed) but transmits more the more it absorbs, eventually it will be "open", it recover (closes) as the inversion is emptied and the pulse dies out and the pump starts repopulate inversion, it takes many passes to bleach the absorber ⇒ gain for different modes becomes more pronounced ⇒ easier to obtain single longitudinal mode operation

chp. 8.4.4 considers active Q-switching (fast switch) for 4-level laser, assumes single mode operation such that eq. 7.2.16 can be used:

$$\frac{dN}{dt} = R_p + B\phi N - \frac{N}{\tau}$$

$$\frac{d\phi}{dt} = \left(BV_a N - \frac{1}{\tau_c}\right)\phi$$

assumes switching happens $t=0$ when the maximal inversion has been accumulated ⇔ $\frac{dN}{dt} = 0$ & $\phi = 0$ because the shutter has initially been closed
⇒ $N_i = \tau R_p(0)$ (initial inversion), assume $R_p(t)$ always looks the same ⇒ $R_p(0) \propto \int R_p(t)\,dt \propto E_p$ ← pump energy for pulsed pump ⇒ $\boxed{N_i \propto E_p}$

assume that $N(t)$ & $\phi(t)$ evolve much faster than the pump rate & spontaneous emission so:

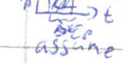
$$\frac{dN}{dt} = B\phi N$$

$$\frac{d\phi}{dt} = \left(BV_a N - \frac{1}{\tau_c}\right)\phi$$

when the switch is open ⇒ $E_{out} = \int_0^\infty P(t)\,dt = \frac{\gamma_2 c}{2L_e} h\nu \int_0^\infty \phi\,dt = \ldots = \frac{\gamma_2}{2}\frac{N_i}{N_p}\eta_E \frac{A_b}{\sigma} h\nu$ ← eq. 8.4.20

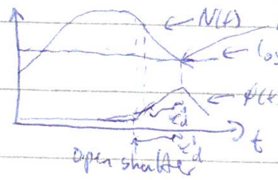
(∞: limits to where ϕ has died out; A_b: beam area; $\eta_E = \frac{N_i - N_f}{N_i}$ ← final inversion after pulse has died out; inversion utilization factor, $N_i - N_f$ = amount of inversion that goes into the pulse)

the peak of the pulse happens when the inversion has depleted until the gain = losses, i.e. at $N = N_c = N_p = \frac{\gamma}{\sigma l}$ ← eq. 8.4.9 (threshold; inversion at peak)



assume square pulse ⇒ $\Delta\tau_p = \frac{E}{P_p}$ ⇒ $\Delta\tau_p = \tau_c \frac{N_i/N_p \cdot \eta_E}{N_i/N_p - \ln(N_i/N_p) - 1}$ ← eq. 8.4.21 (cavity life time)

amount by which the threshold energy is exceeded: $x = \frac{E_p}{E_{pc}} = [E_p \propto N_i] = \frac{N_i}{N_{ic}}$ ← eq. 8.4.7 (threshold with open shutter)

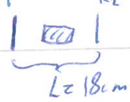


eq. 8.4.23: $\tau_d = \frac{\tau_c}{x-1}\ln\left(\frac{\phi_p}{10}\right)$ ← # photons at the pulse peak

assumes that $N(t)$ is almost unchanged from N_i, i.e. $N(t) \neq N_c$ which it is at the pulse peak, the real delay between the opening of the shutter and the pulse peak is τ_d', τ_d is the time from opening of the shutter until $\frac{1}{10}$ of the peak value!

↘

→ **8.1** $R_1 = 100\%$ $R_2 = 20\%$



flash pump Nd:YAG ↑ pulsed!

from ex. 8.4! $\sigma = 2.8 \cdot 10^{-19}\,cm^2$, $\gamma_i = 0.162$, $A_b = 0.14\,cm^2$, $L_e = 22\,cm$ ← accounts for refractive index in Nd:YAG!, $E_{pca} = 3.4\,J$ ← with $R_2 = 30\%$, $\gamma_a = 0.765$

$E_{in} = 10\,J$ threshold ↓ $E_{pcb}, E_{out}, \Delta\tau_p$?

eq. 8.4.7: $x = \frac{N_i}{N_{ic}} = \frac{E_p}{E_{pc}} \Rightarrow \frac{x_b}{x_a} = \left[N_i \propto E_{in} \text{ which is the same in both cases}\right] = \frac{N_i / N_{icb}}{N_i / N_{ica}} \left[N_c = N_p = \frac{\gamma}{\sigma L} \text{ eq. 8.4.9}\right] = \frac{\gamma_a}{\gamma_b}$ *

$\hookrightarrow = \frac{E_p/E_{pcb}}{E_p/E_{pca}} = \frac{E_{pca}}{E_{pcb}} \Rightarrow \underline{E_{pcb} = \frac{\gamma_b}{\gamma_a} E_{pca}} = \left[\gamma_b = \frac{\gamma_1}{2} + \frac{\gamma_2}{2} + \gamma_i\right] \simeq \underline{4.3\,J}$ ($\gamma_1 = 0$, $\gamma_2 = -\ln R_2$, $= 0.962$)

$\Rightarrow x_b = \frac{E_p}{E_{pcb}} \simeq 2.3$; Fig. 8.11 ($\eta_E$ vs. N_i/N_p) $\Rightarrow \eta_E(2.3) \simeq 0.85 \Rightarrow \underline{E_{out}} = \frac{\gamma_2}{2} \underbrace{\frac{N_i}{N_p}}_{\simeq x} \eta_E \frac{A_b}{\sigma} h\nu = [\text{assume } \lambda = 1.06\,\mu m] \simeq \underline{149\,mJ}$ ← eq. 8.4.20

eq. 7.7.12: $\frac{1}{\tau_c} = \frac{\gamma_{ic}}{L_e} + \frac{\gamma_1 c}{2L_e} + \frac{\gamma_2 c}{2L_e} = \frac{\gamma c}{L_e}$, $\Delta\tau_p = \tau_c \frac{N_i/N_p \cdot \eta_E}{N_i/N_p - \ln(N_i/N_p) - 1} \simeq \underline{3.2\,ns}$ (eq. 8.4.21)

$\hookrightarrow \tau_c \simeq 0.76\,ns$

CW-pumped Q-switch: when pumping between two pulses (closed shutter) N_i need to be recovered from N_f →

$\hookrightarrow$ eq. 2.2.16 $\phi = 0 \Rightarrow \frac{dN}{dt} = R_p - \frac{N}{\tau} \Rightarrow N(t) = R_p\tau - (R_p\tau - N_f)e^{-t/\tau}$ ← time from when was closed, in eq. 8.4.20 → $N(0) = N_f$

$\Rightarrow x \frac{N_p}{N_i}\left(1 - e^{-1/f'}\right) = 1 - \frac{N_f}{N_i} e^{-1/f'}$ ← eq. 8.4.31

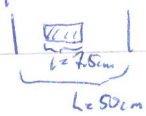
↑ amount threshold is exceeded

use Fig. 8.14

$f' = \tau f = \left[f = \frac{1}{\tau_p}\right] = \frac{\tau}{\tau_p}$

approximates close-open with rep. rate → Showe set $t = \tau_p$ ← corresponds to time between closed and opened shutter < time between two pulse peaks as shown in Fig. 8.10!

8.4 $R_1 = 100\%$ $R_2 = 85\%$



CW-pumped Nd:YAG Q-switched by AOM

from ex. 7.2: $\tau = 230\,\mu s$, $\sigma = 2.8 \cdot 10^{-19}\,cm^2$, $A_b = 0.23\,cm^2$, $\lambda = 1.06\,\mu m$, $\gamma = 0.12$, $P_{pc} = 2.2\,kW$

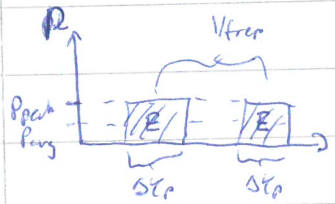
$P_{in} = 10\,kW$, $f = 10\,kHz$, $E_{out}, \Delta\tau_p, P_{peak}, P_{avg}$?

$\gamma_2 = -\ln R_2 \simeq 0.1625$

$x = \frac{P_{in}}{P_{pc}} \simeq 4.5$, Fig. 8.14 for $f' = \tau f = 2.3 \simeq 2 \Rightarrow \frac{N_i}{N_p} \simeq 1.9 \Rightarrow \eta_E = 0.75 \Rightarrow \underline{E_{out}} = \frac{\gamma_2}{2}\frac{N_i}{N_p}\eta_E \frac{A_b}{\sigma} h\nu \simeq \underline{18\,mJ}$ (Fig. 8.14, eq. 8.4.20)

$\frac{N_i}{N_p} \neq x$ for cw-pumped!

eq. 7.7.17 $\frac{1}{\tau_c} = \frac{\gamma c}{L_e}$, $L_e = L - l + nl = [n \simeq 1.8 \text{ for Nd:YAG @ } 1.06\,\mu m] = 56\,cm$ $\Rightarrow \tau_c \simeq 16\,ns$ → eq. 8.4.21 $\underline{\Delta\tau_p} = \tau_c \frac{N_i/N_p \cdot \eta_E}{N_i/N_p - \ln(N_i/N_p) - 1} \simeq \underline{86\,ns}$



$P_{peak} = \frac{E}{\Delta\tau_p} \simeq 0.2\,MW$ $P_{avg} = E \cdot f = 180\,W$

Discrepancy between exp. & theo.: * multimode osc. – different modes can have different build up times ⇒ broaden pulse; * fast switching might not be a good approx. – slow switching broadens pulse

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Gain switching – creates inversion >> threshold value by short pump pulse (comparable to build up time for cavity photons to reach maximum), can get pulses < 1 ns if gain material has short (~ns) life times

$\sin(k_m x - \omega_m t + \varphi_m)$

Mode-locking: in a cw-laser, the phases of different modes are random ⇒ intensity has a random spiky time behaviour that repeats with a period of $\tau_p = \frac{1}{\Delta\nu}$ ← freq. between cavity modes (from Fourier series), the spikes have durations of $\sim \frac{1}{\Delta\nu_L}$ ← gain bandwidth ⇒ shorter pulses for broader gain - ex. solid-state lasers

total electric fields $E(t) = \sum_{m=-n}^{n} E_m e^{j(\omega_0 + m\Delta\omega)t + j\varphi_m}$ ← center freq., mode amplitude, phase for mode m; densely packed modes (small $\Delta\omega$) & $n \to \infty$ ⟶ $E(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{E}(\omega) e^{j\varphi(\omega)} e^{j\omega t} d\omega = \mathcal{F}^{-1}\{\hat{E}(\omega) e^{j\varphi(\omega)}\}$ ($\hat{E}(\omega) \leftarrow E_m$, $\varphi(\omega) \leftarrow \varphi_m$, $\omega \leftarrow \omega_0 + m\Delta\omega$, inverse Fourier transform)

⇒ what we get in time depends on the phases between the cavity modes!

if $\varphi_m = m\varphi \Leftrightarrow \varphi(\omega) = \omega \cdot \varphi$ (constant) ⇒ $\mathcal{F}^{-1}\{\hat{E}(\omega) e^{j\varphi(\omega)}\} = E(t+\varphi)$ ← shifted time profile but the same shape as if $\varphi = 0$

for Gaussian $\hat{E}(\omega)$ ⇒ Gaussian time profile!, if $\varphi_m = m\varphi_1 + m^2\varphi_2 \Leftrightarrow \varphi(\omega) = \omega\varphi_1 + \omega^2\varphi_2 \Rightarrow E(t) \propto e^{-\alpha t^2} e^{j(\omega_0 t + \beta t^2)}$

⇒ instantaneous freq. $\omega(t) = \frac{d}{dt}(\omega_0 t + \beta t^2) = \omega_0 + 2\beta t$ ← varies with time - chirped pulse

time-bandwidth product $\Delta\nu\Delta\tau \gtrsim K$ ← depends on pulse shape $K = \begin{cases} 0.441 & \text{Gaussian} \\ 0.315 & \text{sech} \end{cases}$

FWHM of pulse in freq. & time; equal sign if the pulse is unchirped - "transform-limited"

Time domain picture: $\Delta z = c\Delta\tau = [\Delta\tau = 1\,ps] \sim 10^{-4}\,m$ << cavity length ⇒ can view it as a single pulse circulating the cavity that hits the output mirror once every repetition period τ_p, the pulse is set up by using a fast shutter that only lets the short spikes pass through

- 1 circulating pulse = fundamental mode locking
- n circulating pulses = harmonic modelocking ← τ_{22} separated by $\frac{\tau_p}{n}$ ← depends on shutter opening & placement of ~~shutter~~ shutter (see Fig. 8.19) ← only in linear cavities, not in ring cavities!

Active ML: synchronously pumped = match pump modulation to fundamental freq. of the laser - only for gain media with ns relaxation times - difficult to match very precise ⇒ ~ps pulse durations

phase modulation = modulate the refractive index with a period (equal to mode separation) somewhere in the cavity ⇒ modes coupled to each other - can be viewed as changing cavity length - i.e. position of 1 mirror so pulses can hit it at either extreme ⇒ 2 possible states that are often times jumped back and forth between ⇒ unstable

Amplitude modulation = modulate losses at freq. = mode separation ⇒ modes get coupled, pulses arriving before or after minimum loss at modulator will have their leading or trailing edges cut ⇒ approaches point of minimum loss ⇒ shorter pulse and broader spectrum - limited by gain bandwidth - the whole gain bandwidth is essentially oscillating far above threshold in inhom. materials whereas only the modes closest to the peak are oscillating in hom. materials ⇒ cannot support as broad pulses for the same gain bandwidth!

8.10 $\Delta\nu_L = 1\,GHz$, $\Delta\nu = 100\,MHz$ (mode separation), Gaussian spectrum, inhomogeneous broadening, in HeNe-laser, $\Delta\tau$?

inhom. eq. 8.6.18: $\Delta\tau = \frac{0.441}{\Delta\nu_L} = 441\,ps$ ← Gaussian

if it was a hom. material eq. 8.6.19: $\Delta\tau = \frac{0.45}{\sqrt{\nu_m \Delta\nu_L}} = [\nu_m = \Delta\nu$ to couple the modes$] \approx 1.4\,ns$ ← ν_m: modulation freq.

passive ML: additive pulse ML - 2 cavities coupled together (with equal lengths) - pulses from them meet & interfere

slow saturable absorber (SA) - leading edge of pulse saturates SA ⇒ losses < gain & trailing part of pulse saturates gain (losses > gain) (*) ⇒ pulse shortens, SA must recover before the next pulse come (within a round trip), need short lifetime active medium

(*) gain medium has higher saturation fluence than SA ⇒ more of pulse need to pass before it saturates

fast SA - low saturation intensity & relaxation time comparable to ML pulse, gain media with long lifetime >> pulse duration ns ⇒ gain doesn't change much across the pulse, give less losses for high intensity ~~losses~~ ⇒ pulsed operation, semiconductors have relaxation rates ~ few ps - some ns that help start pulsing, ~~and~~ shorter relaxation times ~ps & ~10 fs which help shortening the pulse.

→ $\sigma \propto g \propto \frac{1}{\Delta\nu_L}$ ← gain bandwidth (line shape) threshold pump power $\propto \frac{1}{\delta\tau}$ ⇒ media that can support short pulses tend to have high pump thresholds!

Kerr lens ML (KLM) - refractive index is intensity dependent, for Gaussian beam ⇒ more phase acquired along the optical axis: $\frac{2\pi}{\lambda} L n_2 I_p e^{-\frac{2r^2}{w^2}} \approx \frac{2\pi}{\lambda} L n_2 \left(1 - 2\frac{r^2}{w^2}\right)$ ← quadratic phase variation over beam acts as a positive lens if $n_2 > 0$ ⇒ self-focusing, can insert aperture in cavity - high intensity parts are focused and pass more ⇒ less losses, very fast response time in few fs

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8.16 Ti:Sa KLM, round trip losses: $2\gamma_t = 2\gamma - kP$ (*) $k = 5 \cdot 10^{-8} W^{-1}$, saturated round trip gain $2g_0' = 0.1$

$\Delta\nu_L = 100\,THz$, intracavity energy $E = 40\,nJ$, neglect SPM & dispersion $\Delta\tau$?

no SPM & dispersion use eq. 8.6.22: $\Delta\tau = \frac{0.79}{\Delta\nu_L}\sqrt{\frac{g_0'}{\gamma'}\frac{I_s}{I_p}}$ ← Saturation intensity, peak power $= [I \propto P] = \frac{0.79}{\Delta\nu_L}\sqrt{\frac{g_0'}{\gamma'}\frac{P_s}{P_p}}$

compare (*) to eq. 8.6.20: $2\gamma_t = 2\gamma - 2\gamma'\frac{I}{I_s}$ ← KLM

$2\gamma_t = 2\gamma - kP$ ⇒ $k = \frac{2\gamma'}{P_s}$ ⇒ $\Delta\tau = \frac{0.79}{\Delta\nu_L}\sqrt{\frac{2g_0'}{kP_p}}$

for sech.: $E = \int_{-\infty}^{\infty} P_p \operatorname{sech}\left(\frac{t}{t_0}\right)dt = 2P_p t_0 = \left[t_0 = \frac{\Delta t}{1.7627}\right]$ ← FWHM ⇒ $P_p = \frac{E \cdot 1.7627}{2\Delta t}$ ⇒ $\Delta t = \frac{0.79}{\Delta\nu_L}\sqrt{\frac{2g_0'}{k\frac{E \cdot 1.7627}{2\Delta\tau}}}$ →

↳ ⇒ $\Delta\tau = \left(\frac{0.79}{\Delta\nu_L}\right)^2 \frac{4g_0'}{kE \cdot 1.7627} \approx 3.5\,fs$

compare $\Delta\tau = \frac{0.315}{\Delta\nu_L} \approx 3.2\,fs$

Monochromatic waves $E = E_0 e^{j(\omega t - \beta z)}$, $\phi = \omega t - \beta z$; $\frac{d\phi}{dt} = 0$ ⇒ $\omega - \beta\frac{dz}{dt} = 0$ → $\frac{dz}{dt} = \frac{\omega}{\beta} = v_p$ ← phase velocity

when the freq. & propagation constant varies (more than 1 monochromatic wave) differentiate! $v_p\beta - \omega = 0$ →

↳ $v_g\, d\beta - d\omega = 0$ → $v_g = \frac{d\omega}{d\beta} = \frac{1}{\frac{d\beta}{d\omega}}$ ← group velocity - speed at which envelope (added waves) propagates

$GVD = \frac{d^2\beta}{d\omega^2}$ group velocity dispersion - how the inverse group velocity changes with freq. if ≠ 0 different parts of spectrum moves at different velocities ⇒ can broaden & compress, important for short pulses

the final pulse duration will depend on:

- cavity dispersion ← need to propagate wavelength different lengths to compensate for - can use prisms & gratings
- gain bandwidth
- shortening on "shutter"

Group delay dispersion = GDD = GVD · (length of propagation)

8.17 $GVD = 50\,fs^2/mm$ $\Delta\tau_1 = 10\,fs$ $\Delta\tau_2 < 1.2\,\Delta\tau_1$ ← after passing length L

use eq. 8.6.35 $\frac{\delta\tau_p}{\tau_p} = 8\ln^2 2\frac{(\phi'')^2}{\Delta\tau^4} = 8\ln^2 2\frac{GVD^2 \cdot L^2}{\Delta\tau^4}$ ⇒ $L = \sqrt{\frac{\delta\tau_p}{\tau_p}\frac{\Delta\tau^4}{8\ln^2 2\, GVD^2}} \approx 456\,\mu m < 0.5\,mm!$

ϕ'' = GDD; $\delta\tau_p = \tau_p(L) - \tau_p$; $\frac{\delta\tau_p}{\tau_p} = 0.2$; $\tau_p \propto \Delta\tau$

Quartz plate @ $\lambda = 800\,nm$ (Ti:Sa)

broadens very fast!

Cavity dumping - have no (or very low) output ⇒ high intracavity power, the output is then coupled out by increasing the output coupling - works for CW, Q-switch & ML lasers

the rep. rate is set by the dumping freq.

lower laser level, pulse duration, upper level

chap. 12.3 considers amplification of pulses, for 4 level when $\tau_1 \ll \tau_p \ll \tau \Rightarrow N_1 = 0$, assume pump & spont. emission negligible during pulse propagation & homogenous broadening $\Rightarrow \frac{\partial N}{\partial t} = -WN = -\frac{NI}{\Gamma_s}$ (1) ← eq. 12.3.

$N(z,t)$ & $I(z,t)$, pulse ~~propagation~~ amplification: $\frac{1}{c}\frac{\partial I}{\partial t} + \frac{\partial I}{\partial z} = \sigma N I - \alpha I$ (2) ← eq. 12.3.9

accounts for time variation of pulse; change per unit length; stim. em.; losses

$\Gamma_s = \frac{h\nu}{\sigma}$ ← saturation fluence ($\frac{\text{energy}}{\text{area}}$)

(1) & (2) are solved with initial condition $N(z,0) = N_0$ (upper level population before pulse – assumes equal population throughout amplifier!)

boundary condition $I(t,0) = I_0(t)$ (input pulse)

↳ neglecting loss ($\alpha = 0$) $\Rightarrow I(z,\tau) = \frac{I_0(\tau)}{1-(1-e^{-gz})\, e^{-\int_{-\infty}^{\tau} I_0(\tau')d\tau'/\Gamma_s}}$ ← eq. 12.3.10, $g = \sigma N_0$

$$\Gamma(l) = \Gamma_s \ln\left\{1 + \left(e^{\frac{\Gamma_{in}}{\Gamma_s}} - 1\right) G_0\right\}$$

$G_0 = e^{gl} = e^{\sigma N_0 l}$ = unsaturated gain of amplifier

↳ $\approx \begin{cases} G_0 \Gamma_{in}, & \Gamma_{in} \ll \Gamma_s \leftarrow \text{low saturation} \\ \Gamma_{in} + \underbrace{gl\Gamma_s}_{= N_0 l h\nu}, & \Gamma_{in} \gg \Gamma_s \leftarrow \text{deep saturation} \end{cases}$

$= N_0 l h\nu$ ← every excited atom ⇒ ~~[illegible]~~ contributes to the gain

- for Quasi 3 level & if $\tau_p \ll \tau_1$ for 4 level $\Gamma_s = \frac{h\nu}{\sigma_e + \sigma_a}$ & $G_0 = e^{\sigma_e N_0 l}$
- if G_0 is too high can lead to amplified spontaneous emission (ASE) – spontaneously emitted photons ⇒ stim. em. & parasitic lasing because of reflections from end facets of the amplifier
- Center part of beam (Gaussian) will experience more saturation than wings (i.e. less gain) ⇒ distorted profile – not as much of a problem in fibers due to the guided mode
- Leading edge of pulse can depopulate energy level so that trailing edge experiences less gain, can lead to pulse distortion – depends on the pulse shape

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<u>12.2</u> Q-switched Nd:YAG amplified, $E_{in} = 100\,\text{mJ}$, $\tau_p = 20\,\text{ns}$, $D = 6.3\,\text{mm}$ (diameter of amplifier), $G_0 = 100$, $\tau_1 \ll \tau_p$, uniform beam profile, $\sigma = 2.8\cdot 10^{-19}\,\text{cm}^2$, E_{out}, amplification, $\frac{\text{extracted energy}}{\text{stored energy}}$?

output fluence: eq 12.3.12: $\Gamma_{out} = \Gamma_s \ln\left\{1 + \left(e^{\Gamma_{in}/\Gamma_s} - 1\right)G_0\right\} = \left[\Gamma_{in} = \frac{E}{\left(\frac{D}{2}\right)^2\pi} \approx 320\,\text{mJ/cm}^2,\ \Gamma_s = \frac{h\nu}{\sigma} = [\text{assume } \lambda = 1.06\,\mu\text{m}] \approx 667\,\text{mJ/cm}^2\right]$

$= 2.76\,\text{J/cm}^2 \Rightarrow \underline{E_{out}} = \Gamma_{out}\cdot\left(\frac{D}{2}\right)^2\pi = \underline{860\,\text{mJ}}$

amplification: $\frac{E_{out}}{E_{in}} = \frac{860}{100} = \underline{8.6}$

stored energy: $\underline{E_{stored}} = \underbrace{h\nu}_{\text{photon energy}} \underbrace{N}_{\text{population}} \underbrace{V}_{\text{volume}} = \left[G_0 = e^{\sigma N l} \rightarrow Nl = \frac{\ln G_0}{\sigma}\right] = h\nu\frac{\ln G_0}{\sigma}\left(\frac{D}{2}\right)^2\pi \approx \underline{959\,\text{mJ}}$ ($\lambda = 1.06\,\mu\text{m}$)

$\Rightarrow \frac{E_{extracted}}{E_{stored}} = \frac{\overset{\leftarrow\text{out}}{860} - \overset{\leftarrow\text{in}}{100}}{959} \approx \underline{79\,\%}$

<u>12.5</u> E_{out} & amplification in 12.2 if $\tau_p \ll \tau_1$? $\Gamma_s = \frac{h\nu}{\sigma_e + \sigma_a}$, $f_{13} = 0.187$ (fractional population of lower laser level), from ex. 2.10: $\sigma^e_{23} = f_{22}\,\sigma^e_{23} = 2.8\cdot 10^{-19}\,\text{cm}^2$ ⇒ ($f_{22} = 0.4$)

(σ^e_{23} ← effective σ; σ_{23} ← σ for stimulated emission from sublevel 2 in upper to sublevel 3 in lower level)

⇔ $\sigma_{23} = \frac{\sigma^e_{23}}{0.4} = 7\cdot 10^{-19}\,\text{cm}^2$, $\sigma_a = f_{13}\,\sigma_{32} = 1.3\cdot 10^{-19}\,\text{cm}^2$ (*) $\Rightarrow \Gamma_s \approx [\lambda = 1.06\,\mu\text{m}] = 0.14\,\text{J/cm}^2$

⇒ eq. 12.3.12: $\Gamma_{out} = 2.11\,\text{J/cm}^2 \Rightarrow \underline{E_{out} = 0.66\,\text{J}} \Rightarrow \frac{E_{out}}{E_{in}} = \frac{0.66}{0.1} = \underline{6.6}$

(*) assume that upper level 2 & lower level 3 have the same degeneracy ⇒ $\sigma_{23} = \sigma_{32}$ (σ_{23} ← emission, σ_{32} ← absorption)