8.1. For the Nd:YAG Q-switched laser considered in Fig. 8.12, calculate the expected threshold energy, output energy, and pulse duration, for $E_{in} = 10$ J, when output coupling is reduced to 20%.

8.4. The Nd:YAG laser in Figs. 7.4 and 7.5 is pumped at a level of $P_{in} = 10$ kW and repetitively Q-switched at a 10-kHz repetition rate by an acoustooptic modulator (whose insertion losses are assumed negligible). Calculate the energy and duration of the output pulses as well as the peak and average powers expected for this case.

8.10. The oscillation bandwidth (FWHM) of a mode-locked He-Ne laser is 1 GHz, the spacing between consecutive modes is 150 MHz, and the spectral envelope can be approximately described by a Gaussian function. For fundamental mode locking, calculate the corresponding duration of the output pulses and the pulse repetition rate.

8.16. Consider a Kerr lens mode-locked Ti:sapphire laser; according to Eq. (8.6.20), assume that total round-trip cavity losses can be written as $2\gamma_t = 2\gamma - kP$, where P is the peak intracavity laser power and the nonlinear loss coefficient k, due to the Kerr lens mode-locking mechanism, can be taken as $\approx 5 \times 10^{-8}\ \mathrm{W}^{-1}$. Assume a saturated round-trip gain of $2g_0' \cong 0.1$, a gain bandwidth of 100 THz, and an intracavity laser energy of $W = 40$ nJ. Calculate the pulse duration achievable in the limiting case where the effects of cavity dispersion and self-phase modulation can be neglected.

8.17. Assuming a GVD for quartz at $\lambda \cong 800$ nm of 50 $\mathrm{fs}^2/\mathrm{mm}$, calculate the maximum thickness of a quartz plane that an initially unchirped 10-fs pulse, of Gaussian intensity profile, can traverse if the output pulse duration is not to exceed input pulse duration by more than 20%.

12.2. The output of a Q-switched Nd:YAG laser ($E = 100\,\mathrm{mJ}$, $\tau_p = 20\,\mathrm{ns}$) is amplified by a 6.3-mm diameter Nd:YAG amplifier having a small signal gain of $G_0 = 100$. Assume that: (a) The lifetime of the lower level of the transition is much shorter than τ_p; (b) the beam transverse intensity profile is uniform; (c) the effective peak cross section for stimulated emission is $\sigma \cong 2.8 \times 10^{-19}\,\mathrm{cm}^2$. Calculate the energy of the amplified pulse, the corresponding amplification, and the fraction of the energy stored in the amplifier that is extracted by the incident pulse.

12.5. Referring to Problem 12.2., assume now that the input pulse duration is much shorter than the lifetime τ_1 of the lower laser level ($\tau_1 \cong 100\,\mathrm{ps}$). Using data obtained in Example 2.10 and knowing that the fractional population of the lower laser sublevel of the $^4I_{11/2}$ state is $f_{13} \cong 0.187$, calculate the energy of the amplified pulse and the corresponding amplification. Compare results with those obtained in Problem 12.2.

Example 8.4. *Output energy, pulse duration, and pulse buildup time in a typical Q-switched Nd:YAG laser.* Figure 8.12 shows a typical plot of laser output energy E versus input energy E_p to the flash lamp for a Q-switched Nd:YAG laser. Rod and cavity dimensions are also indicated in the figure inset.[13] The laser is operated in a pulsed regime, and Q-switched by a KD*P (deuterated potassium dihydrogen phosphate, i.e., KD_2PO_4) Pockels cell. From the figure we observe that the laser has a threshold energy $E_{cp} \cong 3.4$ J, and it gives, e.g., an output energy $E \cong 120$ mJ for $E_p \cong 10$ J. At this value of pump energy, the laser pulse width is found experimentally to be ~ 6 ns.

We now compare these experimental results with those predicted from previous equations. We neglect mirror absorption and so put $\gamma_2 \cong -\ln R_2 = 1.2$ and $\gamma_1 \cong 0$. Internal losses of the polarizer-Pockels-cell combination are estimated to be $L_i \cong 15\%$, while, in comparison, internal losses of the rod can be neglected. We thus get $\gamma_i = -\ln(1 - L_i) \cong 0.162$ and $\gamma = [(\gamma_1 + \gamma_2)/2] + \gamma_i = 0.762$. The predicted value of laser energy at $E_p = 10$ J is obtained from Eq. (8.4.20) once we observe that, for our case, $(N_i/N_p) = (E_p/E_{cp}) = 2.9$. We now assume $A_b \cong A = 0.19\ \text{cm}^2$, where A is the cross-sectional area of the rod. Since $(N_i/N_p) = 2.9$, we find from Fig. 8.11 that $\eta_E \cong 0.94$. From Eq. (8.4.20), assuming an effective value of the stimulated emission cross section of $\sigma = 2.8 \times 10^{-19}\ \text{cm}^2$ (see Example 2.10), we obtain $E \cong 200$ mJ. The somewhat larger value predicted by the theory can be attributed to two factors: (1) The area of the beam is smaller than that of the rod. (2) Due to the short cavity length, the condition for fast switching, namely that switching time is much shorter than the buildup time of the laser pulse, may not be well-satisfied in our case. Later in this example we show, in fact, that the predicted buildup time for the Q-switched pulse τ_d is about 20 ns. It is difficult to switch the Pockels cell in a much shorter time than this; as a consequence some energy is lost through the polarizer during the switching process. (In some cases, with pulses of this short a duration, as much as 20% of the output energy is switched out of the cavity by the polarizer during the Q-switching process).

To calculate the predicted pulse duration we observe that, according to Eq. (7.2.11), the effective resonator length is $L_e = L + (n-1)l \cong 22$ cm, where $n \cong 1.83$ for Nd:YAG; thus from Eq. (7.2.14) we obtain $\tau_c = L_e/c\gamma \cong 1$ ns. The laser pulse width is obtained from Eq. (8.4.21) as $\Delta\tau_p = \tau_c \eta_E x/[x - \ln(x-1)] \cong 3.3$ ns, where Fig. 8.11 is used to calculate η_E. The discrepancy between this value and the experimental value $\Delta\tau_p = 6$ ns is attributed to two factors: (1) Multimode oscillation. In fact the buildup time is expected to differ for different modes due to their slightly different gain, and this should appreciably broaden the pulse duration. (2) As already mentioned, the condition for fast switching may not be completely satisfied in our case, so pulse width is expected to be somewhat broadened by slow switching.

The buildup time for the Q-switched pulse is obtained from Eq. (8.4.23) once ϕ_p is known. If we take $N_p = \gamma/\sigma l \cong 5.44 \times 10^{17}\ \text{cm}^{-3}$ and assume $V_a = A_b l \cong Al \cong 1\ \text{cm}^3$, from Eq. (8.4.14) we obtain $\phi_p \cong 4.54 \times 10^{17}$ photons. Then from Eq. (8.4.23) with $\tau_c = 1$ ns and $x = 2.9$, we obtain $\tau_d \cong 20$ ns.

Example 8.7. *AM mode-locking for cw Ar and Nd:YAG lasers.* We first consider a mode-locked Ar-ion laser oscillating on its $\lambda = 514.5$-nm green transition; this transition is Doppler-broadened to a width of $\Delta\nu_0^* = 3.5$ GHz. From Eq. (8.6.18) we then obtain $\Delta\tau_p \cong 126$ ps. We consider next a mode-locked Nd:YAG laser oscillating on its $\lambda = 1.064$-μm transition, whose linewidth is phonon-broadened to $\Delta\nu_0 \cong 4.3\ \text{cm}^{-1} = 129$ GHz at $T = 300$ K. We consider a laser cavity with optical length $L_e = 1.5$ m whose AM modulator is located at one cavity end (Fig. 8.19a). We then get $\nu_m = c/2L_e = 100$ MHz, and, from Eq. (8.6.19), $\Delta\tau_p \cong 125$ ps. Note that, on account of the different expressions for $\Delta\tau_p$ for a homogeneous or inhomogeneous line, the pulsewidths for the two cases are almost the same, although the linewidth of Nd:YAG is almost 30 times wider than that of Ar-ion.

Example 12.2. *Maximum energy that can be extracted from an amplifier.* It is assumed that the maximum value of gl is limited by parasitic oscillations such that $(gl)^2_{max} \cong 10$, and the rather low gain coefficient of $g = 10^{-2}\ \text{cm}^{-1}$ is also assumed. For a damage energy-fluence of the amplifier medium of $\Gamma_d = 10\ \text{J/cm}^2$, we obtain from Eq. (12.3.19) $E_m \cong 1$ MJ. Note that this represents an upper limit to the energy, since it requires a somewhat impractical amplifier dimension on the order of $l_m \cong (gl)_m/g \cong 3$ m.