Example 6.3. *Diode-array beam focusing onto a multimode optical fiber.* We consider the simple configuration in Fig. 6.13, where a cylindrical lens of sufficiently short focal lens f is used to collimate the beam along the fast axis (*dashed lines*). The beam diameter after the lens and along this axis is then given by $D_\perp = 2f \tan\theta_\perp$. Along the slow axis, the cylindrical lens behaves like a plane-parallel plate, so the beam (*continuous line*) is essentially unaffected by the lens. (To draw attention to this circumstance, the cylindrical lens is drawn as a dashed line in the figure to indicate that it focuses only on the fast-axis plane.) The beam diameter in the slow-axis plane after the lens is approximately: $D_\parallel \approx L_a + 2f \tan\theta_\parallel$ where L_a is the length of the array. If we now set the condition $D_\parallel = D_\perp$, then we have $f = L_a/2(\tan\theta_\perp - \tan\theta_\parallel)$. Taking $L_a = 200\ \mu m$, $\theta_\perp = 20°$, and $\theta_\parallel = 5°$, we obtain $f = 350\ \mu m$, a focal length that can be obtained with fiber microlenses. With such a small value of focal length, the beam diameter after the lens is $D = D_\parallel = D_\perp = 2f \tan\theta_\perp = 254\ \mu m$ which can easily be accepted into, e.g., a 300-μm diameter multimode fiber butt-coupled to the microlens. For a well-corrected fiber microlens, beam divergence after the lens mostly arises from the uncompensated divergence of the slow-axis beam. The fiber NA must then be $NA = \sin\theta_f \geq \sin\theta_\parallel \cong 0.09$. Beam divergence of light leaving the fiber, for a sufficiently long fiber, is then equal to the fiber NA.

6.4. A Nd:YAG rod 6 mm in diameter, 7.5 cm long, with 1 atom.% Nd is cw-pumped by a high-pressure Kr lamp in a close-coupled diffusively-reflecting pumping chamber. Energy separation between the upper laser level and the ground level corresponds to a wavelength of 940 nm. The measured threshold pump power, when the rod is inserted in some given laser cavity, is $P_{th} = 2$ kW. Assuming, for this pump configuration, that the rod is uniformly pumped with an overall pump efficiency of $\eta_p = 4.5\%$, calculate the corresponding critical pump rate.

6.8. A 1 cm long Ti^{3+}:Al_2O_3 rod is longitudinally pumped, from one end, by an argon laser at a 514.5-nm wavelength in a configuration otherwise similar to that in Fig. 6.11c. The absorption coefficient at the pump wavelength for the rod can, in this case, be taken as $\alpha_p \cong 2\ cm^{-1}$. Transmission at the pump wavelength of the cavity mirror through which the pump beam enters the cavity can be taken as $\eta_t = 0.95$. The wavelength corresponding to the minimum pump frequency ν_{mp} (see Fig. 6.17) for Ti:sapphire is $\lambda_{mp} = 616$ nm. Calculate the overall pumping efficiency. If the pump beam is focused to a spot size of $w_p = 50\ \mu m$ in the laser rod, the laser-mode spot size equals the pump spot size, and a single-pass cavity loss $\gamma = 5\%$ is assumed, calculate the optical output power required from the Ar laser at threshold.

6.10. An Yb:YAG laser rod 1.5 mm long with 6.5 atomic % Yb doping is longitudinally pumped in a laser configuration such as that in Fig. 6.11a by the output of an InGaAs/GaAs QW array at a 940-nm wavelength, focused to a spot size approximately matching the laser-mode spot size $w_0 = 45\ \mu m$. Effective cross sections for stimulated emission and absorption at the $\lambda = 1.03\ \mu m$ lasing wavelength at room temperature can be taken as $\sigma_e \cong 1.9 \times 10^{-20}\ cm^2$ and $\sigma_a \cong 0.11 \times 10^{-20}\ cm^2$, while the effective upper state lifetime is $\tau \cong 1.5$ ms. Transmission of the output coupling mirror is 3.5%, so that, including other internal losses, single-pass loss can be estimated as $\gamma \cong 2\%$. Calculate the threshold pump power under the stated conditions.

7.5. Referring to Fig. 7.4 and Example 7.2, suppose that the Nd:YAG rod is replaced by a Nd:YLF rod of the same dimensions (YLF$\equiv$$YLiF_4$). Oscillation can then occur at either $\lambda = 1.047\ \mu m$ (extraordinary wave or π-transition) or at $\lambda = 1.053\ \mu m$ (ordinary wave or σ-transition). The largest value for the effective stimulated emission cross section is for the π-transition ($\sigma_e \cong 1.8 \times 10^{-19}\ cm^2$). The upper state lifetime is the same for the two transitions, i.e., $\tau = 480\ \mu s$. Assuming that the internal loss in the cavity and lamp-pumping efficiency remain the same as for Nd:YAG, calculate the threshold inversion and the threshold pump power, then compare the results with those of Nd:YAG. Assuming that the energy separation between the $^4F_{3/2}$ upper laser level and the ground level remains the same as for Nd:YAG and taking the same value for the beam area A_b, calculate the slope efficiency.

7.10. Consider the Nd:YAG laser in Example 7.4 and assume that optimum output coupling can be calculated by the formula established by the space-dependent case in Sect. 7.5. Calculate the optimum output coupling, and using this value for γ_2, calculate with the help of Eq. (7.3.34) the expected value of the output power at a diode laser pump power of $P_p = 1.14$ W.

Example 7.2. *CW laser behavior of a lamp-pumped high-power Nd:YAG laser.* We consider the laser system in Fig. 7.4, where a 6.35-mm diameter, 7.5 cm long Nd:YAG rod, with 1% atomic concentration of active Nd ions is pumped, in an elliptical pump chamber, by a high-pressure Kr lamp. The laser cavity consists of two plane mirrors separated by 50 cm. The reflectivity of one mirror is $R_1 = 100\%$, while that of the output coupling mirror is $R_2 = 85\%$. A typical curve of the output power P_{out} through mirror 2 (in multimode operation) versus electrical pump power P_p to the Kr lamp is shown in Fig. 7.5.[7] Note that one is dealing with a reasonably high-power cw Nd:YAG laser, with an output power exceeding 200 W. We observe that, since the laser is oscillating on many transverse and longitudinal modes, then, according to the discussion in Sect. 7.2.1, it is reasonable to compare experimental results with theoretical predictions using the preceding space-independent rate equations. In fact, except for input powers just above threshold, experimental points in Fig. 7.5 show a linear relationship between output and input powers, as predicted by Eq. (7.3.9). From the linear part of the curve, an extrapolated threshold of $P_{th} = 2.2$ kW is obtained. Above threshold, the output versus input power relation can be fitted by the equation

$$P_{out} = 53\left[\left(\frac{P_p}{P_{th}}\right) - 1\right] \tag{7.3.15}$$

where P_{out} is expressed in watts. The slope efficiency is then easily obtained from Eq. (7.3.15) as $\eta_s = (dP_{out}/dP_p) = 53/P_{th} = 2.4\%$. Equation (7.3.15) can be readily compared to Eq. (7.3.9) once we remember that, as discussed in Example 2.10, the effective values of the cross section and upper level lifetime for the $\lambda = 1.06\ \mu$m transition in Nd:YAG can be taken as $\sigma = 2.8 \times 10^{-19}\,\text{cm}^2$ and $\tau = 230\ \mu$s, respectively. The energy of the photon at this wavelength is obtained as $h\nu = 3.973 \times 10^{-19} \times (0.5/1.06) = 1.87 \times 10^{-19}$ J, where 3.973×10^{-19} J is the energy of a photon with a wavelength of 0.5 μm (see Appendix I). We then obtain the value of the saturation intensity as $I_s = h\nu/\sigma\tau = 2.9\,\text{kW/cm}^2$. We now take $R_2 = (1 - a_2 - T_2) \cong (1 - T_2)$ since, for a good multilayer coating, mirror absorption, a_2, may be less than 0.1%. Then $\gamma_2 = -\ln R_2 = 0.162$. Comparing Eqs. (7.3.15) and (7.3.9) yields $A_b \cong 0.23\,\text{cm}^2$, to be compared with the cross-sectional area of the rod $A \cong 0.317\,\text{cm}^2$.

To compare measured slope efficiency and extrapolated threshold with values predicted by calculation, we must know γ, i.e., γ_i. Since $\gamma_1 = 0$, Eq. (7.3.12), with the help of Eq. (7.2.8), can be rearranged as:

$$\frac{-\ln R_2}{2} + \gamma_i = \eta_p\left(\frac{\sigma}{A}\right)\left(\frac{P_{th}\tau}{h\nu_{mp}}\right) \tag{7.3.16}$$

Thus, if several measurements are taken of the threshold pump power at different mirror reflectivities R_2, a plot of $\gamma_2 = -\ln R_2$ versus P_{th} should yield a straight line. In fact this is found experimentally, as shown in Fig. 7.6. The intercept of this straight line with the γ_2-axis gives, according to Eq. (7.3.16), the value of internal losses (Findlay and Clay analysis[9]). From Fig. 7.6 we then get $\gamma_i \cong 0.038$, which gives a total loss $\gamma = (\gamma_2/2) + \gamma_i \cong 0.12$.

Once total losses are known, we can use Eq. (7.3.14) to compare the measured slope efficiency $\eta_s = 2.4\%$ with theoretical predictions. We take $\eta_c = \gamma_2/2\gamma \cong 0.68$. We also take $\eta_q = \lambda_{mp}/\lambda = 0.89$, where $\lambda_{mp} = 0.94\ \mu$m is the wavelength corresponding to the transition from the upper laser level to the ground level (see Fig. 2.15) in Nd:YAG; according to the previous calculation we also take $\eta_t = A_b/A \cong 0.72$. From Eq. (7.3.14) we obtain $\eta_p = 5.5\%$, which appears to be a reasonable value for pump efficiency for Kr pumping (see also Table 6.1). The predicted value of P_{th} can now be readily obtained from Eq. (7.3.12) once we take into account that $h\nu_{mp} \cong 2.11 \times 10^{-19}$ J. We obtain $P_{th} \cong 2.26$ kW in good agreement with the experimental result. A knowledge of total losses also allows us to calculate the threshold inversion. From Eq. (7.3.2) we find $N_c \cong 5.7 \times 10^{16}\ \text{ions/cm}^3$. For a 1% atomic doping, total Nd concentration is $N_t = 1.38 \times 10^{20}\ \text{ions/cm}^3$. Thus $N_c/N_t = 4.1 \times 10^{-4}$, which shows that population inversion is a very small fraction of the total population.

Example 7.4. *Threshold and output powers in a longitudinally diode-pumped Nd:YAG laser.* As a representative example of longitudinal diode pumping, we consider the laser configuration in Fig. 7.12, where a 1 cm long Nd:YAG rod is pumped by a 100 μm wide laser array at 805–808 nm wavelength.[15] Coupling optics consists of a 6.5 mm focal-length, 0.615 NA, collecting lens, an anamorphic prism pair providing a 4-times beam magnification, and a 25-mm lens to focus the pump light on the rod (see Fig. 6.12). The Nd:YAG resonant cavity is formed by a plane mirror directly coated on one face of the rod and a 10-cm radius, 95% reflecting mirror separated by approximately 5.5 cm from the plane mirror. About 93% of the pump power is transmitted to the rod through the plane mirror. In this geometry, the TEM_{00} mode waist occurs at the planar reflector, and its spot size can be calculated as $w_0 \cong 130\,\mu$m. (Thermally induced lensing in the rod is neglected). The spot size of the pump beam provides good mode matching with this TEM_{00} laser mode. The laser-operating characteristics are indicated in Fig. 7.13. The threshold pump power is $P_{th} \cong 75$ mW. At an optical pump power of $P_p = 1.4$ W, an output power of $P_{out} = 370$ mW is obtained. At this output power, the measured optical-to-optical slope efficiency is $\eta_s \cong 40\%$.

To compare threshold pump power with the expected value, we assume that the transverse pump beam distribution can be approximated by a Gaussian function and take $w_p \cong w_0 = 130\,\mu$m. From Eq. (6.3.22), with $h\nu_p = 2.45 \times 10^{-19}$ J, $\sigma_e = 2.8 \times 10^{-19}\,\text{cm}^2$, and $\tau = 230\,\mu$s, we obtain $(\gamma/\eta_p) \cong 3.7 \times 10^{-2}$. For 5% transmission of the output mirror, we get $\gamma_2 \cong 5 \times 10^{-2}$, and assuming an internal loss per pass $\gamma_i = 0.5 \times 10^{-2}$, we obtain $\gamma = \gamma_i + (\gamma_2/2) = 3 \times 10^{-2}$. From the previously obtained value of γ/η_p, we then get $\eta_p \cong 81\%$, which includes overall transmission of the coupling optics and the transmission of the plane mirror at the pump wavelength. Note that the absorption efficiency of the pump radiation in the laser rod in a single pass, $\eta_a = \{1 - \exp -[(\alpha l)]\}$, can be taken as unity for an average absorption coefficient of $\sim 6\,\text{cm}^{-1}$ in the 805–808 nm band (see Fig. 6.8a) and for a rod length of $l = 1$ cm. We can now compare the measured slope efficiency with the expected value. Since the threshold power for $w_0 \cong w_p$ is 75 mW, the minimum threshold power, which occurs when $w_0 \to 0$, is expected to be half this value, i.e., $P_{mth} \cong 38$ mW. Thus, at 1.14 W input pump power, we have $x \cong 30$. At this value of x, from Eq. (7.3.34) we obtain $y = 26$, and, from Eq. (7.3.35), $\eta_t \cong 0.97$. We then have $\eta_c = (\gamma_2/\gamma) = 0.83$ and $\eta_q = (807/1060) = 0.76$. Expected overall optical-to-optical slope efficiency is thus $\eta_s = \eta_p \eta_c \eta_t \eta_q = 0.49$, in fair agreement with the measured one. Note that the longitudinal efficiency is not taken into account because the laser is oscillating on many longitudinal modes whose various standing wave patterns add to produce a fairly uniform energy density distribution along the laser rod. According to Eqs. (7.3.27) and (7.3.28), the expected output power at 1.14 W input power is $P_{out} = yP_s = 500$ mW, i.e., somewhat larger than the experimental one. This discrepancy can perhaps be attributed to thermal effects in the laser rod, which, at the highest pump powers, increase losses and decrease spot size w_0.

Note that the quoted 40% efficiency refers to optical-to-optical efficiency. To obtain the overall electrical-to-optical slope efficiency, we must multiply optical efficiency by the radiative efficiency η_r of the array. Again from Fig. 7.13 we obtain $\eta_r \cong 29\%$, so that overall electrical-to-optical slope efficiency is about 11.6%.

Example 7.6. *Optimum output coupling for a lamp-pumped Nd:YAG laser.* We consider the laser configuration discussed in Example 7.2 (see Figs. 7.4 and 7.5) and calculate the optimum transmission of the output mirror when the laser is pumped by a lamp input power of $P_p = 7$ kW. Since the threshold power P_{th} in Fig. 7.5 was measured to be 2.2 kW, then according to Eq. (7.5.1) with $\gamma_1 = 0$, we obtain $P_{mth} = P_{th}(\gamma_i/\gamma) \cong 697$ W, where the values $\gamma_i = 0.038$ and $\gamma = 0.12$, obtained in Example 7.2, are used for internal loss and the total loss, respectively. We then get $x_m = P_p/P_{mth} \cong 10$, so that from Eq. (7.5.5), $S_{op} \cong 2.17$. From Eq. (7.5.3) we finally obtain $(\gamma_2)_{op} \cong 0.165$, which corresponds to an optimum transmission of $(T_2)_{op} = 1 - \exp[-(\gamma_2)_{op}] \cong 15\%$, i.e., agreeing with the value actually used in Fig. 7.4.