Session 2

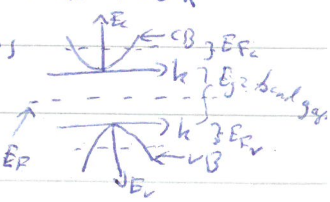
Background - recap

Energy levels of individual atoms form bands in solids when they are close enough to interact, the bands for the outer electrons are called valance band (VB) and conduction band (CB). For insulators, VB & CB are separated by a big energy gap; in metals VB overlaps CB and in semiconductors they are separated by a small energy gap which can be bridged by thermal or other excitations.

The electron populations follow Fermi-Dirac statistics: $f(E) = \frac{1}{1+e^{(E-E_f)/kT}}$ ← probability to occupy energy E

E_f = Fermi level is the energy below which all electrons would reside at T = 0 K

The energies in CB and VB are: $\begin{cases} \overset{CB}{E_c} = \frac{\hbar^2 k^2}{2m_c} \leftarrow \text{eq. 3.2.2 a} \\ \overset{VB}{E_v} = \frac{\hbar^2 k^2}{2m_v} \leftarrow \text{eq 3.2.2 b} \end{cases}$ effective electron masses

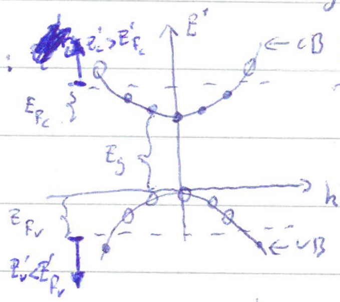


From Boltzmann statistics, we know that there will not be a population inversion between CB and VB at thermal equilibrium (since $E_c > E_v$) ⇒ need pumping

intraband relaxations (electron-phonon collisions) ~ps ≪ interband relaxations (electron-hole recombination) ~ns ⇒ intraband thermal equilibrium when pumping ⇒ we define quasi-Fermi levels, E_{F_c} & E_{F_v} above/below which states are vacant/occupied at a given pump level at T = 0 K

depend on pumping!

Drawing CB & VB with the same axes:



$E_c' = E_c + E_g$

$E_v' = -E_v$

$E_{F_c}' = E_{F_c} + E_g$

$E_{F_v}' = -E_{F_v}$

primed quantities refer to the coordinate system where CB & VB are drawn together

3.7 eq. 3.2.11 a, b: $f_c(E_c') = \frac{1}{1+e^{(E_c'-E_{F_c}')/kT}}$, $f_v(E_v') = \frac{1}{1+e^{(E_v'-E_{F_v}')/kT}}$

$f_c(E_2') \geq f_v(E_1') \Rightarrow \frac{1}{1+e^{(E_2'-E_{F_c}')/kT}} \geq \frac{1}{1+e^{(E_1'-E_{F_v}')/kT}} \Rightarrow e^{(E_1'-E_{F_v}')/kT} \geq e^{(E_2'-E_{F_c}')/kT} \rightarrow$

$\Leftrightarrow \frac{E_1'-E_{F_v}'}{kT} \geq \frac{E_2'-E_{F_c}'}{kT} \Rightarrow E_{F_c}'-E_{F_v}' \geq E_2'-E_1' = h\nu$ ← photon energy cannot be greater than the energy difference of the population inversion!

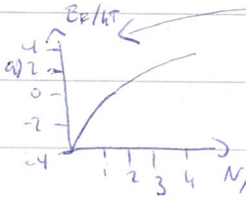
more likely to find electron in state with E_2' than E_1' - inversion between these states

for T > 0 K: $f_{c,v}(E')$: $E' < E_{F_{c,v}}'$ > 1/2, $E' = E_{F_{c,v}}'$ = 1/2, $E' > E_{F_{c,v}}'$ < 1/2

this part can be understood from (X) as $E_{F_c}'-E_{F_v}'$ is the greatest energy difference where there is population inversion! at T = 0 K

cannot have inversion between states of greater energy difference for T > 0 K

3.8 Fig. 3.15 a) ← plot of eq. 3.2.15: $\frac{N}{N_c} = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{\sqrt{\varepsilon}\, d\varepsilon}{1+e^{\varepsilon-\varepsilon_f}}$, $\varepsilon_f = \frac{E_{F_c}}{kT}$



↖ quasi-Fermi energy at different levels of CB population (injection carriers)

b) is adapted for GaAs, from the figure ⇒ $\frac{E_{F_c}}{kT} = 2.35$ (electrons), $\frac{E_{F_v}}{kT} = -1.45$ (holes) for $N = 1.6 \cdot 10^{18}\,cm^{-3}$

$\Rightarrow E_{F_c}' - E_{F_v}' = E_{F_c} + E_g - (-E_{F_v}) = E_g + E_{F_c} + E_{F_v} = E_g + 0.9kT$

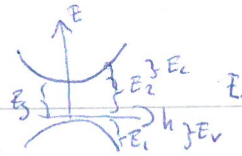
$E_g \leq h\nu \leq E_g + 0.9kT \Rightarrow \underline{\Delta E} = 0.9kT = \begin{bmatrix} k = 1.38 \cdot 10^{-23}\,J/K \\ T = 300\,K \end{bmatrix} \approx 3.7 \cdot 10^{-21}\,J$

E_g in GaAs is 1.424 eV = $1.424 \cdot 1.602 \cdot 10^{-19}\,J$ ($\approx 2.3 \cdot 10^{-19}\,J$)

$E_{photon} = h\nu = h\frac{c}{\lambda} \rightarrow \lambda = \frac{hc}{E} \Rightarrow \lambda_1 = \frac{hc}{E_g} \approx 871\,nm$, $\lambda_2 = \frac{hc}{E_g + \Delta E} \approx 857\,nm \Rightarrow \underline{\Delta\lambda} = 14\,nm$

$\underline{\Delta\nu} = \left| \frac{1}{871 \cdot 10^{-9} \cdot 10^{2}} - \frac{1}{857 \cdot 10^{-9} \cdot 10^{2}} \right| \approx 188\,cm^{-1}$

3.9



$E_2 = E_c + E_g$, $E_1 = -E_v$ $\Rightarrow$ $E_2 - E_1 = E_g = 0.45kT \Leftrightarrow E_c + E_v = 0.45kT$ (1)

eq. 3.2.2 a,b $\Rightarrow$ $\frac{E_c}{E_v} = \frac{\frac{\hbar^2k^2}{2m_c}}{\frac{\hbar^2k^2}{2m_v}} = \frac{m_v}{m_c} = \left[\text{table 3.1 GaAs } \frac{m_c}{m_0} = 0.067,\ \frac{\overbrace{m_{hh}}^{=m_v}}{m_0} = 0.46\right] \Rightarrow$

$\Leftrightarrow \frac{0.46}{0.067} \Rightarrow E_c = \frac{0.46}{0.067} E_v \rightarrow$

$\Leftrightarrow (1): E_v\left(1 + \frac{0.46}{0.067}\right) = 0.45kT \Rightarrow \overbrace{\frac{0.45}{1+\frac{0.46}{0.067}}}^{E_{v2}} kT = 0.0572\,kT \approx 2.4\cdot10^{-22}\,\text{J} = 1.5\,\text{meV}$

$E_c = \frac{0.45}{1+\frac{0.46}{0.067}} \cdot \frac{0.46}{0.067} kT = 0.3928\,kT \approx 1.6\cdot10^{-21}\,\text{J} = 10.2\,\text{meV}$ divide by $1.602\cdot10^{-19}\,\frac{\text{J}}{\text{eV}}$

3.10

eq. 3.2.24: $\overset{\text{absorption}}{\alpha} = \alpha_0\left[f_v(E_1') - f_c(E_2')\right] = \alpha_0\left\{\overbrace{\underbrace{f_v(E_1')}_{\text{probability of occupied VB-state}}\underbrace{[1-f_c(E_2')]}_{\text{probability of vacant CB state}}}^{\text{absorption}} - \overbrace{f_c(E_2')[1-f_v(E_1')]}^{\text{stimulated emission}}\right\}$

following example 3.6 we get $\alpha_0 = 19760\sqrt{h\nu - E_g}$ [cm^{-1}]

$h\nu - E_g = 0.45kT \approx 12\,\text{meV}$, in Fig 3.16 $\Rightarrow$ $\alpha_0 = 1.8\cdot10^3\,\text{cm}^{-1}$

eq. 3.2.10 a,b: $\begin{cases} f_c(E_c) = \frac{1}{1+e^{(E_c-E_{F_c})/kT}} \\ f_v(E_v) = \frac{1}{1+e^{(E_{F_v}-E_v)/kT}} \end{cases}$ from 3.8: $\begin{cases} E_{F_c} = 2.35kT \\ E_{F_v} = -1.45kT \end{cases}$ from 3.9: $\begin{cases} E_c = 0.0572kT \\ E_v = 0.3928kT \end{cases}$

non-primed but gives the same values as 3.2.11 a,b

$\Rightarrow f_c = \frac{1}{1+e^{0.0572-2.35}}$ (≈ 0.877)

$f_v = \frac{1}{1+e^{-1.45-0.3928}}$ (≈ 0.8186)

gain = - absorption $\Rightarrow$ $g = \alpha_0(f_c - f_v) \approx 104\,\text{cm}^{-1}$

4.4

given: $\Delta\nu_{fsr} = 3\cdot10^9\,\text{Hz}$, $\Delta\nu_c = 60\,\text{MHz}$, determine L, F, R & A if peak transmission is 50%

eq. 4.5.8: $\Delta\nu_{fsr} = \frac{c}{2L} \Rightarrow L = \frac{c}{2\Delta\nu_{fsr}} = 0.05\,\text{m}$

eq. 4.5.13: $F = \frac{\Delta\nu_{fsr}}{\Delta\nu_c} = 50$

eq. 4.5.14: $F = \frac{\pi(R_1R_2)^{1/4}}{1-R_1R_2} = [R_1 = R_2 = R] = \frac{\pi\sqrt{R}}{1-R} \rightarrow 1 - 2R + R^2 = \frac{\pi^2}{F^2}R \rightarrow R^2 - \left(2 + \frac{\pi^2}{F^2}\right)R + 1 = 0 \rightarrow$

$\Leftrightarrow R = \frac{2+\frac{\pi^2}{F^2}}{2} \pm \sqrt{\left(\frac{2+\frac{\pi^2}{F^2}}{2}\right)^2 - 1} \approx 1.002 \pm 0.063$ $\quad R \leq 1 \Rightarrow R \approx 0.939$

eq. 4.5.14a: $T_{FP} = \left(\frac{\overbrace{1-R-A}^{T}}{1-R}\right)^2 \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2\phi}$, $\max(T_{FP})$ when $\sin^2\phi = 0$ $\uparrow$ peak transmission $\rightarrow$

$\Leftrightarrow \frac{1}{2} = \left(\frac{1-R-A}{1-R}\right)^2 \rightarrow \frac{1-R}{\sqrt{2}} = 1 - R - A \rightarrow A = (1-R)\left(-\frac{1}{\sqrt{2}} + 1\right) \approx 0.018 = 1.8\%$

4.8

$P = \int I\,dA$, $\overset{\text{always true}}{I \propto |E|^2}$, eq. 4.7.4 & the text below $\Rightarrow$ $I = I_{max}e^{-2\frac{x^2+y^2}{w^2}}$, use polar coordinates: $x^2+y^2 = r^2$, $dx\,dy = r\,dr\,d\theta$

$\Rightarrow P = \int_{\theta=0}^{2\pi}\int_{r=0}^{\infty} I_{max}e^{-\frac{2r^2}{w^2}}r\,dr\,d\theta = \left[s = \frac{2r^2}{w^2} \Rightarrow ds = \frac{4r}{w^2}dr,\ s: 0 \rightarrow \infty\right] = [\theta]_0^{2\pi}\int I_{max}e^{-s}ds\cdot\frac{w^2}{4} = \frac{I_{max}w^2\pi}{2}\left[-e^{-s}\right]_0^{\infty} = \Rightarrow$

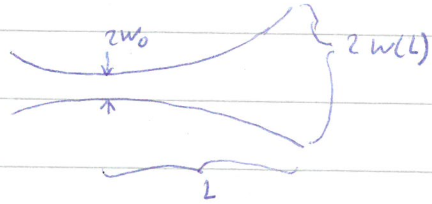
$\Leftrightarrow = \frac{I_{max}w^2\pi}{2}$

$\rightarrow$

↘

4.10 given: pure Gaussian TEM_{00}, $\lambda = 514.5\,nm$, $P = 1W$, $w_0 = 2mm$, $L = 100m$

determine: $w(L)$, $R(L)$, $I_{max}(L)$



eq. 4.7.13 a: $w(L) = w_0\sqrt{1+\left(\frac{\lambda L}{\pi w_0^2}\right)^2} \approx 8.4mm$

eq. 4.7.13 b: $R(z) = z\left[1+\left(\frac{\pi w_0^2}{\lambda z}\right)^2\right] \approx 106m$

from 4.8 $I_{max}(L) = \frac{2P}{\pi w^2(L)} \approx 9\,kW/m^2$

Remarks about Gaussian beams:
- The formulas above apply to perfect Gaussian beams, in practice there are usually some imperfections causing the laser beams to not be ~~Gaussian~~ perfect Gaussians. For these beams there is a parameter called M^2 that quantifies how much the beam deviates from a perfect Gaussian ($M^2 = 1$), for these beams the equations above can be used if λ is replaced by $M^2\lambda$
- other times the parameters $\frac{\lambda}{\pi w_0^2}$ are called z_R = Rayleigh range and quantifies the distance from the beam waist where the beam radius has increased by a factor of $\sqrt{2}$ ⇒ twice the size of the crossectional area
- Some times people talk about a confocal parameter, ~~which~~ is defined as $b = 2z_R$

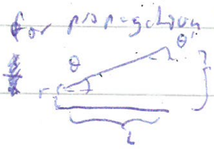
4.13 complex beam parameter: $\frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{\pi w^2}$ ← eq. 4.7.8 transforms as $q = \frac{Aq_1 + B}{Cq_1 + D}$ after passing through element in table 4.1

For free-space propagation: $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ ($L \to L/n$) 3 2 1 ← order of encountering elements

For free space → block → free space we have: $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = (\text{vacuum} \leftarrow \text{block})(\text{block})(\text{block} \leftarrow \text{vacuum}) \Rightarrow$

$\Rightarrow$ [vacuum ⇄ block use spherical dielectric interface with $R = \infty$ i.e. plane interface $\begin{pmatrix} 1 & 0 \\ \frac{n_2 - n_1}{n_2}\frac{1}{R} & \frac{n_1}{n_2} \end{pmatrix}$] ⇒

$\Rightarrow$ [for propagation in a medium with constant refractive index:



$r' = r + L\theta$, $\theta' = 0 \cdot r + \theta$ → $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ (XX)] $= \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix}\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{n} \end{pmatrix} \Rightarrow$

↑ from n to vacuum ↑ from vacuum to n

$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix}\begin{pmatrix} 1 & L/n \\ 0 & 1/n \end{pmatrix} = \begin{pmatrix} 1 & L/n \\ 0 & 1 \end{pmatrix}$ which is the same matrix as (XX) if $L \to \frac{L}{n}$

since these matrices are equal, they will give the same q-parameters and hence the same spot size and radius of curvature!

↘

→ Comments about Fabry-Perots:

- They are commonly used to resolve fine spectral details by measuring the transmission while changing the mirror separation. Changing the separation obviously changes the resonance condition, which is why different wavelengths are transmitted, but also changes $\Delta\nu_{fsr} = \frac{c}{2L}$ & $\Delta\nu_c = \frac{c}{2L}\frac{1-\sqrt{R_1R_2}}{\pi(R_1R_2)^{1/4}}$, for $L=1cm$ and changing L by $\sim 1\mu m$ (order of the wavelength) $\Rightarrow \frac{\frac{c}{2L}-\frac{c}{2(L+\Delta L)}}{\frac{c}{2L}} \approx 10^{-4}$ and is thus not that much affected.
- To resolve a laser spectrum with a Fabry-Perot, $\Delta\nu_{fsr}$ has to be greater than the lasers bandwidth - otherwise there will be ambiguities as parts of the spectra will be transmitted through different transmission peaks!
- The resolving power: $\frac{\nu}{\Delta\nu_c} = \frac{\frac{c}{\lambda}}{\frac{c}{2L}\frac{1-\sqrt{R_1R_2}}{\pi(R_1R_2)^{1/4}}} = [R_1=R_2] = \frac{2L\pi\sqrt{R}}{\lambda(1-R)}$

  for $L=2cm$ $\lambda=1\mu m$ $R=0.99 \Rightarrow \frac{\nu}{\Delta\nu_c} \sim 10^7$ which is greater than good grating spectrometers which have $\frac{\nu}{\Delta\nu_c} \sim 10^6$!

<u>ex. 4.6</u> Gaussian beam with w_0 @ plane wavefront $\rightarrow R = \infty \Rightarrow \frac{1}{q} = \frac{1}{R} - j\frac{\lambda}{\pi w_0^2} = -j\frac{\lambda}{\pi w_0^2} = \left[z_R = \frac{\pi w_0^2}{\lambda}\right] =$ (eq. 4.2.8)

$\hookrightarrow = -\frac{j}{z_{R_0}}$

focused through lens:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix}}_{\text{propagation}} \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}}_{\text{thin lens}} = \begin{pmatrix} 1-\frac{z}{f} & z \\ -\frac{1}{f} & 1 \end{pmatrix} \rightarrow \overset{\text{eq. 4.7.4}}{q = \frac{Aq_1+B}{Cq_1+D}} \rightarrow \frac{1}{q} = \frac{C + D/q_1}{A + B/q_1} = \frac{-\frac{1}{f} - \frac{j}{z_{R_0}}}{1-\frac{z}{f} - j\frac{z}{z_{R_0}}} \cdot \frac{1-\frac{z}{f} + j\frac{z}{z_{R_0}}}{1-\frac{z}{f} + j\frac{z}{z_{R_0}}}$$

$$\hookrightarrow = \frac{1}{\left(1-\frac{z}{f}\right)^2 + \frac{z^2}{z_{R_0}^2}}\left[-\frac{1}{f}\left(1-\frac{z}{f}\right) + \frac{z}{z_{R_0}^2} - j\frac{1}{z_{R_0}}\right]$$

Smallest beam waist when $\frac{1}{q} = \overset{R=\infty}{\frac{1}{R}} - j\frac{\lambda}{\pi w^2} = -j\frac{\lambda}{\pi w^2} = \frac{1}{\left(1-\frac{z}{f}\right)^2 + \frac{z^2}{z_{R_0}^2}}\left[\underbrace{-\frac{1}{f}\left(1-\frac{z}{f}\right) + \frac{z}{z_{R_0}^2}}_{=0} - j\frac{1}{z_{R_0}}\right] \rightarrow$

$$\hookrightarrow \frac{\lambda}{\pi w^2} = \frac{\frac{\lambda}{\pi w_0^2}}{\left(1-\frac{z}{f}\right)^2 + \frac{z^2}{z_{R_0}^2}} \rightarrow w = w_0\sqrt{\left(1-\frac{z}{f}\right)^2 + \frac{z^2}{z_{R_0}^2}} \qquad w' = 0: \; 0 = \frac{w_0}{2}\frac{2\left(1-\frac{z}{f}\right)\left(-\frac{1}{f}\right) + \frac{2z}{z_{R_0}^2}}{\sqrt{\left(1-\frac{z}{f}\right)^2 + \frac{z^2}{z_{R_0}^2}}} \rightarrow$$

$$\hookrightarrow z\left(\frac{1}{f^2} + \frac{1}{z_{R_0}^2}\right) = \frac{1}{f} \rightarrow \underline{z_m} = \frac{1/f}{1/f^2 + 1/z_{R_0}^2} = \underline{\frac{f}{1 + f^2/z_{R_0}^2}} < f! \qquad z_{R_0} \gg f \Rightarrow z_m \rightarrow f$$

$$\Rightarrow w_m = w_0\sqrt{\underbrace{\left(1 - \frac{1}{f}\cdot\frac{f}{1+f^2/z_{R_0}^2}\right)}_{=\frac{1+f^2/z_{R_0}^2 - 1}{1+f^2/z_{R_0}^2}}^2 + \frac{1}{z_{R_0}^2}\cdot\left(\frac{f}{1+\frac{f^2}{z_{R_0}^2}}\right)^2} = \sqrt{\frac{\left(f^2/z_{R_0}^2\right)^2 + f^2/z_{R_0}^2}{\left(1+f^2/z_{R_0}^2\right)^2}} = w_0\frac{f}{z_R}\frac{1}{\sqrt{1+\frac{f^2}{z_{R_0}^2}}} =$$

$$\hookrightarrow = \frac{\lambda f}{\pi w_0}\frac{1}{\sqrt{1+\frac{f^2}{z_{R_0}^2}}}$$

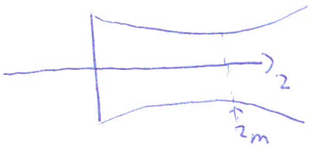
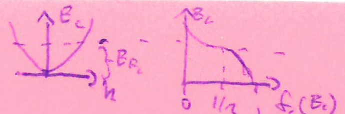


Fig. 3.14



E_{F_c} shows where the "mid point" is located, electrons can populate states above and below this point in the "tails" when T > 0K, if T = 0K they reside up to the E_{F_c}.
If $E_{F_c} < 0$, only electrons in the upper tail with energies above the band edge are found within the band. At T = 0K they would drop out.